Physics 120 Mid-Term Exam 1

Winter 2012, Jan.26

Name: Solutions Section: 01 & 02

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example

$$|\vec{p}| \approx m\,|\vec{v}| = (5\,kg)\,(2\,m/s) = 10\,kg \cdot m/s$$

- Give standard SI units with your results.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet, including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.

Part	Score
Multiple-Choice and Short-Answer Questions (40 pts)	
Free-Response Problem 1 (14 pts)	
Free-Response Problem 2 (10 pts)	
Free-Response Problem 3 (18 pts)	
Free-Response Problem 4 (8 pts)	
Total (90 pts)	

In keeping with the Union College policy on academic honesty, it is assumed that you will neither accept nor provide unauthorized assistance in the completion of this work.

Part 1: *Multiple-choice and short-answer questions. Please circle the best answer to each multiple-choice question and write a brief answer to each short-answer question.*

1 Point Each

1. (5 points) Write "T" next to each true statement below, and write "F" for every false statement.

   _F_ The change in an object's position can be in a different direction than its average velocity.

   _F_ An object's momentum is always in the same direction as the net force on that object.

   _T_ The change in an object's momentum can be in a different direction than its momentum.

   _F_ If the net force on an object is constant, then the rate of change of its position is constant.

   _T_ If the net force on an object is constant, then the rate of change of its momentum is constant.

2. (4 points) On a map, your house is at position $< 30, 50, 0 >$ m and a nearby convenience store is at position $< 90, -40, 0 >$ m. You walk from your house to the convenience store, which takes you 2 minutes. What is your average velocity vector from your house to the convenience store?

   $\vec{v}_{avg} = \frac{\Delta \vec{r}}{\Delta t} = \frac{\langle 90, -40, 0 \rangle m - \langle 30, 50, 0 \rangle m}{120 s} = \frac{\langle 60, -90, 0 \rangle m}{120 s}$

   $\vec{v}_{avg} = \langle 0.5, -0.75, 0 \rangle m/s$

Treat Like True/False 1 pt each

3. (5 points) A spaceship far from all other objects uses its rockets to attain a speed of 1×10^4 m/s. The crew then shuts off the power. According to Newton's first law, which of the following statements about the motion of the spaceship after the power is shut off are correct? (Choose all statements that are correct.)

   (a) The spaceship will move in a straight line.
   (b) The spaceship will travel on a curving path.
   (c) The speed of the spaceship will not change.
   (d) The spaceship will gradually slow down.
   (e) The spaceship will stop suddenly.

4. (5 points) Which of the following are unit vectors? (Numerical values are given to only 3 significant figures.)

Treat Like True/FAlse 1 pt each

(a) <0, 0, -1> m/s

(b) <0.333, 0.333, 0.333>

(c) <1, -1, 0>

(d) <0.577, 0.577, 0.577>

(e) <0.949, 0, -0.316>

5. (2 points) Two objects, with masses M_A and M_B respectively, are separated by a distance r. They are at a location in space far away from all other objects and these two objects exert a gravitational force on each other with magnitude Fstart.

If we replace object A by an object with twice the mass, and we replace object B by an object 6 times its mass, and these objects are brought closer so that they are now separated by $\frac{1}{2}$ r, the new gravitational force would be given by

a. Fstart

b. 3 Fstart

c. 6 Fstart

d. 24 Fstart

e. 48 Fstart

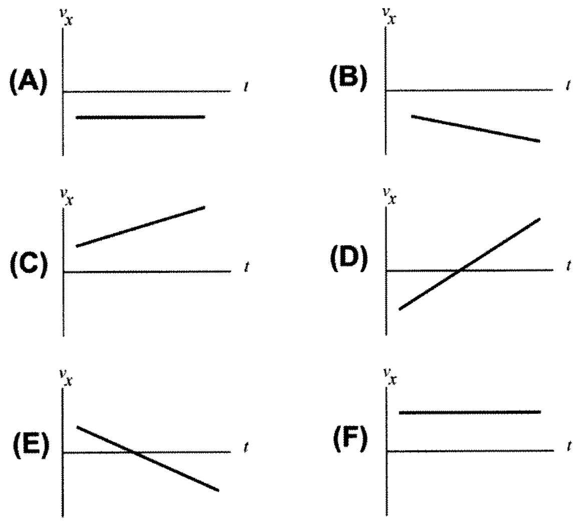
$$F_{old} = \frac{G M_A M_B}{r^2} \qquad F_{new} = \frac{G(2 M_a)(6 M_B)}{(\frac{1}{2} r)^2} = \frac{G M_A M_B}{r^2}\left(\frac{12}{1/4}\right) = 48 F_{old}$$

6. (4 points) A proton ($m_p = 1.67 \times 10^{-27}$ kg) is moving at a constant velocity given by $2.4 \times 10^8 < -1, 0, 0 >$ m/s. What is its momentum?

$$\vec{p} = \gamma m \vec{v} = \frac{1}{\sqrt{1 - \left(\frac{2.4\times10^8}{3\times10^8}\right)^2}} \cdot 1.67\times10^{-27}\,\text{kg} \cdot 2.4\times10^{9}\,\frac{m}{s} \langle -1,0,0\rangle$$

$$= 6.68\times10^{-19}\,\text{kg}\frac{m}{s} \langle -1,0,0\rangle = \langle -6.68\times10^{-19}, 0, 0\rangle \,\text{kg}\frac{m}{s} = \vec{p}$$

7. (7 points) The following graphs plot the x-component of velocity versus time for a cart on a track that lies on x-axis. The graphs are labeled (A) through (F). On the following page are several statements which describe possible motions for the cart on the track. For each statement below, circle the letter (or letters) that correspond to the graph (or graphs) that fit each description. ***There may be more than one correct answer–circle all that apply.*** (A larger copy of the graphs is on the other side of the detachable formula sheet at the back).



The cart moves in the +x direction at constant speed.

(A) (B) (C) (D) (E) (F) None of the graphs

The cart moves in the -x direction at constant speed.

(A) (B) (C) (D) (E) (F) None of the graphs

The cart is stationary; its position does not change.

(A) (B) (C) (D) (E) (F) None of the graphs

1 point each

The cart moves in the +x direction (during the entire time), gradually speeding up.

(A) (B) (C) (D) (E) (F) None of the graphs

The cart moves in the -x direction, gradually slowing down, stops, and then moves in the +x direction, speeding up.

(A) (B) (C) (D) (E) (F) None of the graphs

The x-component of the net force on the cart is changing.

(A) (B) (C) (D) (E) (F) None of the graphs

$\frac{1}{4}$ pt each correct

The x-component of the net force on the cart is constant but nonzero.

(A) (B) (C) (D) (E) (F) None of the graphs

7 pts total

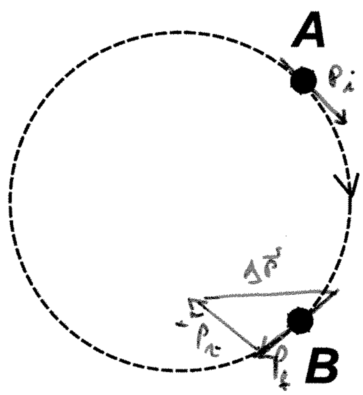
8. (4 points) Consider the following code from a while loop in a VPython program that is designed to make a cart move on a track.

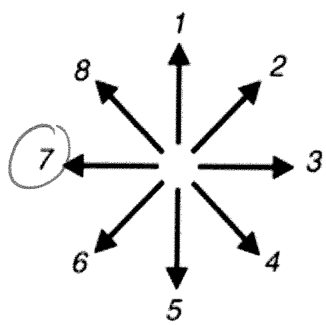
```
while t < 5:
rate(50)
Fnet = vector(-5,0,0)
cart.m = cart.p + Fnet*deltat
cart.pos = cart.pos + (cart.p*cart.m)*deltat
t = t + deltat
```

Some of the code above is incorrect. How could you fix the code so that the program runs correctly? Circle the lines of code above that are incorrect and then provide the correct code next to the incorrect lines. Cart.p ; cart.p/cart.m

2pts 2pts

9. (4 points) A child rides a merry-go-round, traveling from location A to location B, in the clockwise direction as shown, at a constant speed. ***Use the direction diagram to answer parts (a) and (b) of this question.***





9 zero magnitude

(a) What is the direction of $\Delta\vec{p}$, the change in the child's momentum, between locations A and B?

(7)

(b) What is the direction of $\vec{F}_{net}$, the average vector value of the net force acting on the child as she moves from location A to location B?

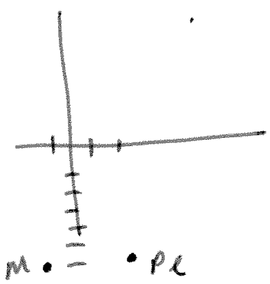
(7) $\vec{F}_{net} \propto \Delta\vec{p}$

Part 2: *Free-response problems.*

1. (14 points) Consider a location in space, far removed from all other objects, in which a planet ($M_{pl} = 5 \times 10^{24}$ kg) located at a position $< 2, -6, 0 > \times 10^6$ m is orbited by a small moon ($M_m = 3 \times 10^{12}$ kg). At a time when the moon is located at a position $< -1, -6, 4 > \times 10^6$ m,

(a) What is the position vector pointing in the direction from the moon to the planet and the magnitude of this position vector?

5pts



$\vec{r} = \vec{r}_{pl} - \vec{r}_m = (\langle 2,-6,0\rangle - \langle -1,-6,4\rangle)\times 10^6 m$

$\vec{r} = \langle 3, 0, -4\rangle \times 10^6 m$

$|\vec{r}| = \sqrt{[3^2 + (-4)^2 + (0)^2](\times 10^6 m)^2} = 5 \times 10^6 m$

(b) What is the unit vector that points in the direction of the gravitational force acting on the moon due to the planet?

2pts

$\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \langle 0.6, 0, -0.8\rangle$

(c) What is the gravitational force (vector) on the moon?

5pts

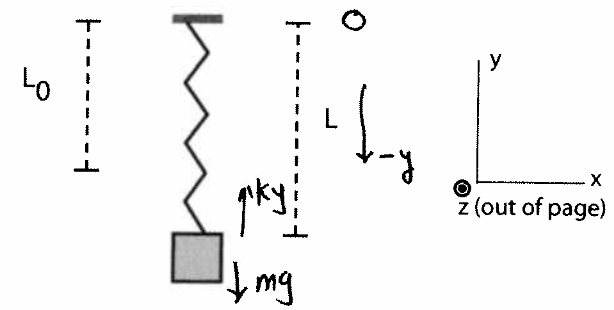
$\vec{F}_{g_{M,pl}} = \frac{G M_m M_p}{r^2}\hat{r} = \frac{(6.67\times 10^{-11} \frac{Nm^2}{kg^2})(3\times 10^{12} kg)(5\times 10^{24} kg)}{(5\times 10^6 m)^2} \langle .6, 0, -.8\rangle$

$\vec{F}_{Mpl} = \langle 2.4, 0, -3.2\rangle \times 10^{13} N$

(d) What is the gravitational force acting on the planet?

2pts

$\vec{F}_{P,m} = -\vec{F}_{M,pl} = \langle -2.4, 0, 3.2\rangle \times 10^{13} N$



2. (10 points) A block with a mass of 0.12 kg hangs vertically on a spring that has a relaxed (or natural) length of $L_0 = 0.30$ m. The length of the spring with the block attached is $L = 0.36$ m when in equilibrium.

(a) What is the spring stiffness (spring constant)?

in the y dir: $\Delta \vec{p}_y = 0 = \vec{F}_{net} \Delta t = 0$

4pts

$\therefore \vec{F}_{net,y} = -ky - mg = 0 \rightarrow k = \frac{mg}{-y} = \frac{mg}{-(-L-(-L_0))}$

where $y = -L-(-L_0)$ by coordinate system given.

$k = \frac{(0.12\,kg)(9.8\,m/s^2)}{-(-0.36+0.30)m} \Rightarrow \boxed{k = 19.6\ N/m}$

(b) The block is pulled down and then released. At some instant, the spring's length is $L = 0.38$ m, and the block is moving downward at a speed of 2 m/s. What is the net force acting on the block at this time? ***Express your answer as a three-component vector.***

4pts

$\vec{F}_{net} = \langle 0, -mg - ky, 0 \rangle$; $y = -L-(-L_0)$

$= \langle 0, -(0.12\,kg)(9.8\,m/s^2) - 19.6\frac{N}{m}(-0.38+0.30)m, 0 \rangle$

$\boxed{F_{net} = \langle 0, +0.392, 0 \rangle N}$

(c) At this instant, the speed of the block is (circle one):

2pts

increasing | **[decreasing]** | not changing

Since $\vec{F}_{net} \uparrow$ and block moving $\downarrow$, $\vec{v} \downarrow$

$\vec{F}_{net}$ opposes $\vec{v}$

and $\Delta \vec{p} < 0$

$\therefore \vec{p}_f - \vec{p}_i < 0$

$\boxed{p_f < p_i}$ → block slows down

3. (18 points) You throw a small rock with a mass of 0.400 kg into the air. At the instant it leaves your hand, the rock's velocity is $\langle 2.00, 3.46, 0.00\rangle$ m/s. (Here, the y-axis is vertical and the x-axis is horizontal.) You may ignore the force of air resistance.

(a) What is the rock's initial momentum, just after it leaves your hand? Express your answer as a vector.

4pts

$\vec{P}_i = \langle mV_{ix}, mV_{iy}, mV_{iz} \rangle$

$= 0.4\,kg \langle 2, 3.46, 0 \rangle$ m/s

$\vec{P}_i = \langle 0.8, 1.38, 0 \rangle$ kgm/s

(b) What is the rock's momentum 0.30 seconds after it leaves your hand? ***Start from a fundamental principle (or else you will not receive full credit).*** Express your answer as a vector.

5pts

$\vec{P}_f = \vec{P}_i + \vec{F}_{net}\Delta t$

$\vec{P}_f = \langle 0.8, 1.38, 0 \rangle \frac{kgm}{s} + \langle 0, -mg, 0 \rangle \Delta t$

$\vec{P}_f = \langle 0.8, 1.38, 0 \rangle \frac{kgm}{s} + \langle 0, -(0.4kg)(9.8 m/s^2), 0 \rangle (0.3s)$

$\vec{P}_f = \langle 0.8, 0.20, 0 \rangle$ kgm/s

(c) Calculate the average velocity of the rock from just after it leaves your hand to 0.30 seconds later. Express your answer as a vector.

5pts

$\vec{V}_{avg} = \frac{\vec{V}_i + \vec{V}_f}{2} = \frac{\vec{P}_i + \vec{P}_f}{2m}$

$= \frac{\langle 0.8, 1.38, 0 \rangle \frac{kgm}{s} + \langle 0.8, 0.2, 0 \rangle \frac{kgm}{s}}{2(0.4)kg}$

$\vec{V}_{avg} = \langle 2, 1.98, 0 \rangle$ m/s

(d) If the rock's initial position just as it leaves your hand is $\langle 0, 1.2, 0\rangle$ m, find the vector position of the ball after 0.30 seconds.

$$\vec{r}_f = \vec{r}_i + \vec{v}_{avg}\,\Delta t$$

$$= \langle 0, 1.2, 0\rangle \text{m} + \langle 2, 1.98, 0 \rangle \tfrac{\text{m}}{\text{s}}\,(0.3\text{s})$$

4pts

$$\boxed{\vec{r}_f = \langle 0.6, 1.79, 0\rangle \text{m}}$$

4. (8 points) Two lumps of clay are thrown into the air and stick together. Lump 1 has mass of 2.00 kg and velocity <0.500,1.50, 0.00> m/s, while lump 2 has mass 3.00 kg and velocity <-1.00, 0.00, 5.00> m/s just prior to colliding. What is the velocity of the combined lump immediately after sticking?

net External force = 0.

$$\therefore \Delta \vec{P}_{system} = 0 \Rightarrow \vec{P}_{i,system} = \vec{P}_{f,system}$$

$$P_{i,system} = m_1 \vec{V}_{1i} + m_2 \vec{V}_{2i}$$

$$= 2\text{kg}\,\langle 0.5, 1.5, 0\rangle \text{m/s} + 3\text{kg}\,\langle -1, 0, 5\rangle \text{m/s}$$

$$= \langle 1, 3, 0\rangle \text{kg m/s} + \langle -3, 0, 15\rangle \text{kg m/s}$$

$$\boxed{\vec{P}_{i\,system} = \langle -2, 3, +15\rangle \text{ kg m/s}}$$

$$\boxed{\vec{P}_{f\,system} = (m_A + m_B)\vec{V}_f = 5\text{kg}\,\vec{V}_f}$$

$$\therefore \vec{P}_{i\,system} = \vec{P}_{f\,system} \Rightarrow \vec{V}_f = \frac{\langle -2, 3, +15\rangle \text{kg m/s}}{5\text{kg}}$$

$$\boxed{\vec{V}_f = \langle -0.4, +0.6, +3\rangle \text{m/s}}$$

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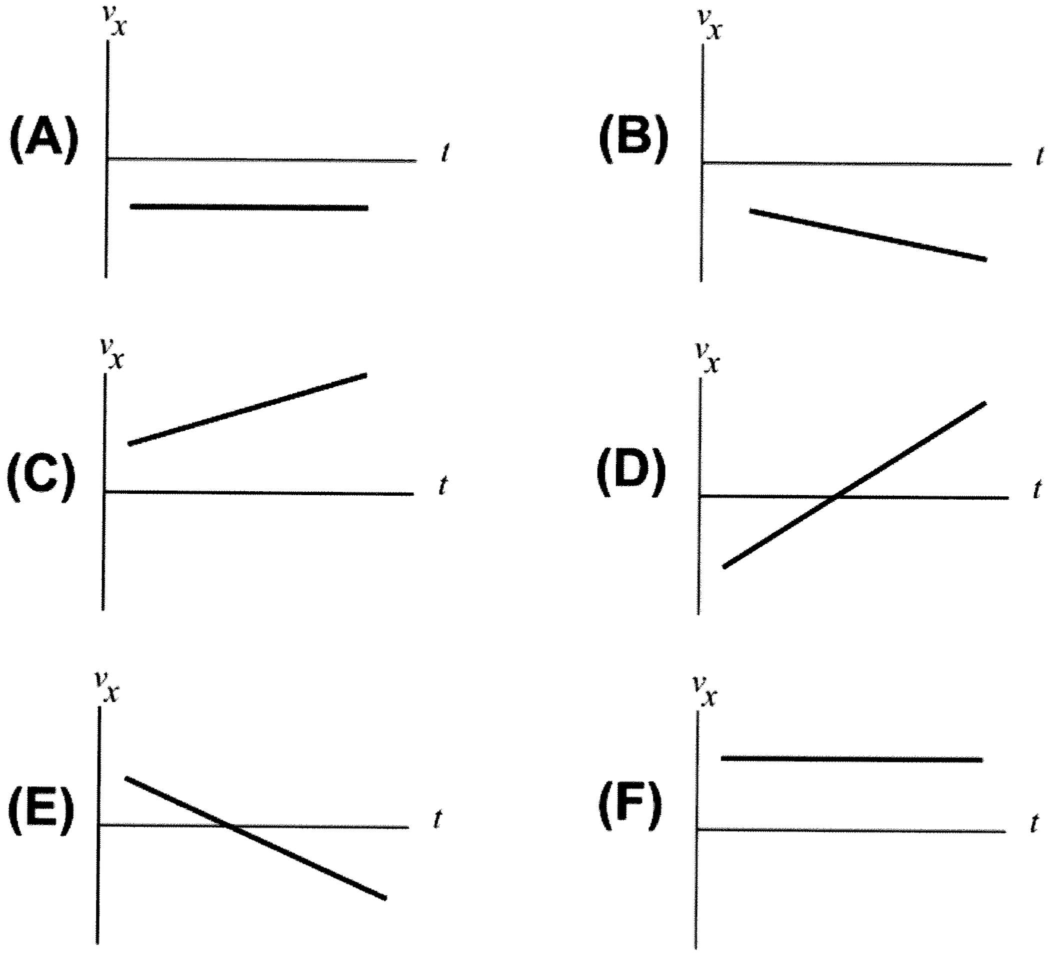
Extra copy of figure for short answer question 7. May be detached.



Equations and Constants

$$\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i} \qquad \vec{r}_f = \vec{r}_i + \vec{v}_{avg}\Delta t$$

$$\vec{p} = \gamma m\vec{v} \qquad \gamma = \frac{1}{\sqrt{1 - \left(\frac{|\vec{v}|}{c}\right)^2}},\ where\ c = 3\times10^8\, m/s$$

$$\Delta\vec{p} = \vec{F}_{net}\Delta t \qquad \vec{p}_f = \vec{p}_i + \vec{F}_{net}\Delta t$$

$$\Delta\vec{p}_{total} = \vec{F}_{net,ext}\Delta t \qquad \Delta\vec{p}_{system} + \Delta\vec{p}_{surroundings} = 0$$

$$\vec{F}_{grav\,on\,2\,by\,1} = -G\frac{m_1 m_2}{|\vec{r}|^2}\hat{r},\ where\ \vec{r} = \vec{r}_2 - \vec{r}_1\ and\ G = 6.7\times10^{-11}N\cdot m^2/kg^2$$

$$\left|\vec{F}_{grav}\right| = mg,\ where\ g = 9.8\,N/kg \qquad \left|\vec{F}_{spring}\right| = k_s\,|s|\,,\ where\ s = L - L_0$$

$$\vec{F}_{elec\,on\,2\,by\,1} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{|\vec{r}|^2}\hat{r},\ \vec{r} = \vec{r}_2 - \vec{r}_1\ and\ \frac{1}{4\pi\varepsilon_0} = 9\times10^9 N\cdot m^2/C^2$$

$$\frac{d\vec{p}}{dt} = \vec{F}_{net}$$

$$\frac{d\vec{P}_{tot}}{dt} = \vec{F}_{net,surr} \qquad \vec{P}_{tot} \approx M\vec{v}_{cm}\ (if\ v \ll c,\ and\ constant\ mass)$$

$$\vec{r}_{cm} \equiv \frac{m_i\vec{r}_i + m_2\vec{r}_2 + m_3\vec{r}_3 + \ldots}{m_1 + m_2 + m_3 + \ldots}$$