# Physics 120

## Exam #2

## February 28, 2020

Name___________________________________

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example, $|\vec{p}| \approx m|\vec{v}| = (5kg)\times(2\frac{m}{s}) = 10\frac{kg\cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- All multiple-choice questions are worth 3 points and each free-response part is worth 7 points

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|---|---|
| Problem #1 | /24 |
| Problem #2 | /24 |
| Problem #3 | /24 |
| Total | /72 |

*I affirm that I have carried out my academic endeavors with full academic honesty.*

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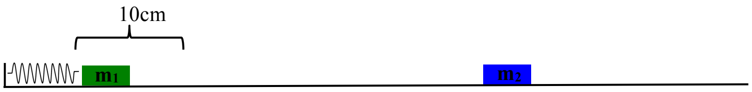
1. A spring of unknown stiffness is compressed by an amount $x = 10cm$ from its equilibrium position at which point mass $m_1 = 3kg$ is placed. The system is released from rest and when the spring reaches its equilibrium position the mass loses contact with the spring. Assume that the horizontal surface is frictionless.



a. Point mass $m_1$ makes a head–on collision with an initially stationary point mass $m_2 = 4kg$. After the collision the two masses move off together with a velocity of $\vec{V} = \langle 4,0,0\rangle\frac{m}{s}$. What is the stiffness of the spring?

$$\vec{p}_i = \vec{p}_f \rightarrow m_1\vec{v}_{i1} + m_2\vec{v}_{i2} = m_1\vec{v}_{f1} + m_2\vec{v}_{f2} \rightarrow m_1\vec{v}_{i1} = (m_1 + m_2)\vec{V}$$

$$\vec{v}_{i1} = \frac{m_1+m_2}{m_1}\vec{V} = \left(\frac{3kg+4kg}{3kg}\right)\langle 4,0,0\rangle\frac{m}{s} = \langle 9.3,0,0\rangle\frac{m}{s}$$

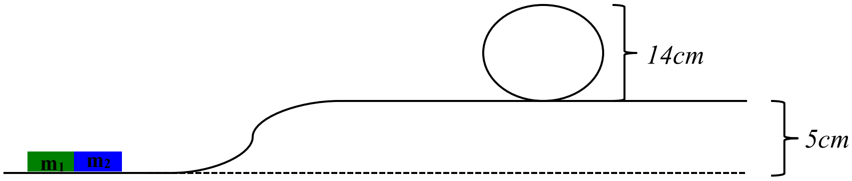
$$\Delta E_{system} = 0 = \Delta K + \Delta U_g + \Delta U_s = \left(\tfrac{1}{2}m_1v_f^2 - \tfrac{1}{2}m_1v_i^2\right) + \left(\tfrac{1}{2}kx_f^2 - \tfrac{1}{2}kx_i^2\right)$$

$$0 = \tfrac{1}{2}m_1v_f^2 - \tfrac{1}{2}kx_i^2 \rightarrow k = \frac{m_1v_f^2}{x_i^2} = \frac{3kg \times \left(9.3\frac{m}{s}\right)^2}{(0.1m)^2} = 2.6 \times 10^4\frac{N}{m}$$

b. The percent of the initial kinetic energy lost to the collision between the system of two point masses $m_1$ and $m_2$ is most likely given by

1. $\% = \left(\frac{m_1}{m_2} - 1\right) \times 100$.
2. $\% = \left(\frac{m_2}{m_1} - 1\right) \times 100$.
3. $\% = \left(\frac{m_1 + m_2}{m_2} - 1\right) \times 100$.
4. [✓ 4.] $\% = \left(\frac{m_1}{m_1 + m_2} - 1\right) \times 100$.
5. None of the above will give the correct expression for the energy lost to the collision.

c. Suppose that after the collision the system of point masses $m_1$ and $m_2$ slide up a $5cm$ hill tall and then around the loop-the-loop with diameter $14cm$. What is the net force on the two point mass system $m_1$ and $m_2$ at a point halfway up the loop-the-loop on the right hand side? Assume that all of the surfaces are frictionless.



$$\Delta E_{system} = 0 = \Delta K + \Delta U_g + \Delta U_s$$
$$0 = \tfrac{1}{2}(m_1 + m_2)\left(v_f^2 - v_i^2\right) + (m_1 + m_2)g\left(y_f - y_i\right)$$
$$v_{f,side}^2 = v_{i,bottom}^2 - 2gy_{side} = \left(4\tfrac{m}{s}\right)^2 - 2 \times 9.8\tfrac{m}{s^2} \times (0.05m + 0.07m) = 13.7\tfrac{m^2}{s^2}$$

$$\vec{F}_{net} = \vec{F}_N + \vec{F}_W = \langle F_N, -F_W, 0\rangle = \langle \tfrac{mv^2}{R}, -mg, 0\rangle$$
$$F_W = -mg = -(3kg + 4kg) \times 9.8 = -68.6N$$
$$F_N = m\frac{v_{side}^2}{R} = (3kg + 4kg)\frac{13.7\frac{m^2}{s^2}}{0.07m} = 1364.8N$$
$$\vec{F}_{net} = \langle 1364.8, -68.6, 0\rangle N$$

d. Suppose that we return to the situation in part a, in which point mass $m_1$ is launched from the spring and makes a head on collision with mass $m_2$. Immediately after the collision, point masses $m_1$ and $m_2$ stick together and encounter an area of space in which the masses are subject to a resistive force $\vec{F} = \langle -\rho x^2, 0,0 \rangle$, where $\rho = 100Nm^2$. If the masses encounter this force at a point $\vec{r_i} = \langle x_i, y_i, z_i \rangle = \langle 0,0,0 \rangle$, how far do the masses slide before coming to rest? That is, what is $\vec{r_f} = \langle x_f, y_f, z_f \rangle$, and in particular, what is the value of $x_f$? Hint: You may need $\int x^n dx = \frac{x^{n+1}}{n+1}$.

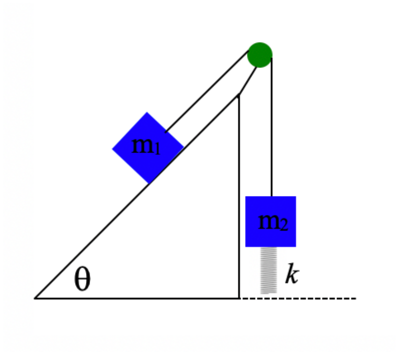
$$W = \int \vec{F} \cdot d\vec{r} = \int_{x_i}^{x_f} (-\rho x^2) dx = -\rho \int_{x_i}^{x_f} x^2 dx = -\frac{\rho}{3}\left(x_f^3 - x_i^3\right) = -\frac{\rho}{3} x_f^3$$

$$W = -\frac{\rho}{3} x_f^3 = \Delta K = \tfrac{1}{2}(m_1+m_2)v_f^2 - \tfrac{1}{2}(m_1+m_2)v_i^2 = -\frac{1}{2}(m_1+m_2)v_i^2$$

$$-\frac{\rho}{3} x_f^3 = -\frac{1}{2}(m_1+m_2)v_i^2 \rightarrow x_f = \sqrt[3]{\frac{3(m_1+m_2)v_i^2}{2\rho}} = \sqrt[3]{\frac{3 \times 7kg \times \left(4\frac{m}{s}\right)^2}{2 \times 100Nm^2}}$$

$$= 1.2m$$

2. Suppose that you have the system of two masses connected to a spring as shown below. The spring is initially at its equilibrium length. The spring-mass system is released from rest and after release, point mass $m_1 = 2kg$ falls and point mass $m_2 = 1kg$ rises. The spring has stiffness $k = 10\frac{N}{m}$ and the track is inclined at $\theta = 55^0$.



a. What is the maximum extension of the spring from its equilibrium length? Assume that there is no friction between the block and the track.

$$\Delta E_{system} = 0 = \Delta K + \Delta U_g + \Delta U_s$$
$$0 = m_1 g\left(y_{1f} - y_{1i}\right) + m_2 g\left(y_{2f} - y_{2i}\right) + \left(\tfrac{1}{2}ky_f^2 - \tfrac{1}{2}ky_i^2\right)$$
$$0 = m_1 g(-y\sin\theta) + m_2 g y + \tfrac{1}{2}ky^2$$

$$\rightarrow y_{min} = 0$$
$$\rightarrow y_{max} = \frac{2(m_1 g\sin\theta - m_2 g)}{k} = \frac{2(2kg\sin 55 - 1kg)\times 9.8\frac{m}{s^2}}{10\frac{N}{m}} = 1.25m$$

b. Suppose that friction existed between block and the ramp. In this case, the maximum extension of the spring would be

[✓ 1.] less than the case without friction.
2. the same as the case without friction.
3. greater than the case without friction.
4. greater in some cases because $m_2$ could be greater than $m_1$.
5. unable to be determined from the information given.

c. Suppose that the system is reset to the initial conditions in part a, except that now there is friction between mass $m_1$ and the ramp with coefficient of friction $\mu_k =$ 0.3. The spring is at its equilibrium length and $m_1$ is released from rest. When the spring has been stretched by an amount $y = 0.3y_{max}$ measured with respect to its equilibrium length, what is the translational speed of mass $m_2$ ? Note, $y_{max}$ is what you determined in part a.

$$\Delta E_{system} = W_{fr} = \Delta K + \Delta U_g + \Delta U_s$$

$$W_{fr} = \int \vec{F}_{fr} \cdot d\vec{r} = \int \langle -\mu_k F_N, 0,0 \rangle \cdot \langle dx, dy, dz \rangle = -(\mu_k m_1 g \cos\theta) y \sin\theta$$

$$W_{fr} = -\mu_k m_1 g y \cos\theta \sin\theta$$
$$W_{fr} = -0.3 \times 2kg \times 9.8\tfrac{m}{s^2} \times 0.3 \times 1.25m \times \cos 55 \sin 55 = -1.03J$$

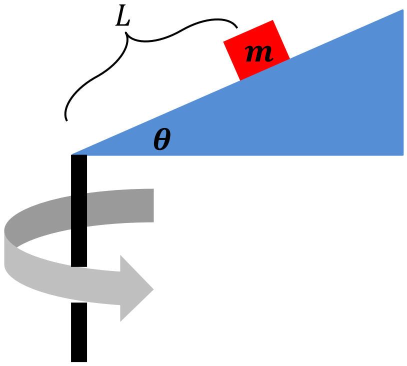
$$\Delta K = \Delta K_1 + \Delta K_2 = \tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2 = \frac{1}{2}(m_1 + m_2) v_f^2 = \frac{1}{2}(2kg + 1kg) v_f^2$$
$$\Delta K = 1.5 v_f^2$$

$$\Delta U_g = \Delta U_{g1} + \Delta U_{g2} = -m_i g y \sin\theta + m_2 g y$$
$$= \left(-2kg \times 9.8\tfrac{m}{s^2} \times 0.3 \times 1.25m \times \sin 55\right) + \left(1kg \times 9.8\tfrac{m}{s^2} \times 0.3 \times 1.25m\right)$$
$$= -2.35J$$

$$\Delta U_s = \tfrac{1}{2} k y_f^2 - \tfrac{1}{2} k y_i^2 = \tfrac{1}{2} k y^2 = \tfrac{1}{2} \times 10\tfrac{N}{m}(0.3 \times 1.25m)^2 = 0.7J$$

$$\Delta E_{system} = W_{fr} = \Delta K + \Delta U_g + \Delta U_s \rightarrow -1.03J = 1.5 v_f^2 - 2.35J + 0.7J$$
$$v_f = \sqrt{\frac{-1.03J + 2.35J - 0.7J}{1.5kg}} = 0.64\tfrac{m}{s}$$

d. Suppose that instead of the masses and spring above, you had the following system A mass $m$ is to stay at rest on the sloping side of the wedge, which is considered to be frictionless. If the wedge is spun at a certain speed by rotating a vertical rod that is firmly attached to the wedge at the bottom end, the mass remains motionless on the wedge. If the mass is to remain at a distance $L$ along the ramp measured from the bottom, derive an expression for the constant speed of the mass?



$$\vec{F}_{net} = \vec{F}_N + \vec{F}_W = \langle F_N \sin\theta, F_N \cos\theta - mg, 0\rangle = \langle m\frac{v^2}{R}, 0,0\rangle$$

y-direction

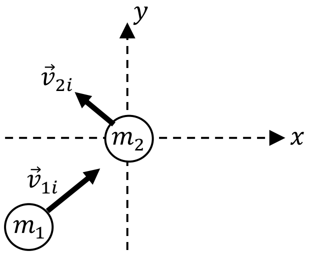
$$F_N \cos\theta - mg = 0 \rightarrow F_N = \frac{mg}{\cos\theta}$$

x-direction

$$F_N \sin\theta = mg\tan\theta = m\frac{v^2}{R} \rightarrow v = \sqrt{Rg\frac{\sin\theta}{\cos\theta}} = \sqrt{gL\cos\theta\left(\frac{\sin\theta}{\cos\theta}\right)}$$

$$\rightarrow v = \sqrt{gL\sin\theta}$$

3. Suppose that two hockey pucks, each of mass $m = 250g$, collide at the origin of a cartesian coordinate system. The first hockey puck $m_1$, is traveling with a velocity $\vec{v}_{1i} = \langle 3,4,0 \rangle \frac{m}{s}$, while the second hockey puck $m_2$ is traveling with velocity $\vec{v}_{2i} = \langle -1,2,0 \rangle \frac{m}{s}$. A schematic of the initial situation is shown on the right.



a. If $m_1$ has a velocity $\vec{v}_{1f} = \langle 2,4,0 \rangle \frac{m}{s}$ after the collision, what speed $|\vec{v}_{2f}|$ does $m_2$ acquire after the collision and at what angle $\phi$ measured with respect to the horizontal, does $m_2$ scatter?

$$\vec{p}_i = \vec{p}_f \rightarrow m_1\vec{v}_{i1} + m_2\vec{v}_{i2} = m_1\vec{v}_{f1} + m_2\vec{v}_{f2}$$
$$\rightarrow \langle 3,4,0 \rangle \frac{m}{s} + \langle -1,2,0 \rangle \frac{m}{s} = \langle 2,4,0 \rangle \frac{m}{s} + \vec{v}_{f2}$$

$$\vec{v}_{f2} = \langle 3,4,0 \rangle \frac{m}{s} + \langle -1,2,0 \rangle \frac{m}{s} - \langle 2,4,0 \rangle \frac{m}{s} = \langle 0,2,0 \rangle \frac{m}{s}$$

$$|\vec{v}_{f2}| = \sqrt{\left(0\frac{m}{s}\right)^2 + \left(2\frac{m}{s}\right)^2} = 12\frac{m}{s}$$
$$\phi = \tan\left(\frac{2\frac{m}{s}}{0\frac{m}{s}}\right) = 90^0$$

b. If the collision time was $\Delta t = 1 \times 10^{-3}s$, what force was exerted on $m_1$ by $m_2$?

$$\vec{F}_{1,2} = \frac{d\vec{p}_1}{dt} = \frac{m_1\vec{v}_{1f} - m_1\vec{v}_{1i}}{\Delta t} = \frac{0.25kg\left[\langle 2,4,0 \rangle \frac{m}{s} - \langle 3,4,0 \rangle \frac{m}{s}\right]}{1 \times 10^{-3}s}$$

$$\vec{F}_{1,2} = \langle -250,0,0 \rangle N$$

Or as a magnitude and a direction, $|\vec{F}_{1,2}| = 250N$ @ $\alpha = 180^0$ with respect to the positive x-axis.

c. During the collision, hockey puck $m_1$exerts a force on hockey puck due to their interaction. Which of the following gives the force on hockey puck $m_2$ during the collision, due to its interaction with hockey puck $m_1$?

1. $\vec{F}_{1,2} = \vec{F}_{2,1}$.
2. $\vec{F}_{1,2} = 2\vec{F}_{2,1}$.
3. $\vec{F}_{1,2} = \frac{1}{2}\vec{F}_{2,1}$.
[✓ 4.] $\vec{F}_{1,2} = -\vec{F}_{2,1}$.
5. None of the above give the correct force relation.

d. What is $\Delta K$ for the collision and based on $\Delta K$ is the collision elastic or inelastic?

$$\Delta K = K_f - K_i$$
$$K_f = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2$$
$$K_f = \frac{1}{2}(0.25kg)\left[\left(\left(2\frac{m}{s}\right)^2 + \left(4\frac{m}{s}\right)^2\right) + \left(\left(0\frac{m}{s}\right)^2 + \left(2\frac{m}{s}\right)^2\right)\right] = 3J$$
$$K_i = \frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2$$
$$K_i = \frac{1}{2}(0.25kg)\left[\left(\left(3\frac{m}{s}\right)^2 + \left(4\frac{m}{s}\right)^2\right) + \left(\left(-1\frac{m}{s}\right)^2 + \left(2\frac{m}{s}\right)^2\right)\right] = 3.75J$$

$\Delta K = K_f - K_i = 3J - 3.75J = -0.75J$.
Thus, since $\Delta K$ is not zero, the collision is inelastic.

# Physics 120 Formula Sheet

## General Definitions of Motion

$$\Delta\vec{r} = \langle \Delta x, \Delta y, \Delta z\rangle = \langle x_f - x_i, y_f - y, z_f - z_i\rangle$$

$$\vec{v} = \tfrac{\Delta\vec{r}}{\Delta t} = \langle \tfrac{\Delta x}{\Delta t}, \tfrac{\Delta y}{\Delta t}, \tfrac{\Delta z}{\Delta t}\rangle$$

$$\vec{a} = \tfrac{\Delta\vec{v}}{\Delta t} = \langle \tfrac{\Delta v_x}{\Delta t}, \tfrac{\Delta v_y}{\Delta t}, \tfrac{\Delta v_z}{\Delta t}\rangle$$

$$d\vec{r} = \langle dx, dy, dz\rangle$$

$$\vec{v} = \langle v_x, v_y, v_z\rangle = \frac{d\vec{r}}{dt} = \langle \frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\rangle$$

$$\vec{a} = \langle a_x, a_y, a_z\rangle = \frac{d\vec{v}}{dt} = \langle \frac{dv_x}{dt}, \frac{dv_y}{dt}, \frac{dv_z}{dt}\rangle$$

## Geometry

$$C = 2\pi r \quad A_{circle} = \pi r^2; \;\; A_{rect} = LW$$

$$A_{triangle} = \tfrac{1}{2}bh; \;\; A_{sphere} = 4\pi r^2$$

$$V_{sphere} = \tfrac{4}{3}\pi r^3; \;\; V_{cyl} = \pi r^2 h; \;\; V_{cone} = \tfrac{1}{3}\pi r^2 h$$

## Motion with constant acceleration

$$\vec{r}_f = \vec{r}_i + \vec{v}_i t + \tfrac{1}{2}\vec{a}t^2 \rightarrow \langle x_f, y, z_f\rangle = \langle x_i + v_{ix}t + \tfrac{1}{2}a_x t^2, y_i + v_{iy}t + \tfrac{1}{2}a_y t^2, z + v_{iz}t + \tfrac{1}{2}a_z t^2\rangle$$

$$\vec{v}_f = \vec{v}_i + \vec{a}t \rightarrow \langle v_{fx}, v_{fy}, v_{fz}\rangle = \langle v_{ix} + a_x t, v_{iy} + a_y t, v_{iz} + a_z t\rangle$$

## Forces/Momentum

$$\vec{p} = m\vec{v}$$

$$\vec{F}_{net} = \tfrac{d\vec{p}}{dt} = m\vec{a}$$

$$\vec{p}_f - \vec{p}_i = \int d\vec{p} = \int \vec{F}_{net}\,dt$$

$$\vec{J} = \int \vec{F}_{net}\,dt$$

$$\vec{F}_{net} = \vec{F}_{\parallel} + \vec{F}_{\perp} = \frac{dp}{dt}\hat{p} + p\frac{d\hat{p}}{dt} = m\vec{a}_{\parallel} + m\vec{a}_{\perp}$$

$$\left|\vec{F}_{\perp}\right| = m\left|\vec{a}_{\perp}\right| = m\frac{v^2}{r}$$

$$\vec{F}_G = G\frac{M_1 M_2}{r_{12}^2}\hat{r}_{12} \rightarrow \left|\vec{F}_G\right| = G\frac{M_1 M_2}{r_{12}^2}$$

$$\vec{F}_G = m\vec{g}; \quad \vec{g} = G\frac{M_{cb}}{(R_{cb} + h)}\hat{r}$$

$$\left|\vec{F}_{fr}\right| = \mu\left|\vec{F}_N\right|$$

$$\vec{F}_s = -k\Delta\vec{r}$$

## Constants

$$g = 9.8\tfrac{m}{s^2}; \;\; G = 6.67 \times 10^{-11}\tfrac{Nm^2}{kg^2}$$

$$v_{sound} = 343\tfrac{m}{s}; \;\; v_{light} = c = 3 \times 10^8\frac{m}{s}$$

$$N_A = 6.02 \times 10^{23}$$

## Vectors

$$\vec{C} = \vec{A} + \vec{B} \rightarrow \langle C_x, C_y, C_z\rangle = \langle A_x + B_x, A_y + B_y, A_z + B_z\rangle + \langle C_x, C_y, C_z\rangle; \;\; \overrightarrow{|C|} = \sqrt{C_x^2 + C_y^2 + C_z^2}$$

$$\vec{A} \cdot \vec{B} = \left|\vec{A}\right|\left|\vec{B}\right|\cos\theta = A_x B_x + A_y B_y + A_z B_z$$

$$\vec{A} \times \vec{B} = \left|\vec{A}\right|\left|\vec{B}\right|\sin\theta = \langle a_y b_z - b_y a_z, a_z b_x - b_z a_x, a_x b_y - b_x a_y\rangle$$

## Work and Energy

$$W_T = \int dW_T = \int \vec{F} \cdot d\vec{r} = \Delta K_T = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 = \frac{p_f^2}{2m} - \frac{p_i^2}{2m}$$

$$W_R = \int dW_R = \int \vec{\tau} \cdot d\vec{\theta} = \Delta K_R = \tfrac{1}{2}I\omega_f^2 - \tfrac{1}{2}I\omega_i^2$$

$$W_{net} = W_T + W_R = \Delta E_{sys} = \begin{cases} 0 \\ W_{fr} \end{cases}$$

$$W_{net} = -\sum \Delta U = \Delta K_T + \Delta K_R$$

$$U_g = mgy$$

$$U_s = \tfrac{1}{2}kx^2$$

## Rotational Motion

$$s = r\theta \rightarrow ds = rd\theta$$

$$\frac{ds}{dt} = r\frac{d\theta}{dt} \rightarrow v = r\omega; \quad \omega = \frac{d\theta}{dt}$$

$$a = \frac{dv}{dt} = r\frac{d\omega}{dt} = r\alpha; \quad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$\theta_f = \theta_i + \omega_i t + \tfrac{1}{2}\alpha t^2$$

$$\omega_f = \omega_i + \alpha t$$

$$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$$

## Rotational Forces/Momentum

$$\vec{\tau} = \vec{r} \times \vec{F} = \frac{d\vec{L}}{dt} = I\vec{\alpha}$$

$$\tau = rF\sin\theta = r_\perp F = rF_\perp$$

$$\vec{L} = I\vec{\omega}$$

$$I = \int r^2 dm$$

$$\vec{L}_f = \vec{L}_i + \int \vec{\tau}_{net} dt$$