Physics 120

Exam #1

April 25, 2014

Name___________________________________

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example $|\vec{p}| \approx m|\vec{v}| = (5kg)\times(2\,\frac{m}{s}) = 10\,\frac{kg\cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- All multiple choice questions are worth 4 points and each free-response part is worth 8 points

Problem #1	/28
Problem #2	/24
Problem #3	/24
Total	/76

I affirm that I have carried out my academic endeavors with full academic honesty.

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1. The human body is capable of sustaining rather large forces without drastic injury. Consider, for example, an automobile accident in which an inflatable safety device (an airbag) is suddenly and quickly inflated to keep your head from hitting the steering wheel.

   a. When the airbag touches your body, the airbag exerts a force on your body. Throughout the time the airbag is in contact with your body,

      1. $F_{head,airbag} = F_{airbag,head}$
      2. $\vec{F}_{head,airbag} = \vec{F}_{airbag,head}$
      3. $\vec{F}_{head,airbag} = -\vec{F}_{airbag,head}$
      4. $\Delta\vec{p}_{head} = \Delta\vec{p}_{airbag}$
      5. $\Delta\vec{p}_{head} = -\Delta\vec{p}_{airbag}$

   b. Derive an expression for the magnitude of the force from the airbag on your body assuming that you were initially traveling at an initial speed of $\vec{v}_i = \langle v_i, 0, 0\rangle \frac{m}{s}$ and your body were brought to rest by undergoing a displacement of $\Delta\vec{r} = \langle \Delta x, 0, 0\rangle m$.

   We calculate the magnitude of the force from the momentum principle. We have $\vec{v}_f = \vec{v}_i + \frac{\vec{F}_{net}}{m}\Delta t$ and from the position-update, $\vec{r}_f = \vec{r}_i + \vec{v}_{avg}\Delta t \rightarrow \Delta\vec{r} = \vec{v}_{avg}\Delta t$. We need the average velocity, so $\vec{v}_{avg} = \frac{\vec{v}_i + \vec{v}_f}{2} = \frac{\vec{v}_i}{2}$, so $\Delta\vec{r} = \vec{v}_{avg}\Delta t = \frac{\vec{v}_i}{2}\Delta t$. The motion only takes place in the x-direction so we can write $\Delta x = \frac{v_i}{2}\Delta t \rightarrow \Delta t = \frac{2\Delta x}{v_i}$. From the momentum principle we can determine the magnitude of the force. So, $v_{f,x} = v_{i,x} + \frac{F_x}{m}\Delta t \rightarrow 0 = v_i + \frac{F_x}{m}\left(\frac{2\Delta x}{v_i}\right) \rightarrow F_x = \left|\frac{mv_i^2}{2\Delta x}\right|$.

   c. If your body (of mass $70kg$) comes to rest over a distance of $30cm$ and if your initial speed $19.6\frac{m}{s}$ ($\sim 44 mph$), what is the magnitude of the force that the airbag exerted on your body?

   From part b, $F_x = \left|\frac{mv_i^2}{2\Delta x}\right| = \frac{70kg \times (19.6\frac{m}{s})^2}{2 \times 0.3m} = 4.48 \times 10^4 N$.

d. There is a probability that the airbag could trigger and inflate while the car is in motion and there was no accident at all. This could impair the vision of the driver and hence the airbag is designed to expand and collapse over a very short time interval. Given the information above, how long would it take the airbag to deflate if the inflation was almost instantaneous and assuming that the airbag is fully deflated as soon as the automobile is brought to rest?

The time is given by $\Delta t = \frac{2\Delta x}{v_i} = \frac{2 \times 0.3m}{19.6\frac{m}{s}} = 0.031s$.

2. A $2.0kg$ model rocket is launched vertically from rest from the ground and its engine produces a constant vertical force of $26N$ for $8s$ after which time the fuel is used up. Assume that the rocket travels in the vertical direction only throughout it's entire motion and that air resistance is negligible.

   a. What maximum height (expressed as a vector) above the ground will the rocket reach?

      There are two phases to the motion. One when the engines are on and working and then one when the engines are off and the rocket coasts against gravity to its maximum height.

      Through the first phase where the engine is on we have $\vec{r}_{f,engines} = \vec{r}_{i,engines} + \vec{v}_{avg,engines}\Delta t_{engines}$. Through the second phase where the engine is off we have $\vec{r}_{f,no engines} = \vec{r}_{i,no engines} + \vec{v}_{avg,no engines}\Delta t'_{no engines}$. Combining the two expressions we have $\vec{r}_f = \vec{r}_{i,no engines} + \vec{v}_{avg,no engines}\Delta t'_{no engines} = \vec{r}_{i,engines} + \vec{v}_{avg,engines}\Delta t_{engines} + \vec{v}_{avg,no engines}\Delta t'_{no engines}$.

      From the momentum principle, the velocity of the rocket when the engine is on is given by $\vec{v}_{f,engine} = \vec{v}_{i,engine} + \frac{\vec{F}_{net,engine}}{m}\Delta t_{engine} = \left\langle 0, \frac{26N}{2kg}, 0 \right\rangle \times 8s = \langle 0,104,0\rangle \frac{m}{s}$.

      Thus the average velocity when the engine is on is $v_{avg,engine} = \frac{v_{i,engine} + v_{f,engine}}{2} = \left\langle \frac{0+0}{2}, \frac{0+104}{2}, \frac{0+0}{2} \right\rangle \frac{m}{s} = \langle 0,52,0\rangle \frac{m}{s}$. This is also the average velocity when the engine turns off and the rocket coast upward to its maximum height. To find the time the rocket coasts upward to its maximum height when the engine shuts off apply the momentum principle. We have

$$\vec{v}_{f,no engine} = \vec{v}_{i,no engine} + \frac{\vec{F}_{net,no engine}}{m}\Delta t'_{no engine}$$

$$\langle 0,0,0\rangle \tfrac{m}{s} = \langle 0,104,0\rangle \tfrac{m}{s} + \langle 0,-9.8,0\rangle \tfrac{m}{s^2} \times \Delta t'_{no engine}$$

$$\Delta t'_{no engine} = \frac{104 \frac{m}{s}}{9.8 \frac{m}{s^2}} = 10.6s$$

      The maximum height is

$$\vec{r}_f = \vec{r}_{i,no engines} + \vec{v}_{avg,no engines}\Delta t'_{no engines} = \vec{r}_{i,engines} + \vec{v}_{avg,engines}\Delta t_{engines} + \vec{v}_{avg,no engines}\Delta t'_{no engines}$$

$$\vec{r}_f = \langle 0,0,0\rangle m + \langle 0,52,0\rangle \tfrac{m}{s} \times 8s + + \langle 0,52,0\rangle \tfrac{m}{s} \times 10.6s = \langle 0,967.2,0\rangle m$$

b. Starting from the position-update and using the momentum principle, what is the time it takes the rocked to fall back to the ground and what is the total flight time of the rocket from lift-off to landing?

The time of flight is the sum of $\Delta t_{engine} + \Delta t_{no\,engine} + \Delta t_{fall}$ Starting from rest at the maximum height, the rocket falls back to the ground. From the position-update we have, $\vec{r}_f = \vec{r}_i + \vec{v}_{avg}\Delta t_{fall}$. The average velocity is

$$\vec{v}_{avg} = \frac{\vec{v}_i + \vec{v}_f}{2} = \frac{\langle 0,0,0\rangle + \langle 0,-g\Delta t_{fall},0\rangle}{2} = \left\langle 0,\frac{-g\Delta t_{fall}}{2},0\right\rangle$$, where the speed just before the ground is determined from the momentum-principle. We have

$$\vec{v}_f = \vec{v}_i + \frac{\vec{F}_{net}}{m}\Delta t_{fall} = \langle 0,-g,0\rangle \Delta t_{fall}$$. Therefore we can calculate Δt_{fall} from

$$\vec{r}_f = \vec{r}_i + \vec{v}_{avg}\Delta t_{fall} \rightarrow \langle 0,0,0\rangle m = \langle 0,967.2,0\rangle m + \left\langle 0,\frac{-g\Delta t_{fall}}{2},0\right\rangle \Delta t_{fall}$$

$$\Delta t_{fall} = \sqrt{\frac{2\times 967.2m}{9.8\frac{m}{s^2}}} = 14.1s$$

The time of flight is $\Delta t_{engine} + \Delta t_{no\,engine} + \Delta t_{fall} = 8s + 10.6s + 14.1s = 32.7s$.

c. Suppose that you could do this same rocket launch experiment on Titan, a moon of Saturn. Titan has a mass $M_T = 0.023M_E$ and a radius $R_T = 0.4R_E$, where M_E and R_E are the mass and radius of the Earth respectively. The acceleration due to gravity for a mass m near the surface of Titan g_T, compared to the acceleration due to gravity for the same mass near the surface of the Earth, g_E is

1. smaller and is given by $g_T = 0.14g_E$.
2. smaller and is given by $g_T = 0.06g_E$.
3. equal to that on Earth.
4. larger and given by $g_T = 6.96g_E$.
5. larger and given by $g_T = 16.7g_E$.

d. Assuming that the acceleration produced by the rocket's engines is the same, if you were to launch this rocket on the surface of Titan, the maximum height reached above the surface of Titan, compared to the height reached on Earth, would be

1. lower because the acceleration due to gravity on Titan is smaller.
2. lower because the acceleration due to gravity on Titan is larger.
3. the same since the acceleration due to gravity on Titan is the same as on Earth.
4. greater because the acceleration due to gravity on Titan is smaller.
5. greater because the acceleration due to gravity on Titan is larger.

3. A daredevil is trying a death-defying stunt in which she will be launched across a $500\,ft \approx 167m$ wide chasm. She will be launched $10m$ from the left edge of the chasm by a cannon at a speed v at an angle of $\theta = 40^o$ above the horizontal.

   a. At what speed (magnitude of the velocity) would the daredevil need to be launched so that she lands on a safety mat located $10m$ from the right edge of the chasm? (Hint: Assume that she is launched $2m$ above the ground and lands in the safety mat that has a height of $1m$ above the ground.)

   We can write the trajectory for the daredevil and we have $\vec{r}_f = \vec{r}_i + \vec{v}_{avg}\Delta t$ where the average velocity is determined from

$$\vec{v}_{avg} = \frac{\vec{v}_i + \vec{v}_f}{2} = \frac{\langle v_i\cos\alpha, v_i\cos\beta, v_i\cos\gamma\rangle + \left(\langle v_i\cos\alpha, v_i\cos\beta, v_i\cos\gamma\rangle + \langle 0, -g\Delta t, 0\rangle\right)}{2}$$

$$\vec{v}_{avg} = \left\langle v_i\cos\alpha, v_i\cos\beta - \frac{g\Delta t}{2}, \cos\gamma\right\rangle$$

   and the final velocity is calculated from the momentum-principle,

$$\vec{v}_f = \vec{v}_i + \frac{\vec{F}_{net}}{m}\Delta t = \langle v_i\cos\alpha, v_i\cos\beta, v_i\cos\gamma\rangle + \left\langle 0, -\frac{mg}{m}, 0\right\rangle\Delta t\,.$$

   We have from the position-update

$$\vec{r}_f = \vec{r}_i + \vec{v}_{avg}\Delta t$$

$$\langle x_f, y_f, z_f\rangle = \langle x_i, y_i, z_i\rangle + \left\langle v_i\cos\alpha, v_i\cos\beta - \frac{g\Delta t}{2}, \cos\gamma\right\rangle\Delta t$$

x-dir: $x_f = x_i + (v_i\cos\alpha)\Delta t$

y-dir: $y_f = y_i + \left(v_i\cos\beta - \frac{g\Delta t}{2}\right)\Delta t$

z-dir: $0 = 0$

   From the horizontal motion we derive an expression for the time of flight and substitute this into the vertical motion to calculate the initial speed. From the horizontal motion, $\Delta t = \frac{x_f - x_i}{v_i\cos\alpha}$. In the vertical direction,

$$y_f = y_i + (v_i\cos\beta)\times\left(\frac{x_f - x_i}{v_i\cos\alpha}\right) - \frac{g}{2}\left(\frac{x_f - x_i}{v_i\cos\alpha}\right)^2$$

$$1m = 2m + \left(\frac{\cos 50}{\cos 40}\times 177m\right) - \left(\frac{9.8\,\frac{m}{s^2}}{2}\right)\left(\frac{177m}{v_i\cos 40}\right)^2$$

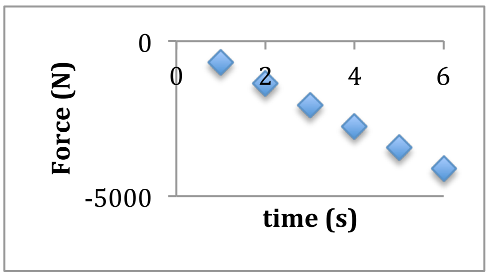
$$v_i = 43\,\tfrac{m}{s}$$

b. How long will the daredevil be in the air enjoying the view?

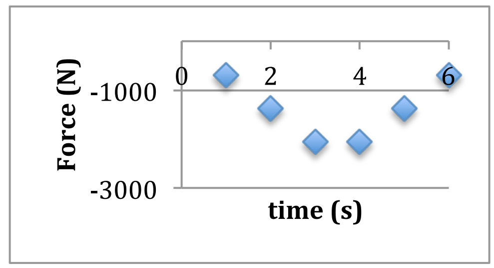
The time of flight is $\Delta t = \dfrac{x_f - x_i}{v_i \cos\alpha} = \dfrac{177m}{43\frac{m}{s}\cos 40} = 5.7s$.

c. Shown below are graphs of the force on the daredevil plotted as a function of time. The graph that best describes the magnitude of the force as a function of time is

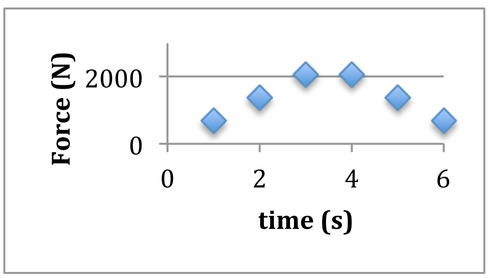
1.



2.



3.



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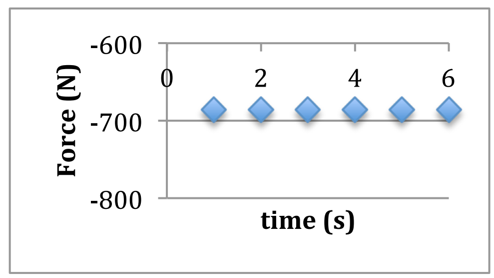
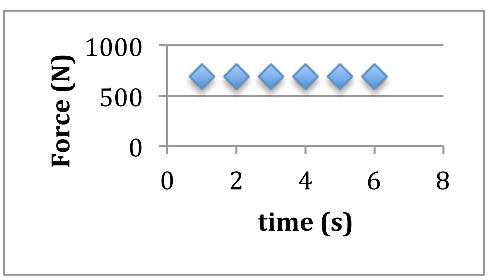


5.



d. Suppose that you decided that you wanted, in light of the concerns over safety, to include air resistance into your calculations. Air resistance is a force that opposes the motion of the object and this force produces opposes the velocity of the object. Compared to the case where there is no air resistance the maximum horizontal and vertical distances reached by the daredevil in the presence of air of resistance will be (assuming that the launch velocity is the same as in part a)

1. both greater in both directions than those reached with no air resistance.
2. both smaller in both directions than those reached with no air resistance.
3. greater and smaller respectively since air resistance only affects the horizontal component of motion.
4. smaller and greater respectively since air resistance only affects the vertical component of motion.

Physics 120 Equations

$$\vec{r} = < r_x, r_y, r_z > = |\vec{r}| \cdot \hat{r}$$

magnitude of a vector : $r = |\vec{r}| = \sqrt{r_x^2 + r_y^2 + r_z^2}$

unit vector : $\hat{r} = \dfrac{\vec{r}}{|\vec{r}|}$

$$\vec{v} = \frac{\Delta \vec{r}}{\Delta t}; \vec{v}_{avg} = \frac{\vec{v}_i + \vec{v}_f}{2}$$

$$\vec{r}_f = \vec{r}_i + \vec{v}_{avg} \Delta t$$

$$\vec{F}_{net} = \frac{\Delta \vec{p}}{\Delta t}$$

$$\vec{p}_f = \vec{p}_i + \vec{F}_{net} \Delta t$$

$$\vec{r}_f = \vec{r}_i + \vec{v}_i \Delta t + \frac{\vec{F}_{net}}{2m} (\Delta t)^2$$

$$\vec{F}_G = -\frac{G M_1 M_2}{r^2} \hat{r}$$

$$\vec{F}_g \sim m\vec{g}$$

Constants:

$$g = 9.8 \tfrac{m}{s^2}$$

$$G = 6.67 \times 10^{-11} \tfrac{Nm^2}{kg^2}$$

$$m_e = 9.11 \times 10^{-31} kg$$

$$m_p = 1.67 \times 10^{-27} kg$$

$$m_E = 6 \times 10^{24} kg$$

$$R_E = 6.4 \times 10^{6} m$$