Physics 120

Exam #1

January 28, 2022

Name___________________________________

Please read and follow these instructions carefully:

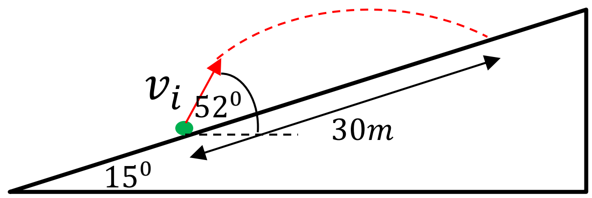
- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So, erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example, $|\vec{p}| \approx m|\vec{v}| = (5kg)\times(2\,\frac{m}{s}) = 10\,\frac{kg\cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- All free-response parts are worth 6 points

Problem #1	/24
Problem #2	/24
Problem #3	/24
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

____

1. Suppose that you are standing on a hill inclined at an angle of 15^0 measured with respect to the horizontal. A ball of mass $m = 0.1kg$ is launched with an initial speed v_i, at an angle of 52^0 (also measured with respect to the horizontal) as shown below. The ball lands $30m$ away along the hill from where it was launched.



a. Assuming a cartesian coordinate system and that $\vec{r}_i = \langle 0.0.0 \rangle m$, what is $\vec{r}_f$?

To use the equations of motion we need to know the horizontal and vertical displacements of the ball, which is the only reason the hill is here.

The horizontal displacement: $\cos 15 = \frac{\Delta x}{30m} \to \Delta x = 30m \cos 15 = 29m$

The vertical displacement: $\sin 15 = \frac{\Delta y}{30m} \to \Delta y = 30m \sin 15 = 7.8m$

$\vec{r}_f = \langle x_f, y_f, z_f \rangle = \langle 29, 7.8, 0 \rangle m$

b. What was the launch speed of the ball?

After the displacements are found, we don't need the hill. This is a projectile launched in the air that travels a horizontal distance x across the ground and a vertical distance y up in the air.

$$\Delta\vec{r} = \vec{v}_i t + \tfrac{1}{2}\vec{a}t^2 = \langle v_{ix}t + \tfrac{1}{2}a_x t^2, v_{iy}t + \tfrac{1}{2}a_y t^2, 0 \rangle$$
$$\Delta x = v_{ix}t + \tfrac{1}{2}a_x t^2 = (v_i \cos\theta)t \to t = \frac{\Delta x}{v_i \cos\theta}$$
$$\Delta y = v_{iy}t + \tfrac{1}{2}a_y t^2 = (v_i \sin\theta)t - \tfrac{1}{2}gt^2 = \Delta x \tan\theta - \frac{g\Delta x^2}{v_i^2 \cos^2\theta}$$

$$7.8m = 29m \tan 52 - \frac{9.8\frac{m}{s^2}(29m)^2}{v_i^2 \cos^2 52} \to v_i = 19.3\frac{m}{s}$$

c. What is the time of flight of the ball from the time it was launched until it impacts the hill?

$$\Delta y = v_{iy}t + \frac{1}{2}a_y t^2 \rightarrow 0 = -\Delta y + v_{iy}t - \frac{1}{2}gt^2 = -7.8 + (19.3\sin 52)t - 4.9t^2$$

$$t = \frac{-15.2 \pm \sqrt{(15.2)^2 - 4(-4.9)(-7.8)}}{-9.8} = \begin{cases} 0.64s \\ 2.45s \end{cases}$$

The time of flight is $2.45s$.

You can also obtain the time of fight from the horizontal motion $\Delta x = v_{ix}t + \frac{1}{2}a_x t^2 = (v_i \cos\theta)t$:

$$t = \frac{\Delta x}{v_i \cos\theta} = \frac{29m}{19.3\frac{m}{s}\cos 52} = 2.44s$$

d. What was the impact velocity of the ball with the hill? You need to either specify the magnitude and direction of the impact velocity or give the components of the final velocity vector.

$$\vec{v}_f = \vec{v}_i + \vec{a}t \rightarrow \langle v_{fx}, v_{fy}, v_{fz} \rangle = \langle v_{ix} + a_x t, v_{iy} + a_y t, 0 \rangle$$

$$v_{fx} = v_{ix} + a_x t = v_{ix} = v_i \cos\theta = 19.3\frac{m}{s}\cos 52 = 11.9\frac{m}{s}$$

$$v_{fx} = v_{ix} + a_x t = (19.3\sin 52) - \left(9.8\frac{m}{s^2}\right) \times 2.45s = -8.9\frac{m}{s}$$

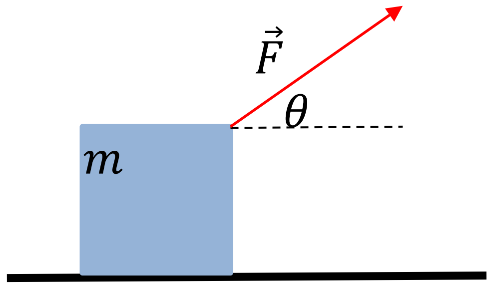
$$\vec{v}_f = \langle v_{fx}, v_{fy}, v_{fz} \rangle = \langle 11.9, -8.9, 0 \rangle \frac{m}{s}$$

Or, as a magnitude and direction:

$$v_f = \sqrt{v_{fx}^2 + v_{fy}^2} = \sqrt{\left(11.9\frac{m}{s}\right)^2 + \left(-8.9\frac{m}{s}\right)^2} = 14.9\frac{m}{s}$$

$$\phi = \tan^{-1}\left(\frac{v_{fy}}{v_{fx}}\right) = \tan^{-1}\left(\frac{-8.9\frac{m}{s}}{11.9\frac{m}{s}}\right) = -36.8^0$$

2. A block of mass m is pulled across a horizontal surface by a constant, externally applied force $\vec{F}$ with magnitude $|\vec{F}| = F$ applied at an angle θ measured with respect to the horizontal as shown on the right. The block accelerates to the right with a magnitude a and friction exists between the block and the horizontal surface with coefficient of friction μ.



a. Write a full vector equation applying Newton's laws to the block above. Make you sure the vector expressions for each vector.

$$\vec{F}_N = \langle F_{Nx}, F_{Ny}, F_{Nz} \rangle = \langle 0, F_N, 0 \rangle$$
$$\vec{F}_W = \langle F_{Wx}, F_{Wy}, F_{Wz} \rangle = \langle 0, -mg, 0 \rangle$$
$$\vec{F}_{fr} = \langle F_{frx}, F_{fry}, F_{frz} \rangle = \langle -F_{fr}, 0,0 \rangle = \langle -\mu F_N, 0,0 \rangle$$
$$\vec{F} = \langle F_x, F_y, F_z \rangle = \langle F \cos\theta, F \sin\theta, 0 \rangle$$

$$\vec{F}_{net} = \vec{F}_N + \vec{F}_W + \vec{F}_{fr} + \vec{F} = m\vec{a}$$
$$\vec{F}_{net} = \langle -\mu F_N + F\cos\theta, F_N - mg + F\sin\theta, 0 \rangle = \langle ma_x, ma_y, ma_z \rangle$$

b. Derive an expression for the coefficient of friction μ between the block and the horizontal surface.

Vertical forces: $F_N + F\sin\theta - mg = ma_y = 0$

$$\rightarrow F_N = mg - F\sin\theta$$

Horizontal forces: $F\cos\theta - \mu F_N = ma_x = ma$

$$\rightarrow \mu = \frac{F\cos\theta - ma}{F_N} = \frac{F\cos\theta - ma}{mg - F\sin\theta}$$

c. If the applied force is large enough (and let's call the magnitude of this largest force F_{max}), the block will lose contact with the horizontal surface. What is the expression for the maximum acceleration a_{max} that the block can attain just before the block loses contact with the horizontal surface? Hint: If the block loses contact with the surface, which force(s) remain and which force(s) do not?

When the block loses contact with the surface, $F_N = 0$.
Vertical forces: $F_N + F_y - F_w = F\sin\theta - mg = ma_y = 0$

$$\rightarrow F = F_{max} = \frac{F_W}{\sin\theta} = \frac{mg}{\sin\theta}$$

Horizontal forces: $F\cos\theta - \mu F_N = F\cos\theta = ma_x = ma$

$$\rightarrow a_{max} = \frac{F_{max}\cos\theta}{m} = \frac{mg\cos\theta}{m\sin\theta} = \frac{g}{\tan\theta}$$

d. Suppose that, at some point in the motion of the block that the constant, externally applied force F is suddenly removed. At the point the applied force F is removed the mass has acquired a speed v_i. Derive expressions for how long will it take the block to come to rest and how far will the block have been displaced when the applied force vanishes? Assume $F < F_{max}$ for this part.

$$\vec{v}_f = \vec{v}_i + \vec{a}t \rightarrow \langle v_{fx}, v_{fy}, v_{fz}\rangle = \langle v_{ix} + a_x t, v_{iy} + a_y t, 0\rangle$$

$$v_{fx} = v_{ix} + a_x t \rightarrow 0 = v_i - \left(\frac{F_{fr}}{m}\right)t \rightarrow t = \frac{mv_t}{F_{fr}} = \frac{mv_i}{\mu F_N} = \frac{mv_i}{\mu mg} = \frac{v_i}{\mu g}$$

Then the trajectory gives:

$$\Delta\vec{r} = \vec{v}_i t + \tfrac{1}{2}\vec{a}t^2 = \langle v_{ix}t + \tfrac{1}{2}a_x t^2, 0, 0\rangle$$

$$\Delta x = v_{ix}t + \tfrac{1}{2}a_x t^2 = v_i\left(\frac{v_i}{\mu g}\right) - \tfrac{1}{2}g\left(\frac{v_i}{\mu g}\right)^2 = \frac{v_i^2}{2\mu g}$$

Or we could use the time-independent equation of motion:

$$v_{fx}^2 = v_{ix}^2 + 2a_x\Delta x \rightarrow 0 = v_{ix}^2 - 2\left(\frac{F_{fr}}{m}\right)\Delta x \rightarrow \Delta x = \frac{mv_{ix}^2}{2F_{fr}} = \frac{mv_{ix}^2}{2\mu F_N} = \frac{mv_i^2}{2\mu mg} = \frac{v_i^2}{2\mu g}$$

3. A train ($m_T = 10000kg$) travels between two stations, A and B. The train starts from rest at station A and accelerates at a constant rate of $\vec{a}_1 = \langle 0.1,0,0 \rangle \frac{m}{s^2}$ for a time t_1 at which point the train operator applies the brakes and the train decelerates to rest at a rate of $\vec{a}_2 = \langle -0.5,0,0 \rangle \frac{m}{s^2}$ for a time t_2 coming to rest at station B. The distance between stations A and B is $5km$.

   a. How long does it take to bring the train to rest from the point the brakes are applied until the train arrives at station B? That is, what is the time t_2? Hints: Start with the expression for the trajectory for the train and express time t_1 in terms of t_2 and assume a standard cartesian coordinate system with the origin at station A.

The train accelerates from rest to speed in a time t_1 at a_1.
Thus $\vec{v}_f = \vec{v}_i + \vec{a}t$ and $\langle v_{fx}, v_{fy}, v_{fz} \rangle = \langle v_{ix} + a_x t, 0,0 \rangle$.
In the x-direction: $v_{fx} = v_{ix} + a_x t \to v_{fx} = v = a_1 t_1$.
The train decelerates from speed to rest in a time t_2 at a_2.
Thus $\vec{v}_f = \vec{v}_i + \vec{a}t$ and $\langle v_{fx}, v_{fy}, v_{fz} \rangle = \langle v_{ix} + a_x t, 0,0 \rangle$.
Again, in the x-direction, $v_f = v_i + a_x t \to 0 = v - a_2 t_2$.

Therefore, $v = a_1 t_1 = a_2 t_2$ and $t_1 = \frac{a_2}{a_1} t_2$.

The trajectory of the train $\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2$. Over t_1, in the x-direction we have $x_{fA} = x_i + v_{ix}t + \frac{1}{2}a_i t_1^2 = \frac{1}{2}a_1 t_1^2$. Over t_2, in the x-direction we have $x_{fB} = x_i + v_{ix}t - \frac{1}{2}a_2 t_2^2 = v_1 t_2 - \frac{1}{2}a_i t_1^2$.

$$x_{total} = x_A + x_B = \left(\tfrac{1}{2}a_1 t_1^2\right) + \left(vt_2 - \tfrac{1}{2}a_2 t_2^2\right) = \tfrac{1}{2}a_1 \left(\frac{a_2}{a_1} t_2\right)^2 + a_2 t_2^2 - \tfrac{1}{2}a_2 t_2^2$$

$$x_{total} = \frac{a_2^2}{2a_1} t_2^2 + \tfrac{1}{2}a_2 t_2^2 = \frac{1}{2}\left(\frac{a_2^2}{a_1} + a_2\right) t_2^2$$

$$t_2 = \sqrt{\frac{2x_{total}}{\frac{a_2^2}{a_1} + a_2}} = \sqrt{\frac{2 \times 5000m}{\frac{\left(0.5\frac{m}{s^2}\right)^2}{0.1\frac{m}{s^2}} + 0.5\frac{m}{s^2}}} = 57.7s$$

b. What was the velocity of the train, $\vec{v}$, just before the brakes were applied and how long (time t_1) did the train accelerate from station A before applying the brakes?

$$v = a_2 t_2 = 0.5\frac{m}{s^2} \times 57.7s = 28.9\frac{m}{s} \rightarrow \vec{v}_f = \langle 28.9,0,0\rangle \frac{m}{s}$$

$$t_1 = \frac{a_2}{a_1} t_2 = \frac{0.5\frac{m}{s^2}}{0.1\frac{m}{s^2}} \times 57.7s = 288.5s$$

c. How far from station A was the train when the brakes were applied? That is, what is $\vec{r}_f$?

$$x_A = x_{iA} + v_{ix,A} t + \frac{1}{2} a_x t^2 = \frac{1}{2} a_1 t_1^2 = \frac{1}{2}\left(0.1\frac{m}{s^2}\right)(288.5s)^2 = 4161.6m$$

d. What is the force that the brakes need to apply, $\vec{F}_B$, to bring the train to rest at station B?

$$\vec{F} = m\vec{a}_2 = 10{,}000kg\langle -0.5,0,0\rangle \frac{m}{s^2} = \langle -5000,0,0\rangle N$$

Physics 120 Formula Sheet

General Definitions of Motion

$\Delta\vec{r} = \langle \Delta x, \Delta y, \Delta z \rangle = \langle x_f - x_i, y_f - y, z_f - z_i \rangle$

$\vec{v} = \frac{\Delta\vec{r}}{\Delta t} = \langle \frac{\Delta x}{\Delta t}, \frac{\Delta y}{\Delta t}, \frac{\Delta z}{\Delta t} \rangle$

$\vec{a} = \frac{\Delta\vec{v}}{\Delta t} = \langle \frac{\Delta v_x}{\Delta t}, \frac{\Delta v_y}{\Delta t}, \frac{\Delta v_z}{\Delta t} \rangle$

$d\vec{r} = \langle dx, dy, dz \rangle$

$\vec{v} = \langle v_x, v_y, v_z \rangle = \frac{d\vec{r}}{dt} = \langle \frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt} \rangle$

$\vec{a} = \langle a_x, a_y, a_z \rangle = \frac{d\vec{v}}{dt} = \langle \frac{dv_x}{dt}, \frac{dv_y}{dt}, \frac{dv_z}{dt} \rangle$

Geometry

$C = 2\pi r \;\; A_{circle} = \pi r^2; \;\; A_{rect} = LW$

$A_{triangle} = \frac{1}{2}bh; \;\; A_{sphere} = 4\pi r^2$

$V_{sphere} = \frac{4}{3}\pi r^3; \;\; V_{cyl} = \pi r^2 h; \;\; V_{cone} = \frac{1}{3}\pi r^2 h$

Motion with constant acceleration

$\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2 \rightarrow \langle x_f, y, z_f \rangle$

$= \langle x_i + v_{ix}t + \frac{1}{2}a_x t^2, y_i + v_{iy}t + \frac{1}{2}a_y t^2, z + v_{iz}t + \frac{1}{2}a_z t^2 \rangle$

$\vec{v}_f = \vec{v}_i + \vec{a}t \rightarrow \langle v_{fx}, v_{fy}, v_{fz} \rangle = \langle v_{ix} + a_x t, v_{iy} + a_y t, v_{iz} + a_z t \rangle$

Forces/Momentum

$\vec{p} = m\vec{v}$

$\vec{F}_{net} = \frac{d\vec{p}}{dt} = m\vec{a}$

$\vec{p}_f - \vec{p}_i = \int d\vec{p} = \int \vec{F}_{net} dt$

$\vec{J} = \int \vec{F}_{net} dt$

$\vec{F}_{net} = \vec{F}_{\parallel} + \vec{F}_{\perp} = \frac{d\vec{p}}{dt}\hat{p} + \vec{p}\frac{d\hat{p}}{dt} = m\vec{a}_{\parallel} + m\vec{a}_{\perp}$

$|\vec{F}_{\perp}| = m|\vec{a}_{\perp}| = m\frac{v^2}{r}$

$\vec{F}_G = G\frac{M_1 M_2}{r_{12}^2}\hat{r}_{12} \rightarrow |\vec{F}_G| = G\frac{M_1 M_2}{r_{12}^2}$

$\vec{F}_G = m\vec{g}; \quad \vec{g} = G\frac{M_{cb}}{(R_{cb} + h)}\hat{r}$

$|\vec{F}_{fr}| = \mu|\vec{F}_N|$

$\vec{F}_s = -k\Delta\vec{r}$

Constants

$g = 9.8\frac{m}{s^2}; \;\; G = 6.67 \times 10^{-11}\frac{Nm^2}{kg^2}$

$v_{sound} = 343\frac{m}{s}; \;\; v_{light} = c = 3 \times 10^8\,\frac{m}{s}$

$N_A = 6.02 \times 10^{23}$

Vectors

$\vec{C} = \vec{A} + \vec{B} \rightarrow \langle C_x, C_y, C_z \rangle = \langle A_x + B_x, A_y + B_y, A_z + B_z \rangle + \langle C_x, C_y, C_z \rangle; \quad \overrightarrow{|C|} = \sqrt{C_x^2 + C_y^2 + C_z^2}$

$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}|\cos\theta = A_x B_x + A_y B_y + A_z B_z$

$\vec{A} \times \vec{B} = |\vec{A}||\vec{B}|\sin\theta = \langle a_y b_z - b_y a_z, a_z b_x - b_z a_x, a_x b_y - b_x a_y \rangle$

Work and Energy

$$W_T = \int dW_T = \int \vec{F} \cdot d\vec{r} = \Delta K_T = \tfrac{1}{2} m v_f^2 - \tfrac{1}{2} m v_i^2 = \frac{p_f^2}{2m} - \frac{p_i^2}{2m}$$

$$W_R = \int dW_R = \int \vec{\tau} \cdot d\vec{\theta} = \Delta K_R = \tfrac{1}{2} I \omega_f^2 - \tfrac{1}{2} I \omega_i^2 = \frac{L_f^2}{2I} - \frac{L_i^2}{2I}$$

$$W_{net} = W_T + W_R = \Delta E_{sys} = \begin{cases} 0 \\ W_{fr} \end{cases}$$

$$W_{net} = -\sum \Delta U = \Delta K_T + \Delta K_R$$

$$U_g = mgy$$

$$U_s = \tfrac{1}{2} k x^2$$

Rotational Motion

$$s = r\theta \rightarrow ds = r d\theta$$

$$\frac{ds}{dt} = r\frac{d\theta}{dt} \rightarrow v = r\omega; \quad \omega = \frac{d\theta}{dt}$$

$$a = \frac{dv}{dt} = r\frac{d\omega}{dt} = r\alpha; \quad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$\theta_f = \theta_i + \omega_i t + \tfrac{1}{2}\alpha t^2$$

$$\omega_f = \omega_i + \alpha t$$

$$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$$

Rotational Forces/Momentum

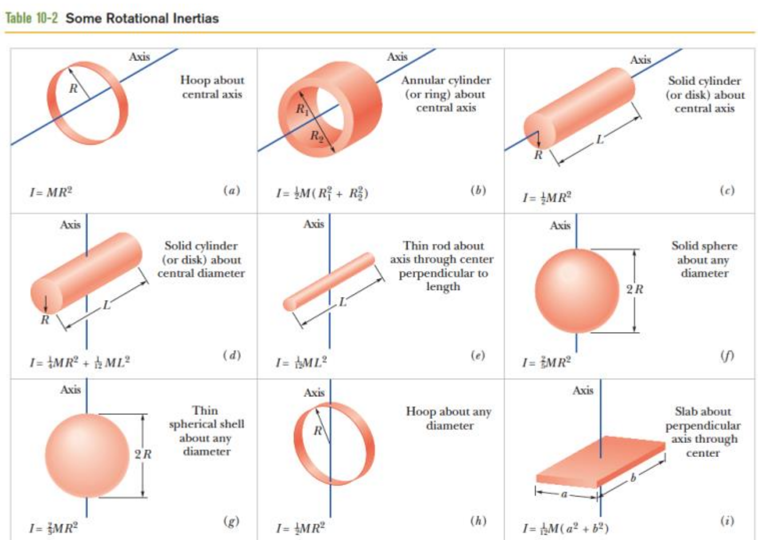
$$\vec{\tau} = \vec{r} \times \vec{F} = \frac{d\vec{L}}{dt} = I\vec{\alpha}$$

$$|\vec{\tau}| = rF\sin\theta = r_\perp F = rF_\perp$$

$$\vec{L} = I\vec{\omega}$$

$$I = \int r^2 dm$$

$$\vec{L}_f = \vec{L}_i + \int \vec{\tau}_{net} dt$$



Some moments of inertia from Halliday, Resnick, & Walker, 10th edition.