Physics 111 Homework Solutions for Friday 3/6 and Monday 3/9

Friday, March 6, 2015

Chapters 26

26.7 α particles are low energy so they do not penetrate very far into tissue. They are stopped by the skin producing burns to the exposed patch.

β particles are higher energy and will penetrate farther into tissue than α particles. Beta particles travel about 100 times farther than alpha particles. Whereas alpha particles seldom pass beyond the outer, dead layer of the skin, the free, fast-moving electrons and positrons that constitute beta radiation penetrate for about a quarter of an inch into living matter.

γ rays are very high-energy photons and are not stopped by the skin. They pass through almost undisturbed. However they will ionize atoms as they pass by. Gamma rays and X rays will pass readily through a large organism; they reach the innermost recesses of the body and injure highly sensitive tissues, but they produce only about one twentieth of the damage inflicted on cells by alpha particles.

26.8 To determine the unknown product we use both conservation of mass and charge.
a) For the following reaction $^{60}_{27}Co \to {}^{60}_{28}Ni + {}^{A'}_{Z'}X$, conservation of charge gives me 27 = 28 + Z' which gives Z' = -1. Conservation of mass gives me 60 = 60 + A' which gives A' = 0. Thus the unknown particle is a beta particle, $^{A'}_{Z'}X = {}^{0}_{-1}e$.
b) For the following reaction $^{A'}_{Z'}X \to {}^{234}_{91}Pa + {}^{0}_{-1}e$, conservation of charge gives me Z' = 91-1' which gives Z' = 90. Conservation of mass gives me A' = 234 + 0 which gives A' = 234. Thus the unknown particle is a thorium nucleus, $^{A'}_{Z'}X = {}^{234}_{90}Th$.
c) For the following reaction $^{A'}_{Z'}X \to {}^{234}_{90}Th + {}^{4}_{2}He$, conservation of charge gives me Z' = 90 + 2 which gives Z' = 92. Conservation of mass gives me A' = 230 + 4 which gives *A' = 234.* Thus the unknown particle is a uranium nucleus, $^{A'}_{Z'}X = {}^{234}_{92}U$.

26.16 The electron-positron annihilation occurs when the positron encounters an electron and both are assumed to be at rest. To conserve momentum, the two photons produced are done so at exactly 180°. Thus photons from the annihilation are detected coincidently 180° apart. If the two photons are not detected coincidentally 180° apart, they are from two different annihilation events.

Multiple-Choice

- None

Problems

26.4 The nuclear binding energy is given through:

$NBE = Zm_p c^2 + Nm_N c^2 - m_{atom}c^2$; where $m_p = 1.00727u$ and $m_n = 1.00867u$.

For radium-226:

$$NBE = [88(1.00727u) + 138(1.00867u) - 225.97709u]c^2 \times \frac{931.5\text{MeV}}{1uc^2} = 1731.8\text{MeV}.$$

The $\frac{NBE}{nucleon} = \frac{1731.8MeV}{226} = 7.66 \frac{MeV}{nucleon}$.

For radium-228:

$$NBE = [88(1.00727u) + 140(1.00867u) - 227.98275u]c^2 \times \frac{931.5\text{MeV}}{1uc^2} = 1742.7\text{MeV}.$$

The $\frac{NBE}{nucleon} = \frac{1742.7MeV}{228} = 7.64 \frac{MeV}{nucleon}$.

For thorium-232:

$$NBE = [90(1.00727u) + 142(1.00867u) - 231.98864u]c^2 \times \frac{931.5\text{MeV}}{1uc^2} = 1766.9\text{MeV}.$$

The $\frac{NBE}{nucleon} = \frac{1766.9MeV}{232} = 7.62 \frac{MeV}{nucleon}$.

26.6 For the β-decay reaction of ^{24}Na,

$$Q = \left(M_{{}^{24}_{11}Na} - M_{{}^{24}_{12}Mg} - M_{{}^{0}_{-1}e}\right)c^2$$

$$\therefore Q = \left(23.98492 - 23.97845 - 5.49\times10^{-4}\right)uc^2 \times \frac{931.5MeV}{uc^2} = 5.52MeV.$$

26.7 For alpha decay: $Q = \left(M_{parent} - M_{daughter} - M_{He}\right)c^2$. If the parent is at rest when it decays, then from conservation of momentum, the daughter gets a recoil velocity in the direction opposite direction to the velocity of the alpha particle. Conservation of momentum gives: $0 = -m_{daughter}v_{daughter} + m_\alpha v_\alpha \rightarrow v_{daughter} = \frac{m_\alpha}{m_{daughter}}v_\alpha$. Next we apply conservation of energy and we find:

$m_{parent}c^2 = \frac{1}{2}m_{daughter}v_{daughter}^2 + \frac{1}{2}m_{He}v_{He}^2 + m_{daughter}c^2 + m_{He}c^2$. Then we can replace the velocity of the recoiling daughter atom in terms of the velocity of the alpha particle, which can be measured. We have

$m_{parent}c^2 = \frac{1}{2}m_{daughter}\left(\frac{m_\alpha}{m_{daughter}}v_\alpha\right)^2 + \frac{1}{2}m_{He}v_{He}^2 + m_{daughter}c^2 + m_{He}c^2$. Bringing all of the rest of the energy terms to one side and combining the terms involving the velocity of the alpha particle we find that

$\left(m_{parent}c^2 - m_{daughter}c^2 - m_{He}c^2\right) = Q = \left[\frac{1}{2}\frac{m_\alpha^2}{m_{daughter}} + \frac{1}{2}m_\alpha\right]v_\alpha$. Factoring out the mass of the alpha particle on the left hand side we have $\mathrm{Q}=\frac{1}{2}m_\alpha v_\alpha^2\left[1+\frac{m_\alpha}{m_{daughter}}\right]$ which is the desired result.

From example 27.2, Q = 4.28 MeV, m_α = 4.0015u, $m_{daughter}$ = $m_{thorium}$ = 233.99409u, and we find the velocity of the alpha particle to be $1.42\text{x}10^7$ m/s.

26.16 Uranium Decay

a. ${}^{238}_{92}U \rightarrow {}^{4}_{2}He + {}^{234}_{90}Th$

b. The daughter nucleus is thorium.

c. The kinetic energy of the emitted alpha particle is given by (ignoring the recoil of the thorium nucleus):

$$KE=[M_U-M_\alpha-M_{Th}]c^2$$

$$KE=\left\{[238.000187u-4.00150u-233.99409u]\times\frac{931.5MeV/c^2}{1u}\right\}c^2=4.282MeV$$

d. To determine the speed of the emitted alpha particle we use the relativistic form of the kinetic energy since we don't know if the alpha particle is relativistic or not. Thus

$$KE=4.282MeV=(\gamma-1)m_\alpha c^2=(\gamma-1)\left|4.00150u\times\frac{931.5MeV/c^2}{1u}\right|c^2\rightarrow\gamma=1.0015$$

$$\gamma=\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\rightarrow v=\sqrt{1-\frac{1}{\gamma^2}}c=\sqrt{1-\frac{1}{(1.0015)^2}}c=0.055c$$

e. Based on part d, the alpha particle is almost at the relativistic cutoff. However, we have actually ignored the recoil of the thorium nucleus, so if we were to include the recoil of the thorium atom, the alpha particle is probably not actually relativistic.

Monday, March 9, 2015

Chapter 26

Questions

26.10 No it does not matter when each experimenter starts their respective experiments. Since the radioactive decay law can, for example, be written in terms of the activity of the radioactive sample, as long as the experimenters know their initial sample activity then the decay constant can be determined. The decay constant will be the same for both experimenters and thus the half-life can be calculated independent of the initial activity the experimenters started with.

Multiple-Choice

26.9 D

26.10 A

26.11 D

Problems

26.8 The total number of events per hour is given as the product of the number of phototubes and the number of events per hour per phototube. Thus the total number of events per hour is

$$\frac{\#\text{events}}{\text{hour}} = 110\text{phototubes} \times 0.027\frac{\text{events}}{\text{hour}\times\text{phototube}} = 2.97\frac{\text{events}}{\text{hour}}.$$ In order to figure out the fraction of the neutrinos that interact with the water in the tank we need to calculate the neutrino flux.

$$\text{neutrino flux} = \frac{\#\text{neutrinos}}{\text{second}} = 1\times10^{13}\frac{\text{neutrinos}}{\text{sec}\times\text{cm}^2}\times 4\text{m}^2 \times \frac{(100\text{cm})^2}{1\text{m}^2} = 4\times10^{17}\frac{\text{neutrinos}}{\text{sec}}.$$

Therefore the fraction of neutrinos that interacted is

$$\text{fraction} = \frac{2.97\text{events}}{3600\,\text{sec}}\times\frac{1\text{sec}}{4\times10^{17}} = 2.06\times10^{-21}.$$

26.13 The Chernobyl accident

a. The half-life is the time needed for ½ of the radioactive nuclei to disintegrate and we can relate the half-life to the decay constant. We have

$\text{N} = \frac{\text{N}_0}{2} = \text{N}_0 e^{-\lambda t_{\frac{1}{2}}} \rightarrow t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$ and the decay constant is therefore

$$\lambda = \frac{0.693}{t_{\frac{1}{2}}} = \frac{0.693}{8days} = 0.0866 day^{-1}$$

b. To calculate the storage time, we use the radioactive decay law with N = 0.15No.

Therefore $0.15N_0 = N_0 e^{-(0.0866\text{days}^{-1})t} \rightarrow t = \frac{\ln(0.15)}{-0.0866\text{days}^{-1}} = 21.9\text{days}.$

26.15 We first determine the decay constant for palladium and we have

$\lambda = \frac{0.693}{t_{½}} = \frac{0.693}{17days} = 0.041 days^{-1}$. Next we calculate the number of nuclei that remain after the 30 days from the radioactive decay law and find

$N = N_o e^{-\lambda t} = N_o e^{-0.041 days^{-1}\times 30 days} = 0.292 N_o$ Therefore the number that have decayed is $N_{decayed} = 1 - 0.292N_o = 0.708N_o$. Further we know how much energy we need to destroy the tumor and we know the energy of the emitted gamma rays. Thus we can determine how many initial atoms we need. This produces

$2.12J = 0.708N_o \times 21000 eV \times \frac{1.6\times10^{-19}J}{1eV} \rightarrow N_o = 8.92\times10^{14}$. Now that we know the initial number of radioactive atoms we can calculate the initial activity of the

palladium and we find $A_o = \lambda N_0 = 0.041 days^{-1} \times 8.92 \times 10^{14} = 3.66 \times 10^{13} day^{-1}$.
Lastly the initial mass of palladium can be calculated from

$$m = N_o m_{pd} = 8.92 \times 10^{14} \times 103u \times \frac{1.66 \times 10^{-27} kg}{1u} = 1.53 \times 10^{-10} kg = 15.3ng.$$

26.17 Here the initial mass M_0 = 1 and the final mass is 0.01% of the initial mass. Thus $M_f = 1x10^{-4}\ M_0$. Referring to problem #7, the decay constant $\lambda = 0.024\ yr^{-1}$. From the radioactive decay law:

$$M_f = M_o e^{-\lambda t} \rightarrow 1 \times 10^{-4} M_0 = M_o e^{-\left(0.024\,\mathrm{yr}^{-1}\right)t} \rightarrow t = 383.4\mathrm{yrs}.$$

26.18 The activity is given as the product λN, where λ is the decay constant and N is the number of nuclei that decay. Given that are sample is radium-226, with a half-life of 1600 years, we can calculate the decay constant.

$$\lambda = \frac{0.693}{t_{\frac{1}{2}}} = \frac{0.693}{1600 yrs} \times \frac{1 yr}{3.2 \times 10^7 \sec} = 1.35 \times 10^{-11} \sec^{-1}.$$ To calculate the number of nuclei present we use the mass given, 1g. There are 226 g of radium per mole and in 1 mole there are $6.02x10^{23}$ nuclei. Thus in 1 g there are $2.66x10^{21}$ nuclei. The activity is therefore λN = $(1.35x10^{-11}\ sec^{-1})(2.66x10^{21}$ nuclei) = $3.6x10^{10}$ decays/sec = $3.6x10^{10}$ Bq.

26.19 The activity when the bone chip is measured is 0.5 decays/sec. The initial activity when the animal died needs to be determined. In the bone there is found 5g of carbon. Since 1 mole of carbon contains $6.02x10^{23}$ atoms and 1 mole of carbon has a mass of 12 g, there are $2.51x10^{23}$ carbon nuclei. Further the ration of $^{14}C/^{12}C$ has remained relatively constant and has a value of $1.3x10^{-12}$. Thus the number of ^{14}C nuclei is given as $(1.3x10^{-12})(2.51x10^{23}$ nuclei) = $3.26x10^{11}\ ^{14}C$ nuclei when the animal died. The initial activity is a product of the decay constant and the number of ^{14}C nuclei present when the animal died. The decay constant is found from the half-life of carbon (5730yrs).

$$\lambda = \frac{0.693}{t_{\frac{1}{2}}} = \frac{0.693}{5730 yrs} \times \frac{1 yr}{3.2 \times 10^7 \sec} = 3.78 \times 10^{-12} \sec^{-1}.$$ The initial activity is λN = $(3.78x10^{-12}\ sec^{-1})(3.26x10^{11}\ ^{14}C$ nuclei) = 1.23 Bq. To calculate the age of the bone we use the radioactive decay law

$$A = A_o e^{-\lambda t} \rightarrow 0.5Bq = 1.23Bqe^{-(3.78 \times 10^{-12} \sec^{-1})t} \rightarrow \ln(\frac{0.5}{1.23}) = -(3.78 \times 10^{-12} \sec^{-1})t$$

$$\rightarrow -0.902 = -(3.78 \times 10^{-12} \sec^{-1})t \rightarrow t = 2.39 \times 10^{11} \sec = 7456 yrs.$$

Since the bone is only about 7500 years old and knowing that the dinosaurs disappeared over 65 million years ago, it is probably not the bone of a dinosaur. Further since the activity for this bone chip is smaller than the bone chip found in problem #16, the age of the bone chip must be greater that that found in problem #16, so the answer seems reasonable, but not for a dinosaur bone.