Physics 120

Exam #1

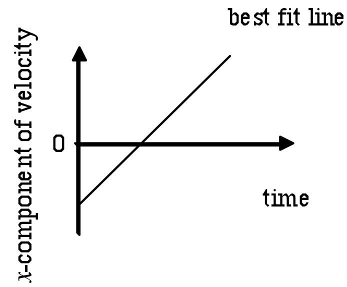
April 15, 2011

Name___________________________________

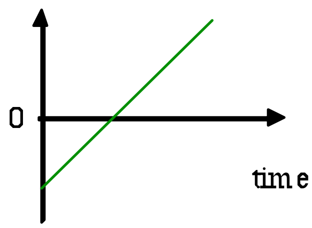
Multiple Choice	/16
Problem #1	/28
Problem #2	/28
Problem #3	/28
Total	/100

Part I: Multiple-Choice: *Circle the best answer to each question. Any other marks will not be given credit. Each multiple-choice question is worth 4 points for a total of 16 points.*

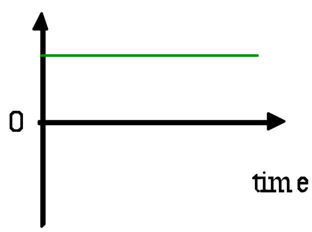
1. The x-component of a particle's velocity is sampled every 5 seconds. The data are fit with a straight line as shown in the figure to the right. Assuming the fit is a good approximation to the motion, which of the following best represents the x-component of the displacement of the particle?



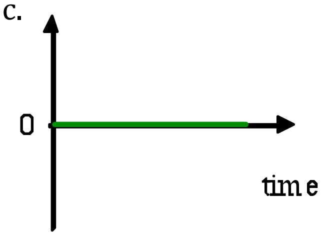
a.



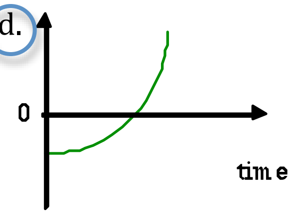
b.



c.



d.



2. You observe three carts moving to the left.
   Cart *A* moves with constant speed.
   Cart *B* is speeding up.
   Cart *C* is slowing down.
   Which cart or carts are experiencing a net force to the left?

   a. Cart *A* b. Cart *B* c. Cart *C* d. Carts *B* and *C*

3. Suppose that instead of the Earth being created in its present size and shape that it were somehow actually created twice as massive as it really is with twice the radius. The approximate acceleration due to gravity on this "new" Earth would be equal to

   a. g b. $g/4$ c. $g/2$ d. $4g$

4. A proton is accelerated from rest to a speed of $8x10^7$*-m/s*. The momentum of the proton is most closely

   a. $1.34\times10^{-19}\,\frac{kgm}{s}$ b. $7.29\times10^{-23}\,\frac{kgm}{s}$ c. $1.39\times10^{-19}\,\frac{kgm}{s}$ d. $7.57\times10^{-23}\,\frac{kgm}{s}$

Part II: Free Response Problems: *The three problems below are worth 84 points total and each subpart is worth 7 points each. Please show all work in order to receive partial credit. If your solutions are illegible or illogical no credit will be given. A number with no work shown (even if correct) will be given no credit. Please use the back of the page if necessary, but number the problem you are working on.*

1. An object with a mass of *2.0kg* and an initial velocity $\vec{v}_i = \langle 30,-42,0 \rangle \frac{m}{s}$ slides across the floor (taken as the x-y coordinate system) passes a point taken to be the origin at a time $t_i = 0$. As the object passes the origin enters a region of space where it is subject to a constant frictional force of $\vec{F} = \langle -7, 9.8, 0 \rangle N$.

   a. What is the speed of the object and what is the unit vector that points in the direction of the velocity?

$$|\vec{v}| = v = \sqrt{v_x^2 + v_y^2 + v_z^2} = \sqrt{\left(30\tfrac{m}{s}\right)^2 + \left(-42\tfrac{m}{s}\right)^2 + \left(0\tfrac{m}{s}\right)^2} = 51.6\tfrac{m}{s}$$

$$\hat{v} = \frac{\vec{v}}{v} = \frac{\langle 30,-42,0 \rangle \frac{m}{s}}{51.6\frac{m}{s}} = \langle 0.58,-0.81,0 \rangle$$

   b. What is the average velocity of the object between the time it crosses the point that is the origin and the time it comes to rest?

$$\vec{v}_{avg} = \frac{\vec{v}_i + \vec{v}_f}{2} = \frac{\langle 30,-42,0 \rangle \frac{m}{s} - \langle 0,0,0 \rangle \frac{m}{s}}{2} = \langle 15,-21,0 \rangle \tfrac{m}{s}$$

   c. How long does it take the object to come to rest?

$$\vec{p}_f = \vec{p}_i + \vec{F}_{net}\Delta t$$

$$\langle 0,0,0 \rangle \tfrac{kgm}{s} = \langle 60,-84,0 \rangle \tfrac{kgm}{s} + \langle -7,9.8,0 \rangle N \times \Delta t$$

$$\rightarrow 0 = 60 - 7\Delta t \Rightarrow \Delta t = 8.6s$$

$$\rightarrow 0 = -84 + 9.8\Delta t \Rightarrow \Delta t = 8.6s$$

   d. What is the location of the object, with respect to the origin, when the object comes to rest?

$$\vec{r}_f = \vec{r}_i + \vec{v}_{avg}\Delta t = \langle 0,0,0 \rangle m + \langle 15,-21,0 \rangle \tfrac{m}{s} \times 8.6s = \langle 129,-180.6,0 \rangle m$$

2. In a local drinking establishment a customer slides an empty mug down the counter for a refill. Momentarily distracted, the bartender does not see the mug, which slides off of the counter and strikes the floor *1.4m* from the base of the counter.

   a. Suppose that we take the *x-direction* along the counter in the direction the mug is traveling and the *y-direction* perpendicular to the counter pointing towards the ceiling. Starting with the momentum principle and using the position-update, with what velocity did the mug leave the counter if it hits the floor at a location $\vec{r}_f = \langle 1.4, -0.86, 0 \rangle m$ from the edge of the counter?

$$\vec{v}_f = \langle v_{fx}, v_{fy}, v_{fz} \rangle = \vec{v}_i + \frac{\vec{F}_{net}}{m}\Delta t = \langle v_{ix}, 0, 0 \rangle + \langle 0, -g, 0 \rangle \Delta t = \langle v_{ix}, -g\Delta t, 0 \rangle$$

$$\vec{r}_f = \langle x_f, y_f, z_f \rangle = \vec{r}_i + \vec{v}_{avg}\Delta t = \langle 0,0,0 \rangle + \left\langle v_{ix}, \frac{v_{fy}}{2}, 0 \right\rangle \Delta t$$

$$\text{where, } \vec{v}_{avg} = \frac{\vec{v}_i + \vec{v}_f}{2} = \frac{\langle v_{ix}, 0, 0 \rangle + \langle v_{fx}, v_{fy}, v_{fz} \rangle}{2} = \left\langle v_{ix}, \frac{v_{fy}}{2}, 0 \right\rangle$$

$$\Rightarrow x_f = v_{ix}\Delta t \to \Delta t = \frac{x_f}{v_{ix}}; \quad y_f = \frac{v_{fy}}{2}\Delta t = \left(\frac{-g\Delta t}{2}\right)\left(\frac{x_f}{v_{ix}}\right) = \frac{-gx_f^2}{2v_{ix}^2}$$

$$\therefore v_{ix} = \sqrt{\frac{-2x_f^2}{gy_f}} = \sqrt{\frac{-2 \times (1.4m)^2}{9.8\frac{m}{s^2} \times (-0.86m)}} = 3.34\,\tfrac{m}{s}$$

$$\vec{v}_i = \langle 3.34, 0, 0 \rangle \tfrac{m}{s}$$

   b. What is the time of flight of the projectile from the time it leaves the counter until it reaches the position $\vec{r}_f = \langle 1.4, -0.86, 0 \rangle m$?

$$\Delta t = \frac{x_f}{v_{ix}} = \frac{1.4m}{3.34\frac{m}{s}} = 0.42s$$

   c. Just before the mug strikes the floor, what will be the impact velocity?

$$\vec{v}_f = \vec{v}_i + \frac{\vec{F}_{net}}{m}\Delta t = \langle 3.34, 0, 0 \rangle \tfrac{m}{s} + \langle 0, -9.8\tfrac{m}{s^2}, 0 \rangle (0.42s) = \langle 3.34, -4.12, 0 \rangle \tfrac{m}{s}$$

   d. At what angle with respect to the floor with the mug strike the ground?

$$\tan\theta = \frac{v_{fy}}{v_{fx}} = \frac{-4.12}{3.34} = -1.1234 \to \theta = \tan^{-1}(-1.1234) = -51^o$$

3. 65 million years ago a large meteorite hit the Yucatan Peninsula in Mexico. The *Chicxulub Impact*, also called the *KT* impact, was a $1.3x10^{16}$*-kg* meteorite and is believed to be what caused the mass extinction of the dinosaurs. In addition, the meteorite was about *10,000-m* in diameter and hit the Earth (initially at rest) with an impact velocity of $\vec{v}_{i,A} = \langle 28000,0,0 \rangle \frac{m}{s}$. Studying the impact craters and applying some basic physics allows one to deduce these data.

   a. Starting from the momentum principle, if the mass of the Earth is $6x10^{24}$*-kg*, what recoil speed did the Earth acquire? (Hint: Assume that the system is the Earth and the asteroid and that after the asteroid impacts the Earth, the Earth and asteroid move off together at the same final velocity.)

$$\vec{F}_{net,system} = 0 \rightarrow \vec{p}_{f,system} = \vec{p}_{i,system} \rightarrow m_A \vec{v}_A = (m_A + m_E)\vec{v}_{recoil,E}$$

$$\vec{v}_{recoil.E} = \left(\frac{m_A}{m_A + m_E} \right) \vec{v}_A = \left(\frac{1.3 \times 10^{16} kg}{1.3 \times 10^{16} kg + 6.0 \times 10^{24} kg} \right) \langle 28000,0,0 \rangle \tfrac{m}{s} = \langle 6.1 \times 10^{-5},0,0 \rangle \tfrac{m}{s}$$

   b. What was the impulse that the Earth experienced due to the impact of the asteroid?

$$\vec{I}_E = \Delta p_E = \vec{p}_{f,Earth} - \vec{p}_{i,Earth} = \left\langle \left(1.3 \times 10^{16} kg + 6.0 \times 10^{24} kg\right) \times 6.1 \times 10^{-5},0,0 \right\rangle \tfrac{kgm}{s} - \langle 0,0,0 \rangle \tfrac{kgm}{s}$$

$$\vec{I}_E = \langle 3.64 \times 10^{20},0,0 \rangle Ns$$

   c. What was the impact force on the Earth due to the asteroid if the asteroid takes *2s* to collide and come to rest in the Earth's crust?

$$\vec{I}_E = \vec{F}_{E,A} \Delta t \rightarrow \vec{F}_{E,A} = \frac{\vec{I}_E}{\Delta t} = \frac{\langle 3.64 \times 10^{20},0,0 \rangle Ns}{2s} = \langle 1.82 \times 10^{20},0,0 \rangle N$$

   d. What was the impact force on the asteroid due to the Earth if the asteroid takes *2s* to collide and come to rest in the Earth's crust?

   By Newton's 3rd law: $\vec{F}_{A,E} = -\vec{F}_{E,A} = \langle -1.82 \times 10^{20},0,0 \rangle N$

Useful formulas:

$$\vec{p} = \gamma m\vec{v}$$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\vec{v}_{avg} = \frac{\vec{v}_i + \vec{v}_f}{2}$$

Momentum Principle: $\vec{p}_f = \vec{p}_i + \vec{F}_{net}\Delta t$

Position-update:

$$\vec{r}_f = \vec{r}_i + \vec{v}_{avg}\Delta t = \vec{r}_i + \frac{\vec{p}}{m\sqrt{1+\frac{p^2}{m^2c^2}}}\Delta t$$

Vectors

magnitude of a vector: $|\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$

writing a vector: $\vec{a} = \langle a_x, a_y, a_z \rangle = |\vec{a}|\hat{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

Constants

$g = 9.8 \frac{m}{s^2}$

$G = 6.67 \times 10^{-11} \frac{Nm^2}{kg^2}$

$1e = 1.6 \times 10^{-19} C$

$k = \frac{1}{4\pi\varepsilon_o} = 9 \times 10^9 \frac{C^2}{Nm^2}$

$\varepsilon_o = 8.85 \times 10^{-12} \frac{Nm^2}{C^2}$

$1eV = 1.6 \times 10^{-19} J$

$\mu_o = 4\pi \times 10^{-7} \frac{Tm}{A}$

$c = 3 \times 10^8 \frac{m}{s}$

$h = 6.63 \times 10^{-34} Js$

$m_e = 9.11 \times 10^{-31} kg = \frac{0.511 MeV}{c^2}$

$m_p = 1.67 \times 10^{-27} kg = \frac{937.1 MeV}{c^2}$

$m_n = 1.69 \times 10^{-27} kg = \frac{948.3 MeV}{c^2}$

$1amu = 1.66 \times 10^{-27} kg = \frac{931.5 MeV}{c^2}$

$N_A = 6.02 \times 10^{23}$

$Ax^2 + Bx + C = 0 \rightarrow x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

Geometry

Circles: $C = 2\pi r = \pi D \qquad A = \pi r^2$

Triangles: $A = \frac{1}{2}bh$

Spheres: $A = 4\pi r^2 \qquad V = \frac{4}{3}\pi r^3$