Physics 111 Homework Solutions Collected on Tuesday 3/3/15

Friday, February 27, 2015

- none due to the exam.

Monday, March 2, 2015

Chapter 24
Questions
Chapters 19 & 24
Questions

24.1 Based on special relativity we know that as a particle with mass travels near the speed of light its mass increases. In order to accelerate this object from rest to a speed near that of light would require an ever increasing force (one that rapidly becomes larger by a factor of γ.) There are no known forces that could accelerate a particle with mass to the speed of light in a finite amount of time and with a finite amount of energy. So for objects with no rest mass, as they travel at the speed of light, there mass does not increase with increasing speed and we avoid these problems of accelerating the massless particles.

24.6 The Compton shift in wavelength for the proton and the electron are given by $\Delta\lambda_p = \frac{h}{m_p c}(1-\cos\phi)$ and $\Delta\lambda_e = \frac{h}{m_e c}(1-\cos\phi)$ respectively. Evaluating the ratio of the shift in wavelength for the proton to the electron, evaluated at the same detection angle φ, we find $\frac{\Delta\lambda_p}{\Delta\lambda_e} = \frac{m_e}{m_p} = \frac{1.67\times10^{-27}kg}{9.11\times10^{-31}kg} = \frac{1}{1833} = 5\times10^{-4}$. Therefore the shift in wavelength for the proton is smaller than the wavelength shift for the electron.

Multiple-Choice

19.16 D
19.17 A

Problems

19.15 The frequency is given as $f = \frac{c}{\lambda} = \frac{3\times10^8 \frac{m}{s}}{5.5\times10^{-7}m} = 5.5\times10^{14}s^{-1}$.

19.17 A Neodymium-YAG laser

a. The number of photons is given as the total beam energy divided by the energy per photon, or $\# = \frac{5J}{hc/\lambda} = \frac{5J}{6.63\times10^{-34}Js\times3\times10^8\frac{m}{s}}\times1.06\times10^{-6}m = 1.33\times10^{20}$.

b. The power in the beam is the energy delivered per unit time, or

$$P = \frac{\Delta E}{\Delta t} = \frac{5J}{1\times10^{-9}s} = 5\times10^{9}W = 5GW.$$

c. The average power output of the laser is $P = \frac{5J}{pulse}\times\frac{10\,pulses}{1s} = 50W$.

19.18 The range of frequencies is given as

$$f_{400nm} = \frac{c}{\lambda_{400nm}} = \frac{3\times10^{8}\,\frac{m}{s}}{400\times10^{-9}m} = 7.5\times10^{14}Hz$$

through $f_{750nm} = \frac{c}{\lambda_{750nm}} = \frac{3\times10^{8}\,\frac{m}{s}}{750\times10^{-9}m} = 4\times10^{14}Hz.$

The energies are given by $E = \frac{hc}{\lambda} = hf$ which for *750 nm* the energy is

$$E = \frac{\left(6.63\times10^{-34}Js\right)\left(3\times10^{8}\,\frac{m}{s}\right)}{750\times10^{-9}m} = 2.65\times10^{-19}J\times\frac{1eV}{1.6\times10^{-19}J} = 1.66eV$$

while the energy for the *400 nm* photon is

$$E = \frac{\left(6.63\times10^{-34}Js\right)\left(3\times10^{8}\,\frac{m}{s}\right)}{400\times10^{-9}m} = 4.97\times10^{-19}J\times\frac{1eV}{1.6\times10^{-19}J} = 3.11eV.$$

19.20 The energy of the emitted photon is equal to the difference in energies of the two levels: $\Delta E = -1.51eV - \left(-3.4eV\right) = 1.89eV\times\frac{1.6\times10^{-19}J}{1eV} = 3.02\times10^{-19}J$. Then setting this energy equal to $\frac{hc}{\lambda}$ we find

$$\lambda = \frac{hc}{E} = \frac{6.63\times10^{-34}Js\times3\times10^{8}\,\frac{m}{s}}{3.02\times10^{-19}J} = 6.58\times10^{-7}m = 658nm.$$

19.21 Another Neodymium-YAG laser

a. Each photon has an energy given by

$$E = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}Js\times3\times10^{8}\,\frac{m}{s}}{532\times10^{-9}m} = 3.74\times10^{-19}J$$

per photon. Similarly the momentum is $p = \frac{h}{\lambda} = \frac{6.63\times10^{-34}Js}{532\times10^{-9}m} = 1.25\times10^{-27}\,\frac{kgm}{s}$ per photon.

b. Since the total energy is 5J, we calculate the number of photos by dividing the total energy by the energy per photon. Thus we have

$$\# = \frac{E_{total}}{E/photon} = \frac{5J}{3.74\times10^{-19}\ J/photon} = 1.34\times10^{19}$$

photons.

c. In *1 ns*, if all the photons in part *b* are absorbed, each carrying a momentum *p*, then the force is the total change in momentum and is given by

$$F = N\frac{\Delta p}{\Delta t} = 1.34\times10^{19}\times\frac{1.25\times10^{-27}\ \frac{kgm}{s}}{1\times10^{-9}s} = 16.7N$$

24.1 Relativistic energy and momentum for an object of mass m.
For an object with a $m = 1kg$ rest mass, it has a rest energy of $E = mc^2 = 9x10^{16}\ J$.

The Lorentz factor is given by: $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$, with the relativistic momentum $p = \gamma mv$, and the relativistic energy $E^2 = p^2c^2 + m^2c^4$.

a. For a velocity of 0.8c,

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-0.64}} = 1.67$$

The relativistic momentum and energy are therefore,

$$p = \gamma mv = 1.67\times1kg\times0.8\times3\times10^8\ \tfrac{m}{s} = 4.0\times10^8\ \tfrac{kgm}{s}$$ and

$$E = \sqrt{p^2c^2+m^2c^4} = \sqrt{\left(4.0\times10^8\ \tfrac{kgm}{s}\times3\times10^8\ \tfrac{m}{s}\right)^2 + \left(9\times10^{16}J\right)^2} = 1.5\times10^{17}J$$

b. Following the procedure in part a, for a velocity of 0.9c,

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-0.81}} = 2.29$$

, the relativistic momentum is $6.19x10^8$ *kgm/s*, and the relativistic energy is $2.07x10^{17}$ *J*.

c. For a velocity of 0.95c, $\gamma = 3.20$, the relativistic momentum is $9.13x10^8$ *kgm/s*, and the relativistic energy is $2.88x10^{17}$ *J*.

d. For a velocity of 0.99c, $\gamma = 7.09$, the relativistic momentum is $2.11x10^9$ *kgm/s*, and the relativistic energy is $6.387x10^{17}$ *J*.

e. For a velocity of 0.999c, $\gamma = 22.4$, the relativistic momentum is $6.70x10^9$ *kgm/s,* and the relativistic energy is $2.0x10^{18}$ *J*.

24.2 For an electron with rest mass *9.11x10*$^{-31}$ *kg*, it has a rest energy of $E = mc^2 =$ *8.199x10*$^{-14}$ *J = 0.511 MeV*. The Lorentz factor is given by: $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$, and the relativistic momentum and energy are given respectively as $p = \gamma mv$ and $E = \sqrt{p^2c^2 + m^2c^4}$.

a. For the electron with a velocity of *0.8c, g = 1.67*. The relativistic momentum and energy are therefore,
$p = \gamma mv = 1.67 \times 9.11 \times 10^{-31} kg \times 0.8 \times 3 \times 10^8 \frac{m}{s} = 3.65 \times 10^{-22} \frac{kgm}{s}$ and
$E = \sqrt{p^2c^2 + m^2c^4} = \sqrt{\left(3.65 \times 10^{-22} \frac{kgm}{s} \times 3 \times 10^8 \frac{m}{s}\right)^2 + (8.199 \times 10 - 14J)^2} = 1.37 \times 10^{-13} J$
Using the fact that *1.6x10*$^{-19}$ *J = 1 eV*, the relativistic energy is given as $E = 1.37 \times 10^{-13} J \times \frac{1eV}{1.6 \times 10^{-19} J} = 0.855 \times 10^6 eV = 0.855 MeV$. Further it can be shown that the total relativistic energy is given as $E = \gamma mc^2 = \gamma E_{rest}$. Thus the relativistic energy can be computed in a more efficient method.
$E = \gamma E_{rest} = 1.67 \times 0.511 MeV = 0.855 MeV$.

b. Following the method outlined in part a, for a velocity of *0.9c*, γ *= 2.29*. The relativistic momentum is therefore,
$p = \gamma mv = 2.29 \times 9.11 \times 10^{-31} kg \times 0.9 \times 3 \times 10^8 \frac{m}{s} = 5.63 \times 10^{-22} \frac{kgm}{s}$ and
$E = \gamma E_{rest} = 2.29 \times 0.511 MeV = 1.17 MeV$.

c. For a velocity of *0.95c*, γ *= 3.2*. The relativistic momentum is therefore,
$p = \gamma mv = 3.2 \times 9.11 \times 10^{-31} kg \times 0.95 \times 3 \times 10^8 \frac{m}{s} = 8.31 \times 10^{-22} \frac{kgm}{s}$ and
$E = \gamma E_{rest} = 3.2 \times 0.511 MeV = 1.63 MeV$.

d. For a velocity of *0.99c*, γ *= 7.09*. The relativistic momentum is therefore,
$p = \gamma mv = 7.09 \times 9.11 \times 10^{-31} kg \times 0.99 \times 3 \times 10^8 \frac{m}{s} = 1.92 \times 10^{-21} \frac{kgm}{s}$ and
$E = \gamma E_{rest} = 7.09 \times 0.511 MeV = 3.62 MeV$.

e. For a velocity of *0.999c*, γ *= 22.4*. The relativistic momentum is therefore,
$p = \gamma mv = 22.4 \times 9.11 \times 10^{-31} kg \times 0.999 \times 3 \times 10^8 \frac{m}{s} = 6.12 \times 10^{-21} \frac{kgm}{s}$ and
$E = \gamma E_{rest} = 22.4 \times 0.511 MeV = 11.45 MeV$.

24.4 The relativistic momentum and relativistic energy are given as $p = \gamma mv$ and $E = \gamma mc^2$ respectively. To show the relation between energy and momentum, equation (24.8), we start by squaring the relativistic energy. This gives us $E^2 = \gamma^2 m^2 c^4$. Next, we use a mathematical "trick." We add and subtract the same quantity from the right hand side of the equation we just developed. The quantity we want to add and subtract is v^2. This produces factoring out a factor of c^2, $E^2 = \gamma^2 m^2 c^2 (c^2 + v^2 - v^2)$. Expanding this result we get $E^2 = \gamma^2 m^2 c^2 v^2 + \gamma^2 m^2 c^2 (c^2 - v^2)$. Recognizing that the first term is nothing more than p^2c^2 allows us to write $E^2 = p^2c^2 + \gamma^2 m^2 c^2 (c^2 - v^2)$. Factoring out a c^2 from the 2nd term on the right hand side give us $E^2 = p^2c^2 + \gamma^2 m^2 c^4 \left(1 - \frac{v^2}{c^2}\right)$.

The quantity $\left(1 - \frac{v^2}{c^2}\right)$ is simply $\frac{1}{\gamma^2}$. Therefore we arrive at the desired result, $E^2 = p^2c^2 + m^2c^4$.

24.7 For a $1.2\times10^6 eV \times \frac{1.6\times10^{-19} J}{1eV} = 1.92\times10^{-13} J$ photon,

a. its momentum is given by $p = \frac{E}{c} = \frac{1.92\times10^{-13} J}{3\times10^8 \frac{m}{s}} = 6.4\times10^{-22} \frac{kgm}{s}$.
b. its wavelength is given by the de Broglie relation

$$\lambda = \frac{h}{p} = \frac{6.63\times10^{-34} Js}{6.4\times10^{-22} \frac{kgm}{s}} = 1.04\times10^{-12} m.$$

c. its frequency is given by $f = \frac{c}{\lambda} = \frac{3\times10^8 \frac{m}{s}}{1.04\times10^{-12} m} = 2.9\times10^{20} s^{-1}$.

24.8 Photoelectric effect in cesium

We are given the work function for cesium is $\phi = 2.9\ eV = 4.64x10^{-19}\ J$.

a. The maximum wavelength corresponds to the minimum frequency. Therefore,

$KE_{\min} = hf_{\min} - \phi = 0$ which gives $f_{\min} = \frac{\phi}{h} = \frac{4.64\times10^{-19} J}{6.63\times10^{-34} Js} = 7.0\times10^{14} s^{-1}$. From

$$c = f_{\min}\lambda_{\max} \rightarrow \lambda_{\max} = \frac{c}{f_{\min}} = \frac{3\times10^8 \frac{m}{s}}{7.0\times10^{14} s^{-1}} = 4.28\times10^{-7} m = 428nm.$$

b. If *400nm* photons are used, their energy is given by

$$E = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\,Js\times3\times10^{8}\,\frac{m}{s}}{400\times10^{-9}\,m} = 4.97\times10^{-19}\,J\times\frac{1eV}{1.6\times10^{-19}\,J} = 3.11eV\,.$$

Therefore the maximum kinetic energy is given as

$$KE_{\max} = hf - \phi = 3.11eV - 2.9eV = 0.21eV\times\frac{1.6\times10^{-19}\,J}{1eV} = 3.33\times10^{-20}\,J\,.$$

c. A *1 W* beam of photons corresponds to *1 J of photons incident per second*. In *1 J* of photons there are $\frac{1J}{4.97\times10^{-19}\,\frac{J}{photon}} = 2.01\times10^{18}$ photons. If the photo ejection of an electron is 100% efficient, then for each photon lost, one electron is produced. Thus the photocurrent is the amount of charge produced each second, where *1* electron has $1.6x10^{-19}$ *C* of charge. This corresponds to a total charge of $Q = 2.01\times10^{18}\times1.6\times10^{-19}\,C = 0.322C$ of charge in *1 second*. Therefore the photocurrent is *0.322 A = 322 mA*.

d. If green photons are used with a wavelength of *500 nm*, this corresponds to an energy of

$$E = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\,Js\times3\times10^{8}\,\frac{m}{s}}{500\times10^{-9}\,m} = 3.98\times10^{-19}\,J\times\frac{1eV}{1.6\times10^{-19}\,J} = 2.49eV\,.$$

The maximum *f* that will allow for photo ejection of electrons is the one where the electrons are ejected with a maximum kinetic energy equal to zero. Therefore $KE_{\min} = hf_{\min} - \phi = 0$, and solving for ϕ we obtain $hf_{\min} = \phi = 2.49eV$.

24.9 The maximum KE is given by the product of the stopping potential (0.82V) and the electron's charge (e^-). Thus the maximum $KE = eV_{stop} = 0.82eV$. This is equal to the energy of the photons incident minus the work function (the minimum energy needed to eject a photoelectron). In symbols,

$$KE_{\max} = hf - \phi = \frac{hc}{\lambda} - \phi$$

$$\rightarrow 0.82eV = \left(\frac{6.63\times10^{-34}\,Js\times3\times10^{8}\,\frac{m}{s}}{400\times10^{-9}\,m}\right)\times\frac{1eV}{1.6\times10^{-19}\,J} - \phi \rightarrow \phi = 2.29eV$$