Physics 120

Exam #2

May 23, 2014

Name___________________________________

Please read and follow these instructions carefully:

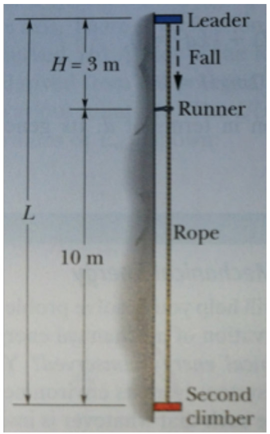
- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example $|\vec{p}| \approx m|\vec{v}| = (5kg) \times (2\,\tfrac{m}{s}) = 10\,\tfrac{kg \cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- All multiple choice questions are worth 4 points and each free-response part is worth 8 points

Problem #1	/20
Problem #2	/28
Problem #3	/20
Problem #4	/28
Total	/96

I affirm that I have carried out my academic endeavors with full academic honesty.

____

1. Rock-climbing is a fun but perhaps very dangerous sport. Suppose that the lead climber (the leader) has an accident and falls from a height H above a runner (a fixed metal loop through which the rope runs) while a second (lower) climber holds fast to limit the fall and does not move. When the falling leader reaches distance H below the runner, the rope begins to stretch. The stretch is a maximum when the leader has fallen an additional distance d and has come to a stop. The force on the rope is then a maximum at magnitude $F_{\max}$. This is the dangerous part of the fall because $F_{\max}$ could be large enough to snap the rope. For any particular rope, the spring constant k depends on the length of rope L and on the elasticity of the rope material to stretch e_{rope}, considered to be constant. Thus, for this example we can write $k = \frac{e_{rope}}{L}$.



Halliday, Resnick, & Walker, Fundamentals of Physics, 7th Ed.

a. What is the maximum stretch of the rope, if $k = 1500\,\frac{N}{m}$ for this rope if your mass is $80kg$?

Assume the system is the rope, person, and the Earth. From the energy principle we have, assuming that $\Delta KE = 0$ is zero because the leader starts and ends at rest and taking the zero of the gravitational potential energy at the runner. Thus we have

$$\Delta E = 0 = \Delta KE + \Delta U_g + \Delta U_s = \left(mgy_f - mgy_i\right) + \left(\tfrac{1}{2}kx_f^2 - \tfrac{1}{2}kx_i^2\right) = \left(-mg(H+d) - mgH\right) + \left(\tfrac{1}{2}kd^2 - 0\right)$$

$$\rightarrow -2mgH - mgd + \tfrac{1}{2}kd^2 = 0$$

$$d = \frac{mg \pm \sqrt{(mg)^2 + 4mgkH}}{k} = \frac{\left(80kg \times 9.8\,\frac{m}{s^2}\right) \pm \sqrt{\left(80kg \times 9.8\,\frac{m}{s^2}\right)^2 + 4\left(80kg \times 9.8\,\frac{m}{s^2}\right) \times 1500 \times 3m}}{1500\,\frac{N}{m}} = \begin{cases} 3m \\ -2m \end{cases}$$

Therefore the maximum stretch is $d = 3m$.

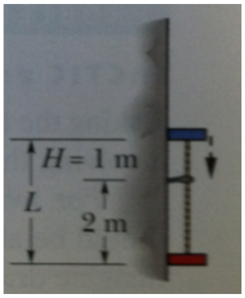
b. What is the magnitude of the maximum force?

The maximum magnitude of the force is given by Hooke's law:

$$F_{\max} = kd = mg + \sqrt{(mg)^2 + 4mgkH} = 1500\,\frac{N}{m} \times 3m = 4500N$$

c. Suppose that you have the situation of a short fall. Compared to the maximum force for the longer fall ($F_{\max,long}$), the maximum force for the short fall ($F_{\max,short}$) is

[✓ 1.] greater than $F_{\max,long}$.
2. less than $F_{\max,long}$.
3. equal to $F_{\max,long}$.
4. unable to be determined from the information given.



Halliday, Resnick, & Walker, Fundamentals of Physics, 7th Ed.

Since $F_{\max} = kd = mg + \sqrt{(mg)^2 + 4mgkH}$, we have as L decreases in the expression for k, k increases as does d. Thus the maximum force increases since both d and k. You can also see this using the numbers in the problem and the result of part b.

2. When one thinks of bones in the human body, one doesn't normally consider bone as a compressible material, but human bones compress by different amounts when various loads (forces) are applied. Consider for example, an average adult male femur or thighbone. The femur has an average length of approximately $L = 48cm$ and a circular cross-sectional area with a diameter of $d = 2.3cm$. When a force is applied over the cross-sectional area of the bone, a stress on the bone is produced and this stress causes the bone to strain under the load. The strain changes the length of the bone of length. We've used Hooke's law do describe elastic materials, or materials that deform under an applied force and when the applied force is removed return to their original shape. Young's modulus for bone is $Y = 16 \times 10^9 \frac{N}{m}$. (*Data are taken from Biomedical Applications of Introductory Physics, by J.A. Tuszynski and J.M. Dixon, Wiley, 2002 and Clinically Oriented Anatomy, Ed. by K.L. Moore & A.F. Dalley, LWW Publishing, 2005*)

   a. Using the generalized expression above for Hooke's law, under normal conditions, by how much do you compress a single femur when you stand upright at rest? Assume that you have a mass of $60kg$.

$$Stress = Y \times Strain \rightarrow \frac{F}{A} = Y \times \frac{\Delta L}{L} \rightarrow \Delta L = \frac{FL}{AY} = \frac{F_W L}{2AY} = \frac{mgL}{2AY}$$

$$\Delta L = \frac{60kg \times 9.8\frac{m}{s^2} \times 0.48m}{2 \times \left(\pi\left(\frac{.0234m}{2}\right)^2\right) \times 16 \times 10^9 \frac{N}{m^2}} = 2.1 \times 10^{-5} m = 0.021mm$$

   Here we are looking at calculating the change in length of a single femur. Each leg supports only one-half of your total weight.

   b. Suppose that you were to somehow compress your femur by $6mm$ (maybe by falling while rock climbing and by the way this amount is more than enough to break your femur). What is the ratio of the force needed to break your leg to your weight?

$$Stress = Y \times Strain \rightarrow \frac{F}{A} = Y \times \frac{\Delta L}{L} \rightarrow F = \frac{Y A \Delta L}{L}$$

$$F = \frac{16 \times 10^9 \frac{N}{m^2} \times \left(\pi\left(\frac{.0234m}{2}\right)^2\right) \times 0.006m}{0.48m} = 8.6 \times 10^4 N = \alpha F_W$$

$$\alpha = \frac{F}{F_W} = \frac{8.6 \times 10^4 N}{60kg \times 9.8\frac{m}{s^2}} = 141$$

c. Starting from rest, through what height would you have to fall landing straight-legged on your feet, so that you could compress a femur by $6mm$? (Hint: Recall Hooke's law from class and you will need to apply this to the generalized form of Hooke's law above to determine a value for the stiffness of your femur.)

From the energy principle we have:

$$\Delta E = 0 = \Delta KE + \Delta U_g + \Delta U_s = \left(\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2\right) + \left(mgy_f - mgy_i\right) + \left(\tfrac{1}{2}kx_f^2 - \tfrac{1}{2}kx_i^2\right)$$

$$0 = \left(\frac{mg}{2}(-d) - \frac{mg}{2}h\right) + \tfrac{1}{2}k(-d)^2$$

$$h = \frac{kd^2}{mg} - d = \frac{1.43\times10^7\,\tfrac{N}{m}\times(0.006m)^2}{60kg\times9.8\,\tfrac{m}{s^2}} - (0.006m) = 0.88m$$

where the effective spring constant of a femur has been determined from the generalized form of Hooke's law,

$$Stress = Y \times Strain \rightarrow \frac{F}{A} = Y \times \frac{\Delta L}{L} \rightarrow F = \frac{AY}{L}\Delta L = k\Delta L$$

$$k = \frac{AY}{L} = \frac{\left(\pi\left(\frac{.0234m}{2}\right)^2\right)\times16\times10^9\,\tfrac{N}{m^2}}{0.48m} = 1.43\times10^7\,\tfrac{N}{m}$$

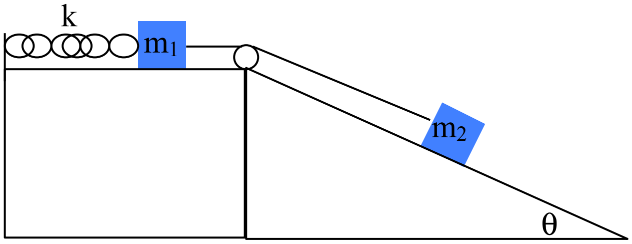
Note also that from problem #1, $e_{rope} = A\Delta y$.

d. Suppose that instead of landing straight-legged as in the previous part you bend your knees while landing. In this case

[✓ 1.] the work done in bringing you to rest would be the same, but the force on your femur would be less because your momentum changes over a larger distance.
2. the work done in bringing you to rest would be the same, but the force on your femur would be greater because your momentum changes over a smaller distance.
3. the work done in bringing you to rest would be the greater, but the force on your femur would be less because your momentum changes over a larger distance.
4. the work done in bringing you to rest would be the smaller, but the force would be greater on your femur because your momentum changes over a smaller distance.

The work done is given by $W = \Delta KE = (-mgh) = F\Delta y$. Dropping from the same height produces the same change in kinetic energy. But bending your legs makes your momentum change over a much larger distance. Therefore the force on each of your femurs is less.

3. Suppose you have the arrangement of masses connected to a spring as shown below where $m_1 = 3kg$, $m_2 = 6kg$, $k = 10\frac{N}{m}$, and the angles are $\theta = 30^0$. The block with mass m_1 sits on a horizontal surface while the block of mass m_2 sits on the incline. Both portions of the track have friction with coefficient of friction $\mu = 0.4$.



a. At what stretch of the spring will the blocks be moving at a speed of $v = 0.8\frac{m}{s}$?

There was an error involving the numbers to this question on the exam. The solutions are actually given by solving the quadratic equation for d - which that part is fine. I would expect two positions for which the speed is $v = 0.8\frac{m}{s}$, one when the blocks are speeding up and one when the blocks are slowing down. However with these numbers (mainly the friction part) there are no real solutions. So, everyone gets full credit on this part and the next.

b. How much energy was lost to the surroundings as heat?

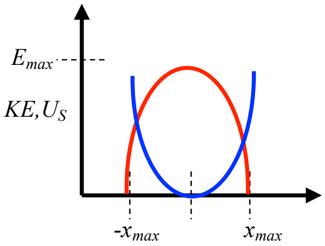
The energy dissipated as heat is due to friction. The work done by friction would be given if d could have been solved for by

$$\Delta E = W_{fr,1} + W_{fr,2} = -\mu F_{N,1} - \mu F_{N,2} = -\mu\left(m_1 gd + m_2 gd\cos\theta\right) = -\mu gd\left(m_1 + m_2\cos\theta\right)$$

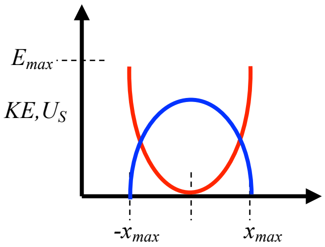
$$\Delta E = -0.4 \times 9.8 \times d \times \left(3kg + 6kg\cos 30\right) = -32.2d$$

c. Suppose that you remove the spring from the system above and place it on a horizontal frictionless surface. You attach one end to a wall and to the other end you put mass m_3. You then stretch the spring by an amount $x_{\max}$ from equilibrium (defined to be $x_{eq} = 0$). When you release the mass from rest, the kinetic and potential energies change according to which of the following graph? (Key: Red = KE and Blue = U_s)

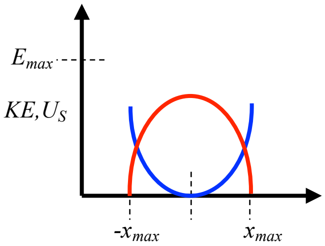
[✓ 1.]



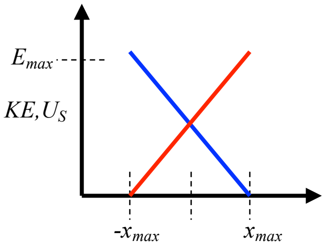
2.



3.



4.

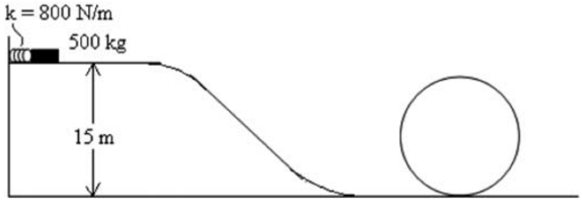


Since there's no friction, at any point the sum of the kinetic and potential energies have to sum to the total energy by:

$$\Delta E = 0 = \Delta Ke + \Delta U_g + \Delta U_s = \left(\tfrac{1}{2}mv^2 - 0\right) + \left(\tfrac{1}{2}kx^2 - \tfrac{1}{2}kx_{\max}^2\right)$$

$$\rightarrow \tfrac{1}{2}kx_{\max}^2 = E_{\max} = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2$$

4. An amusement park thrill ride consists of a cart with some riders (of total mass $500kg$) that are set in motion by a large spring with spring constant $800\,\frac{N}{m}$. The cart and riders travels along the flat horizontal section of track that is located $15m$ above the ground and then down the ramp toward the loop-the-loop, which has an unknown diameter. The entire track is frictionless unless otherwise stated.



a. What will be the speed of the cart and riders half way down the ramp if the spring were initially compressed by $3m$ and there were a friction present only for the launch portion of the ride? (Assume $\mu = 0.2$)

Using the energy principle assuming that the system is the cart, riders, and the earth we have since the only external force is friction

$$\Delta E = W_{fr} = \Delta KE + \Delta U_g + \Delta U_s + \Delta E_{rest} = \left(\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2\right) + \left(mgy_f - mgy_i\right) + \left(\tfrac{1}{2}kx_f^2 - \tfrac{1}{2}kx_i^2\right)$$

$$-\mu F_N = -\mu mgx_i = -\tfrac{1}{2}kx_i^2 + \tfrac{1}{2}mv_f^2 + mgy_f - mgy_i$$

$$v_f = \sqrt{\frac{2\left(mg\left(y_i - y_f\right) + \tfrac{1}{2}kx_i^2 - \mu mgx_i\right)}{m}}$$

$$v_f = \sqrt{\frac{2\left(500kg \times 9.8\,\tfrac{m}{s^2}\left(7.5m - 0.2 \times 3.0m\right) + \tfrac{1}{2} \times 800\,\tfrac{N}{m} \times \left(3.0m\right)^2\right)}{500kg}} = 11\,\tfrac{m}{s}$$

b. If the speed of the cart and riders at the top of the loop were $9\,\frac{m}{s}$, what is the radius of the loop?

Applying the energy principle between the point the cart and riders are half-way down the incline and the point at the top of the loop we have assuming the system is the cart, riders and the earth

$$\Delta E = 0 = \Delta KE + \Delta U_g + \Delta U_s + \Delta E_{rest} = \left(\tfrac{1}{2}mv_t^2 - \tfrac{1}{2}mv_a^2\right) + \left(mgy_t - mgy_a\right)$$

$$y_t = y_a + \tfrac{1}{2g}\left(v_a^2 - v_t^2\right) = 7.5m + \tfrac{1}{2\times 9.8\frac{m}{s^2}}\left(\left(11\tfrac{m}{s}\right)^2 - \left(9\tfrac{m}{s}\right)^2\right) = 9.5m$$

$$radius = \frac{y_t}{2} = 4.8m$$

c. How much work did the gravitational force on the cart and riders?

The work done is the difference in the kinetic energies and we have

$$W = \Delta KE = -\Delta U = \tfrac{1}{2}m\left(v_t^2 - v_b^2\right) = \tfrac{1}{2}\times 500kg\times\left(\left(9\tfrac{m}{s}\right)^2 - \left(16.4\tfrac{m}{s}\right)^2\right) - 4.7\times 10^4 J = -4.70kJ$$

where the speed at the bottom of the loop is calculated using the energy principle and is

$$\Delta E = 0 = \Delta KE + \Delta U_g + \Delta U_s + \Delta E_{rest} = \left(\tfrac{1}{2}mv_b^2 - \tfrac{1}{2}mv_a^2\right) + \left(mgy_b - mgy_a\right)$$

$$v_b = \sqrt{v_a^2 + 2gy_a} = \sqrt{\left(11\tfrac{m}{s}\right)^2 + 2\times 9.8\tfrac{m}{s^2}\times 7.5m} = 16.4\tfrac{m}{s}$$

Or we could explicitly calculate the work done by gravity.

$$W = \int \vec{F}_{net}\cdot d\vec{r} = \int_{y_i}^{y_f}\langle 0,-mg,0\rangle\cdot\langle dx,dy,dz\rangle \int_{y_{i=0}}^{y_{f=2r}} -mg\,dy$$

$$W = -mg\Delta y = -500kg\times 9.8\tfrac{m}{s^2}\times 9.5m = -4.7kJ$$

d. Which of the following expressions gives the normal force on the cart and riders at the top of the loop?

1. $g + \frac{v^2}{R}$
2. $g - \frac{v^2}{R}$
3. 0
4. $mg + \frac{mv^2}{R}$
[✓ 5.] $mg - \frac{mv^2}{R}$
6. $-mg - \frac{mv^2}{R}$

From the momentum principle, we have

$$\vec{F}_{net} = \frac{d\vec{p}}{dt} \rightarrow \langle 0,-mg - F_N,0\rangle = \left\langle 0,-\frac{mv^2}{R}\right\rangle \rightarrow F_N = mg - \frac{mv^2}{R}$$

Physics 120 Equations

$$\vec{r} = < r_x, r_y, r_z > = |\vec{r}| \cdot \hat{r}$$

$$\text{magnitude of a vector}: r = |\vec{r}| = \sqrt{r_x^2 + r_y^2 + r_z^2}$$

$$\text{unit vector}: \hat{r} = \frac{\vec{r}}{|\vec{r}|}$$

$$\vec{v} = \frac{\Delta \vec{r}}{\Delta t}; \; \vec{v}_{avg} = \frac{\vec{v}_i + \vec{v}_f}{2}$$

$$\vec{r}_f = \vec{r}_i + \vec{v}_{avg} \Delta t$$

$$\vec{F}_{net} = \frac{\Delta \vec{p}}{\Delta t}$$

$$\vec{p} = \gamma m \vec{v}; \; \lim_{v<<c}(\vec{p}) \sim m\vec{v}$$

$$\vec{p}_f = \vec{p}_i + \vec{F}_{net} \Delta t$$

$$\vec{r}_f = \vec{r}_i + \vec{v}_i \Delta t + \frac{\vec{F}_{net}}{2m}(\Delta t)^2$$

$$\vec{F}_G = -\frac{GM_1M_2}{r^2}\hat{r}$$

$$\vec{F}_g \sim m\vec{g}$$

Constants:

$$g = 9.8 \tfrac{m}{s^2}$$

$$G = 6.67 \times 10^{-11} \tfrac{Nm^2}{kg^2}$$

$$m_e = 9.11 \times 10^{-31} kg = 0.51 \tfrac{MeV}{c^2}$$

$$m_p = 1.67 \times 10^{-27} kg = 938.5 \tfrac{MeV}{c^2}$$

$$m_E = 6 \times 10^{24} kg$$

$$R_E = 6.4 \times 10^6 m$$

$$N_A = 6.02 \times 10^{23}$$

$$c = 3 \times 10^8 \tfrac{m}{s}$$

$$1eV = 1.6 \times 10^{-19} J$$

$$ax^2 + bx + c = 0 \rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{d\vec{p}}{dt} = \vec{F}_{net}$$

$$\vec{F}_s = -k\vec{s}$$

$$F_{fr} = \mu F_N$$

$$k_{parallel} = \sum_{i=1}^{N} k_i$$

$$\frac{1}{k_{series}} = \sum_{i=1}^{N} \frac{1}{k_i}$$

$$\rho = \frac{m}{V}$$

$$Y = \frac{k_{IAB}}{d}$$

$$\text{Stress} = Y \times \text{Strain}; \; \text{Stress} = \frac{F}{A}; \; \text{Strain} = \frac{\Delta l}{l}$$

$$\vec{F}_{\parallel} = \frac{dp}{dt}\hat{p}$$

$$\vec{F}_{\perp} = p\frac{d\hat{p}}{dt} \rightarrow |\vec{F}_{\perp}| = \frac{mv^2}{R};$$

$$\vec{F}_{Net} = \vec{F}_{\parallel} + \vec{F}_{\perp} = \frac{d\vec{p}}{dt} = \frac{dp}{dt}\hat{p} + p\frac{d\hat{p}}{dt}$$

$$\Delta E = \Delta Q + \Delta W = \Delta KE + \Delta U_g + \Delta U_s + \Delta E_{rest}$$

$$\Delta W = \Delta KE = \int \vec{F} \cdot d\vec{r}; \;\; \vec{F} \cdot d\vec{r} = F_x dx + F_y dy + F_Z dz$$

$$KE = (\gamma - 1)mc^2; \; \lim_{v<<c}(KE) \sim \tfrac{1}{2}mv^2$$

$$E_T = \gamma mc^2; \; E_T^2 = p^2c^2 + m^2c^4; \; E_{rest} = mc^2$$

$$U_g = mgy$$

$$U_s = \tfrac{1}{2}kx^2$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$\text{Inclines: } \; F_N = mg\cos\theta; \; \frac{dv}{dt} = g\sin\theta$$