Physics 111 Homework Solutions Collected on Friday 1/23

Tuesday, January 20, 2015

Chapters 15 & 16

Questions

- None

Multiple-Choice

15.12 C
15.13 D
15.17 D
15.18 A

Problems

15.12 The capacitance is given by

$$C = \frac{\kappa\varepsilon_0 A}{d} = \frac{1\times 8.85\times 10^{-12}\,\frac{C^2}{Nm^2}\times\left(25mi^2\times\left(\frac{1600m}{1mi}\right)^2\right)}{1mi\times\frac{1600m}{1mi}} = 3.54\times 10^{-7}F$$

The charge stored is given by $Q = CV = 3.54\times 10^{-7}F\times 50\times 10^{6}V = 17.7C$. Finally, the energy stored is given as

$$E = \tfrac{1}{2}CV^2 = \tfrac{1}{2}\times 3.54\times 10^{-7}F\times\left(50\times 10^{6}V\right)^2 = 4.43\times 10^{8}J = 443MJ$$

15.17 An air-spaced parallel plate capacitor

a. C = Q/V = 36 μC/12 V = 3 μF

b. The dielectric constant of Pyrex glass is 4.7 (from Table 16.1), so $C = \kappa C_o$ = 14.1 μF

c. Since the capacitance has increased by a factor of 4.7 and V is the same, then Q increases by the same factor to Q = 169 μC.

15.18 Another air-spaced parallel plate capacitor

a. The energy stored in a parallel plate capacitor is given by $E = \frac{1}{2}CV^2$. Here we need the capacitance which is given by C = q/V = $(0.05\text{x}10^{-6}\text{C})/(10\text{V}) = 5\text{x}10^{-9}\text{F}$. Thus the energy is $E = \frac{1}{2}CV^2 = \frac{1}{2}\left(5\times 10^{-9}\,\text{F}\right)\left(10\text{V}\right)^2 = 2.5\times 10^{-7}\,\text{J}$.

b. The electric field is related to the potential difference and the plate separation through $E = \frac{\Delta V}{\Delta d}$ and thus $E_{before} = \frac{10\text{V}}{0.1\times 10^{-3}\,\text{m}} 100{,}000\,\frac{\text{V}}{\text{m}}$

c. The final voltage across the capacitor is the same as the battery, or 10V. The final energy is given again by $E = \frac{1}{2}CV^2 = \frac{1}{2}\left(1\times 10^{-9}\,\text{F}\right)\left(10\text{V}\right)^2 = 8.3\times 10^{-8}\,\text{J}$, where the new capacitance is obtained from the old capacitance through $C_{new} = \frac{1}{3}C_{old} = 1.7\times10^{-9}F$. And the new energy is given by $E = \frac{Q^2}{2C_{new}} = \frac{\left(0.05\times10^{-6}\mathrm{C}\right)^2}{2\times1.7\times10^{-9}\mathrm{F}} = 7.5\times10^{-7}\mathrm{J}$.. Lastly the potential difference changes and is related to the old potential difference through $V_{new} = 3V_{old} = 30V$ and the new field is $E_{new} = \frac{\Delta V}{\Delta d} = \frac{30V}{0.3\times10^{-3}m} = 1\times10^5 \frac{V}{m} + E_{old}$

d. The work done in pulling the plates apart is

$$W = \frac{qEd}{2} = \frac{0.05\times10^{-6}C\times1\times10^5\frac{V}{m}\times0.2\times10^{-3}m}{2} = 5\times10^{-7}J.$$

15.22 The charge that flows due to the sodium ions is

$$Q = \frac{50\text{channels}}{1\mu m^2}\times100\mu m^2\times\frac{1000\text{Na}^{+1}\text{ion}}{1\text{channel}}\times\frac{1.6\times10^{-19}C}{\text{Na}^{+1}\text{ion}} = 8.0\times10^{-13}C.$$

The specific capacitance can be related to the capacitance through

$$C = \left(\frac{C}{A}\right)A = \frac{1\times10^{-6}F}{1cm^2\times\left(\frac{1m}{100cm}\right)^2}\times1000\mu m^2\times\left(\frac{1m}{1\times10^6\mu m}\right)^2 = 1.0\times10^{-12}F.$$

Therefore the voltage change across the membrane due to the ion flow is given by

$$Q = CV \rightarrow \Delta V = \frac{\Delta Q}{C} - 100mV = \frac{8.0\times10^{-13}C}{1.0\times10^{-12}F} - 100mV = 0.8V - 100mV = 700mV.$$

15.23 The energy in a charged capacitor is given by

$$E = \tfrac{1}{2}CV^2 \rightarrow V = \sqrt{\frac{2E}{C}} = \sqrt{\frac{2\times300J}{30\times10^{-6}F}} = 4470V$$

16.18 A RC circuit

a. The time constant is given by the product of the equivalent resistance and equivalent capacitance. Thus, $\tau = RC = (10\times10^3\,\Omega)(100\times10^{-6}F) = 1s$.

b. To find the initial current that flows through the resistor we use Ohm's Law. The voltage drop across the capacitor is equal to the voltage of the source since only the capacitor and battery were in the circuit as the capacitor is charging. When the battery is disconnected and the capacitor discharges through the resistor, the current is given by: $I_0 = \frac{V_C}{R} = \frac{q_0}{RC} = \frac{q_0}{\tau} = \frac{10\times10^{-6}C}{1s} = 1\times10^{-5}A$..

c. After 1τ, the amount of charge that remains on the capacitor is given by:

$$q(t = 1\tau) = q_0 e^{-\frac{\tau}{RC}} = q_0 e^{-1} = \frac{q_0}{e} = 3.68\times10^{-6}C.$$

d. The current after a time equal to 1τ is given by:

$$I(t=1\tau)=I_0 e^{-\frac{\tau}{RC}}=I_0 e^{-1}=\frac{I_0}{e}=3.68\times10^{-6}\,A.$$

e. After 3τ, the amount of charge that remains on the capacitor is given by: $q(t=3\tau)=q_0 e^{-\frac{3\tau}{RC}}=q_0 e^{-3}=\frac{q_0}{e^3}=4.97\times10^{-7}\,C.$ The current after a time equal to 3τ is given by: $I(t=3\tau)=I_0 e^{-\frac{3\tau}{RC}}=I_0 e^{-3}=\frac{I_0}{e^3}=4.97\times10^{-7}\,A.$ This seems intuitively correct since as time goes on the amount of charge left to flow is decreasing and thus the current should also decrease (to zero) with time.

16.21 Lightning

a. The potential difference is give by

$$\Delta V=V_L-V_U=\frac{kQ}{r_L}-\frac{kQ}{r_U}=kQ\left(\frac{1}{r_L}-\frac{1}{r_U}\right)$$

$$\Delta V=9\times10^9\,\tfrac{Nm^2}{C^2}\times5\times10^5\,C\times\left(\frac{1}{6.4\times10^6\,m}-\frac{1}{(6.4\times10^6\,m+5\times10^3\,m)}\right)=5.49\times10^5\,V.$$

b. The capacitance of the earth-cloud system is $C=\frac{Q}{V}=\frac{5\times10^5\,F}{5.49\times10^5\,V}=0.91F$.

c. The time constant is $\tau=RC=300\Omega\times0.91F=273.3s$.

d. At *25 C* per strike the total amount of charge corresponds to $\frac{5\times10^5\,C}{25\,\frac{C}{strike}}=20{,}000$ strikes.

e. Here we have a discharging capacitor and we use our expression for a discharging capacitor to determine the time. We have

$$Q_f=Q_o e^{-\frac{t}{RC}}\rightarrow t=-RC\ln\left(\frac{Q_f}{Q_o}\right)$$

$$t=-273.3s\ln\left(\frac{0.001Q_o}{Q_o}\right)=1888s\times\frac{1hr}{3600s}=0.52hr$$

f. From parts *d* and *e* we have

$$\frac{20{,}000strikes}{0.52hr}\times\frac{24hr}{day}=9.2\times10^5\,\tfrac{strikes}{day}=0.9million\,\tfrac{strikes}{day}.$$

Wednesday, January 21, 2015

Chapter 16

Questions

- None

Multiple-Choice

- None

Problems

16.2 The current is given by the average charge per unit interval of time. Here, 5 μC flows in 2 μs, so $I = \frac{\Delta Q}{\Delta t} = \frac{5\times 10^{-6} C}{2\times 10^{-6} s} = 2.5A.$

16.3 To calculate the average current that flows across this muscle membrane, I need to know how many sodium channels there are over this area. Then knowing the number of ions that flow per millisecond I can calculate the average current. To start I'm going to calculate the total number of sodium channels over this patch of membrane: $\#\,\text{Na Channels} = \frac{50\text{Na Channels}}{\mu m^2}\times 100\mu m^2 = 5000\text{Na Channels}$. If there are *1000 Na* ions per channel flowing per millisecond, then the average current is given by:

$$I_{avg} = (5000\text{channels}\times \frac{1000\,{}^{\text{ions}}/_{\text{channel}}}{1\times 10^{-3} s})\times \frac{1e}{\text{ion}}\times \frac{1.6x10^{-19} C}{1e} = 8\times 10^{-10} A = 0.8nA.$$

16.17 Using $V(t) = \frac{V_0}{2} = V_0 e^{-\frac{t}{RC}}$ gives for a time, called the half time, $\ln\left(\frac{1}{2}\right) = \frac{-t_{\frac{1}{2}}}{RC} \rightarrow t_{\frac{1}{2}} = RC\ln(2)$. Thus, a single measurement of the half-time will give the value of the time constant (RC) in a single measurement.

16.20 A defibrillator

a. The time constant is $\tau = RC = 47\times 10^3\,\Omega\times 32\times 10^{-6} F = 1.5s$

b. The maximum charge is $Q_{\max} = CV_{\max} = 32\times 10^{-6} F\times 5000V = 0.16C$.

c. The maximum current is given by Ohm's Law $I_{\max} = \frac{V_{\max}}{R} = \frac{5000V}{47\times 10^3\,\Omega} = 0.106A = 106mA$

d. The charge as a function of time is given as $Q(t) = Q_{\max}\left(1 - e^{-\frac{t}{\tau}}\right) = 0.160C\left(1 - e^{-\frac{t}{1.5s}}\right)$. The current as a function of time is $I(t) = I_{\max}\left(1 - e^{-\frac{t}{\tau}}\right) = 0.106A\left(1 - e^{-\frac{t}{1.5s}}\right)$

e. The maximum energy is $E = \frac{1}{2}CV^2 = \frac{1}{2}\times 32\times 10^{-6} F\times (5000V)^2 = 400J$.