Physics 120

Exam #1

January 31, 2020

Name___________________________________

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example, $|\vec{p}| \approx m|\vec{v}| = (5kg)\times(2\,\tfrac{m}{s}) = 10\,\tfrac{kg\cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- All multiple-choice questions are worth 3 points and each free-response part is worth 7 points

Problem #1	/24
Problem #2	/24
Problem #3	/24
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

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1. A city subway train accelerates from rest at station A and heads to station B. Stations A and B are separated by a fixed distance of $x_{total} = 860m$. The train leaves station A and accelerates at a constant rate of $a_1 = 1.34\frac{m}{s^2}$ for a time t_1(measured in seconds) at attains a speed v_1 at which point the train applies the brakes and decelerates to rest at a constant rate of $a_2 = 0.8\frac{m}{s^2}$, for a time t_2 (measured in seconds) arriving at station B.

   a. In terms of a_1, a_2, and t_1, what is the expression for the trajectory (x_{total}) of the train from startion A to station B?

   The trajectory is composed of two parts, one piece to accelerate to speed $v_1$at $a_1$for $t_1$and then from that position decelerate to rest from speed $v_1$at $a_2$for t_2.

   $$x_{total} = \Delta x_{accel} + \Delta x_{decel} = \frac{1}{2}a_1t_1^2 + v_1t_2 + \frac{1}{2}a_2t_2^2$$

   The train accelerates from rest and acquires a speed v_1.
   $$v_{fx} = v_{ix} + a_1t_1 \rightarrow v_1 = a_1t_1$$

   The train decelerates from speed $v_1$to rest.
   $$v_{fx} = 0 = v_1 - a_2t_2 \rightarrow v_1 = a_2t_2$$

   Thus, $v_1 = a_1t_1 = a_2t_2$. This lets us express $t_2$in terms of t_1. $t_2 = \frac{a_1}{a_2}t_1$

   The trajectory then can be written as
   $$x_{total} = \frac{1}{2}a_1t_1^2 + v_1t_2 + \frac{1}{2}a_2t_2^2 = \frac{1}{2}a_1t_1^2 + a_1t_1\left(\frac{a_1}{a_2}t_1\right) - \frac{1}{2}a_2\left(\frac{a_1}{a_2}t_1\right)^2$$
   $$x_{total} = \frac{1}{2}a_1t_1^2 + \frac{1}{2}\left(\frac{a_1^2}{a_2^2}\right)t_1^2 = \frac{1}{2}a_1t_1^2\left[1 + \frac{a_1}{a_2}\right]$$

   b. What is the total travel time ($t_{total} = t_1 + t_2$) for the train ride between stations A and B?

   $$t_1 = \sqrt{\frac{2x_{total}}{a_1\left[1+\frac{a_1}{a_2}\right]}} = \sqrt{\frac{2\times 860m}{1.34\frac{m}{s^2}\left[1+\frac{1.34\frac{m}{s^2}}{0.8\frac{m}{s^2}}\right]}} = 21.9s$$

   and $t_2 = \frac{a_1}{a_2}t_1 = \left(\frac{1.34\frac{m}{s^2}}{0.8\frac{m}{s^2}}\right) \times 21.9s = 36.7s.$

   Thus, $t_{total} = t_1 + t_2 = 21.9s + 36.7s = 58.6s$

c. What is the speed $v_1$reached by the train?

$$v_1 = a_1 t_1 = 1.34\frac{m}{s^2} \times 21.9s = 29.4\frac{m}{s}$$

$$v_2 = a_2 t_2 = 0.8\frac{m}{s^2} \times 36.7s = 29.4\frac{m}{s}$$

c. In order for the train to come to rest at station B, the brakes need to applied for some amount of time. Which of the following graphs gives a qualitative representation of the force (from the brakes) as a function of time?

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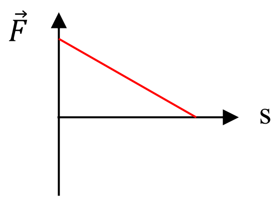

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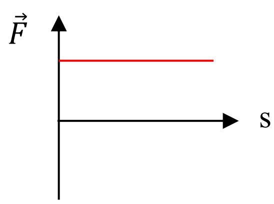

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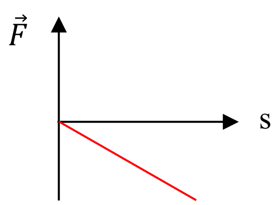

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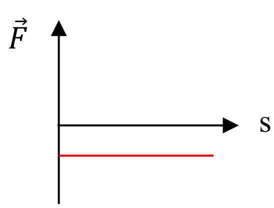


5. None of the above give a correct representation of the force as a function of time.

2. Boeing's newest model passenger airplane is the 777X (shown on the right) with an empty (no passengers/luggage) mass of $181{,}000kg$ ($\sim 440{,}000lbs$) taxis onto a runway. Awaiting the control tower's clearance for takeoff the plane sits on the end of the runway, with overall length of $3200m$ ($\sim 2miles$), at rest. When the control tower gives the plane clearance for takeoff, the pilots increase engine power and when the desired engine power is reached, the brakes are released and the plane accelerates from rest down the runway.

https://samchui.com/2020/01/25/boeing-777x-successfully-performs-maidenflight/

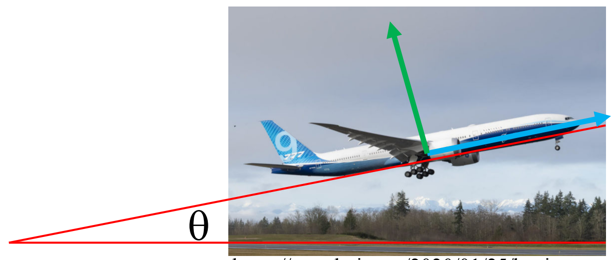
   a. If the pilots want the plane to takeoff after it has traveled a distance of $2000m$ ($\sim 1.3miles$), what minimum acceleration would the airplane need and how long would it take the airplane to takeoff and become airborne? Assume that the plane needs to have a speed of $80\frac{m}{s}$ ($\sim 180\frac{mi}{hr}$) before it can take off from the runway. Ignore air resistance in this problem.

$$v_{fx}^2 = v_{ix}^2 + 2a_x\Delta x = 2a_x\Delta x \to a_x = \frac{v_{fx}^2}{2\Delta x} = \frac{\left(80\frac{m}{s}\right)^2}{2\times 2000m} = 1.6\frac{m}{s^2}$$

$$v_{fx} = v_{ix} + a_x t = a_x t \to t = \frac{v_{fx}}{a_x} = \frac{80\frac{m}{s}}{1.6\frac{m}{s^2}} = 50s$$

Or, $x_f = x_i + v_i t + \frac{1}{2}a_x t^2 = \frac{1}{2}a_x t^2 \to t = \sqrt{\frac{2x_f}{a_x}} = \sqrt{\frac{2\times 2000m}{1.6\frac{m}{s^2}}} = 50s$

b. As the plane accelerates, airflow over the wings produces a force called lift ($\vec{F}_{lift}$) that is perpendicular to the wings. When the plane reaches a speed of $80\frac{m}{s}$ ($\sim 180\frac{mi}{hr}$) the wings have generated enough lift to let the plane fly into the air. Suppose that the plane climbs into the air at a constant velocity at an angle θ measured with respect to the horizontal as shown below. At what angle θ does the plane make with respect to the horizontal. Hint, the engines provide a force called thrust (with magnitude $|\vec{F}_{thrust}| = 4.2 \times 10^5 N$) in line with the engines that makes the plane move forward. $\vec{F}_{lift}$ (in green) and $\vec{F}_{thrust}$ (in blue) are shown on the diagram below. Ignore air resistance in this problem.



https://samchui.com/2020/01/25/boeing-777x-successfully-performs-maidenflight/

$$\vec{F}_{net} = \vec{F}_{lift} + \vec{F}_{thrust} + \vec{F}_{weight} = m\vec{a}$$

$$\langle 0, F_{lift}, 0\rangle + \langle F_{thrust}, 0,0\rangle + \langle -mg\sin\theta\,, -mg\cos\theta\,, 0\rangle = \langle 0,0,0\rangle$$

x-direction

$$F_{thrust} - mg\sin\theta = 0 \rightarrow \sin\theta = \frac{F_{thrust}}{mg} = \frac{4.2\times 10^5 N}{181000kg \times 9.8\frac{m}{s^2}} = 0.2368$$

$$\theta = \sin^{-1}(0.236) = 13.7^0$$

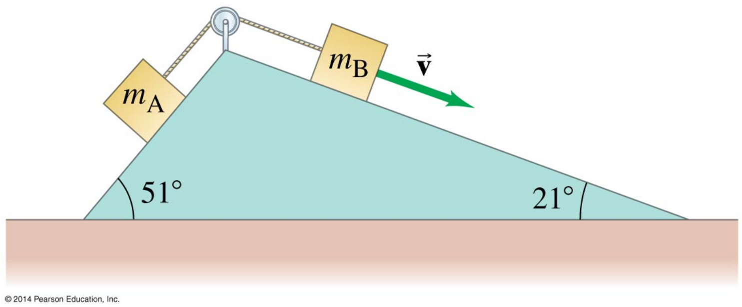
y-direction

$$F_{lift} - mg\cos\theta = 0 \rightarrow F_{lift} = mg\cos\theta$$

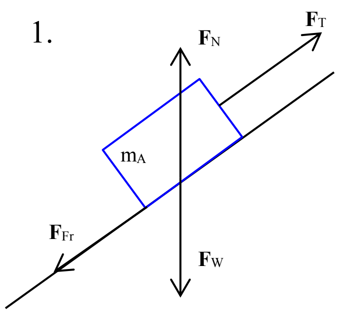
c. What magnitude of the lift force ($\vec{F}_{lift}$) would be needed to keep the plane flying at angle θ?

$$F_{lift} = mg\cos\theta = 181000kg \times 9.8\frac{m}{s^2} \times \cos 13.7 = 1.7 \times 10^6 N$$

d. Suppose instead of the airplane taking off, you have the following situation. A block of mass m_A is connected to a block of mass m_B by a light rope that passes over a “massless” pulley, as shown below. Assume that there is friction between each block and the ramp with coefficient of friction μ. If block with mass m_A slides up the ramp while the block with mass m_B slides down the ramp, which of the following could be a possible free-body (force) diagram for block m_A?



1.



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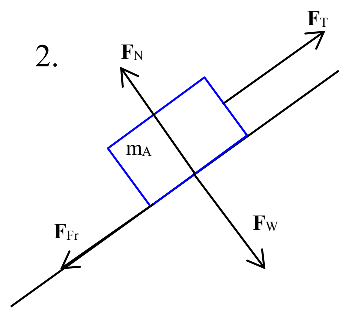

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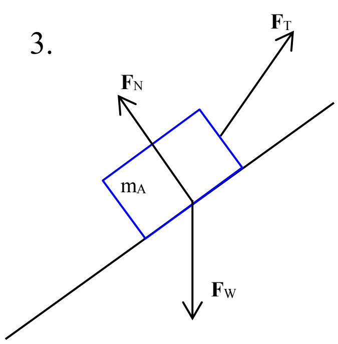

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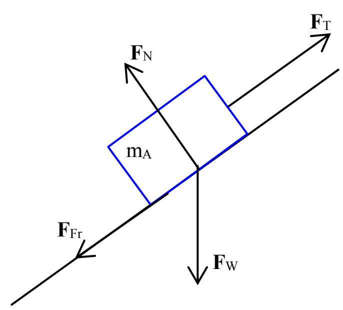
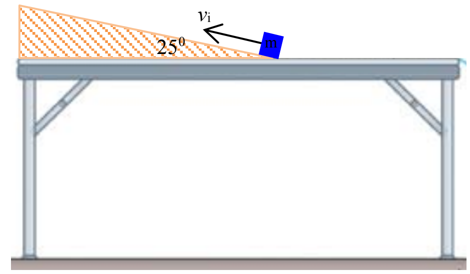


5. None of the above would give the correct free-body (force) diagram for the block of mass m_A.

3. Consider a small ramp located on the edge of a table, where the top of the table is located $y_{table} = 1.5m$ above the ground. A $0.5kg$ block is given an initial speed of $|\vec{v}_i| = 10\frac{m}{s}$ directed along up along the ramp inclined at an angle of 25^0 measured with respect to the top of the table as shown on the right. The block slides along the ramp, which is $1.0m$ long, and is launched from the top of the ramp. Friction between the block and the ramp exists with coefficient of friction $\mu = 0.4$.



a. What is the net acceleration of the block on the ramp? You may express your answer as either a vector specifying the components or as a magnitude and direction. In either case, be sure to specify your coordinate system clearly.

Assuming a tilted coordinate-system with the positive x-direction up the ramp and the positive y-direction perpendicular to the ramp, we have

$$\vec{F}_{net} = \vec{F}_N + \vec{F}_{fr} + \vec{F}_{weight} = m\vec{a}$$

$$\langle 0, F_N, 0\rangle + \langle -F_{fr}, 0,0\rangle + \langle -mg\sin\theta\,, -mg\cos\theta\,, 0\rangle = \langle ma, 0,0\rangle$$

y-direction

$$F_N - mg\cos\theta = 0 \rightarrow F_N = mg\cos\theta$$

x-direction

$$-F_{fr} - mg\sin\theta = -\mu F_N - mg\sin\theta = -\mu mg\cos\theta - mg\sin\theta = ma$$

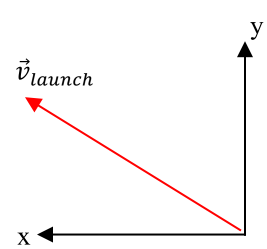
$$a = -g(\sin\theta + \mu\cos\theta) = -9.8\tfrac{m}{s^2}(\sin 25 + 0.4\cos 25) = -7.69\tfrac{m}{s^2}$$

$$\vec{a} = \langle -7.69,0,0\rangle\tfrac{m}{s^2}$$

b. At the top of the ramp the block is launched into the air. Using the coordinate system below, what is the block's launch velocity $\vec{v}_{launch}$?

$$v_{fx}^2 = v_{ix}^2 + 2a_x\Delta x \rightarrow v_{launch} = \sqrt{\left(10\tfrac{m}{s}\right)^2 - 2\times 7.69\tfrac{m}{s^2}\times 1m} = 9.2\frac{m}{s}$$

$$\vec{v}_f = \langle v_{launch}\cos\theta, v_{launch}\sin\theta, 0\rangle = \langle 9.2\cos 25, 9.2\sin 25, 0\rangle\frac{m}{s}$$
$$= \langle 8.3,3.9,0\rangle\frac{m}{s}$$



c. With respect to the end of the ramp, where does the block land? That is, what is $\vec{r}_f = \langle x_f, y_f, z_f \rangle$?

$\vec{r}_f = \langle x_f, y_f, z_f \rangle = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2$
$= \langle v_{launch}\cos\theta, v_{launch}\sin\theta, 0 \rangle t + \frac{1}{2}\langle 0, -g, 0 \rangle t^2$

$\sin 25 = \frac{y_{launch}}{1m} \rightarrow y_{launch} = 1m \sin 25 = 0.42m$

x-direction
$x_f = (v_{launch}\cos\theta)t = 8.3\frac{m}{s} \times 1.14s = 9.5m$

y-direction
$y_f = (v_{launch}\sin\theta)t - \frac{1}{2}gt^2 \rightarrow -1.92 = 3.9t - 4.9t^2$

By the quadratic formula the solutions are: $t = \begin{cases} 1.14s \\ -0.34s \end{cases}$

Therefore $\vec{r}_f = \langle x_f, y_f, z_f \rangle = \langle 9.5, -1.92, 0 \rangle m$

d. Which of the following would allow the block to travel a greater horizontal distance as measured from the end of the ramp?

1. Decreasing the launch angle of the ramp.
2. Decreasing the coefficient of friction between the block and the ramp.
3. Increasing the initial speed of the block at the bottom of the ramp.
4. All of the above would allow the block to travel a greater horizontal distance.
5. None of the above would allow the block to travel a greater horizontal distance.

Physics 120 Formulas

Motion

$\Delta x = x_f - x_i$
$v_{avg} = \frac{\Delta x}{\Delta t}$
$a_{avg} = \frac{\Delta v}{\Delta t}$

Equations of Motion

displacement: $\begin{cases} x_f = x_i + v_{ix}t + \frac{1}{2}a_x t^2 \\ y_f = y_i + v_{iy}t + \frac{1}{2}a_y t^2 \end{cases}$

velocity: $\begin{cases} v_{fx} = v_{ix} + a_x t \\ v_{fy} = v_{iy} + a_y t \end{cases}$

time-independent: $\begin{cases} v_{fx}^2 = v_{ix}^2 + 2a_x \Delta x \\ v_{fy}^2 = v_{iy}^2 + 2a_y \Delta y \end{cases}$

general: $\begin{cases} \vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2 \\ \vec{v}_f = \vec{v}_i + \vec{a}t \\ v_f^2 = v_i^2 + 2\vec{a}\cdot\Delta\vec{r} \end{cases}$

Uniform Circular Motion

$F_r = ma_r = m\frac{v^2}{r}; \quad a_r = \frac{v^2}{r}$

$v = \frac{2\pi r}{T}$

$F_G = G\frac{m_1 m_2}{r^2}$

Geometry /Algebra

Circles *Triangles* *Spheres*

$C = 2\pi r$ $\quad A = \frac{1}{2}bh$ $\quad A = 4\pi r^2$

$A = \pi r^2$ $\quad V = \frac{4}{3}\pi r^3$

Quadratic equation : $ax^2 + bx + c = 0$,

whose solutions are given by : $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Vectors

magnitude of a vector: $v = |\vec{v}| = \sqrt{{v_x}^2 + {v_y}^2}$

direction of a vector: $\phi = \tan^{-1}\left(\frac{v_y}{v_x}\right)$

Useful Constants

$g = 9.8\, m/s^2$ $\quad G = 6.67\times10^{-11}\, Nm^2/kg^2$

$N_A = 6.02\times10^{23}\, atoms/mole$ $\quad k_B = 1.38\times10^{-23}\, J/K$

$\sigma = 5.67\times10^{-8}\, W/m^2K^4$ $\quad v_{sound} = 343\, m/s$

Linear Momentum/Forces

$\vec{p} = m\vec{v}$

$\vec{p}_f = \vec{p}_i + \vec{F}\Delta t$

$\vec{F} = m\vec{a}$

$\vec{F}_s = -k\vec{x}$

$F_f = \mu F_N$

Work/Energy

$K_t = \frac{1}{2}mv^2$

$K_r = \frac{1}{2}I\omega^2$

$U_g = mgh$

$U_S = \frac{1}{2}kx^2$

$W_T = FdCos\theta = \Delta E_T$

$W_R = \tau\theta = \Delta E_R$

$W_{net} = W_R + W_T = \Delta E_R + \Delta E_T$

$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = 0$

$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = -\Delta E_{diss}$

Rotational Motion

$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$

$\omega_f = \omega_i + \alpha t$

$\omega^2{}_f = \omega^2{}_i + 2\alpha\Delta\theta$

$\tau = I\alpha = rF$

$L = I\omega$

$L_f = L_i + \tau\Delta t$

$\Delta s = r\Delta\theta : \; v = r\omega : \; a_t = r\alpha$

$a_r = r\omega^2$

Simple Harmonic Motion/Waves

$\omega = 2\pi f = \frac{2\pi}{T}$

$T_S = 2\pi\sqrt{\frac{m}{k}}$

$T_P = 2\pi\sqrt{\frac{l}{g}}$

$v = \pm\sqrt{\frac{k}{m}}A\left(1 - \frac{x^2}{A^2}\right)^{\frac{1}{2}}$

$x(t) = A\sin\left(\frac{2\pi t}{T}\right)$

$v(t) = A\sqrt{\frac{k}{m}}\cos\left(\frac{2\pi t}{T}\right)$

$a(t) = -A\frac{k}{m}\sin\left(\frac{2\pi t}{T}\right)$

$v = f\lambda = \sqrt{\frac{F_T}{\mu}}$