Physics 120

Exam #2

February 25, 2022

Name___________________________________

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So, erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example, $|\vec{p}| \approx m|\vec{v}| = (5kg) \times (2\,\frac{m}{s}) = 10\,\frac{kg \cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- All free-response parts are worth 6 points

Problem #1	/24
Problem #2	/24
Problem #3	/24
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

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1. A karate expert strikes downward with her fist of mass $m_{fist} = 0.7kg$ breaking a stationary brick with $m_{brick} = 3.2kg$. The stiffness constant for the brick is $k = 2.6 \times 10^6 \frac{N}{m}$ and the brick breaks at a deflection $d = 1.5mm$.

   a. How much work did the brick do bringing your hand to rest, just as the brick breaks?

   $$W = -\Delta U_S$$
   $$W - \left(\tfrac{1}{2}ky_f^2 - \tfrac{1}{2}ky_i^2\right)$$
   $$W = -\tfrac{1}{2}ky_f^2 = -\tfrac{1}{2}kd^2$$
   $$W = -\tfrac{1}{2} \times 2.6 \times 10^6 \tfrac{N}{m} \times (0.0015m)^2$$
   $$W = -2.925J$$

   b. How fast are *the brick and your fist* ($v_{fist+brick}$) moving just after you strike the brick.

   $$W = \Delta K + \Delta U_g$$
   $$= \left(\tfrac{1}{2}m_{fist+brick}v_{f,b+f}^2 - \tfrac{1}{2}m_{fist+brick}v_{i,b+f}^2\right) + \left(m_{fist+brick}gy_f - m_{fist+brick}gy_i\right)$$
   $$W = -m_{fist+bric}gy_f - \tfrac{1}{2}m_{fist+brick}v_{i,b+f}^2$$
   $$v_{i,f+b} = \sqrt{-\frac{2W+2m_{fist+bric}gy_f}{m_{fist+brick}}} = \sqrt{-\frac{2\times(-2.982J)+2\times(0.7kg+3.2kg)\times 9.8\frac{m}{s^2}\times 0.0015m}{0.7kg+3.2kg}}$$
   $$v_{i,f+b} = 1.22\frac{m}{s}$$

c. With what minimum speed would the karate expert's hand be moving (v_{fist}) before it collides with the brick so that she can break this karate brick? (Hint: just after your strike the brick, your hand and brick are moving at the same speed.)

$$\Delta\vec{p} = 0 \rightarrow \vec{p}_f - \vec{p}_i = 0 \rightarrow \vec{p}_i = \vec{p}_f$$
$$\langle 0, -m_{fist}v_{fist}, 0\rangle = \langle 0, -m_{fist}v_{fist+board}, 0\rangle + \langle 0, -m_{board}v_{fist+board}, 0\rangle$$

$$m_{fist}v_{iist} = (m_{fist} + m_{board})v_{ffist+board}$$

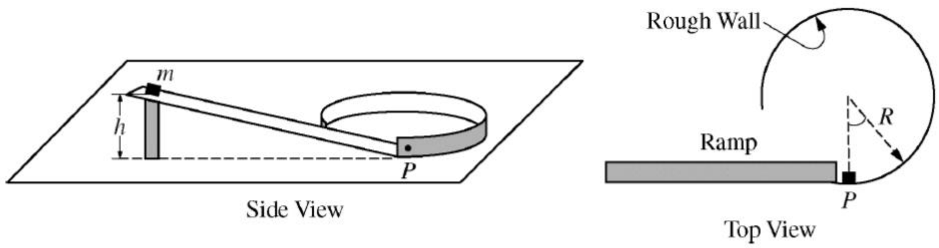
$$v_{fist} = \frac{(m_{fist}+m_{board})v_{fist+board}}{m_{fist}} = \left(\frac{0.7kg+3.2kg}{0.7kg}\right)1.22\frac{m}{s} = 6.8\frac{m}{s}$$

d. During the collision you exert a force on the brick and the brick exerts a force back on your fist. Assuming that this force (during the collision) is the largest force that acts, what was the collision time between your fist and the board?

$$\vec{F}_{fist,board} = \frac{\Delta\vec{p}_{fist}}{\Delta t} \rightarrow \left|\vec{F}_{fist,board}\right| = \left|\frac{\Delta\vec{p}_{fist}}{\Delta t}\right| \rightarrow F_{fist,board} = \left|\frac{p_{f,fist}-p_{i,fist}}{\Delta t}\right|$$

$$\Delta t = \frac{m_{fist}v_{fist}-m_{fist}v_{fist+board}}{kd} = \frac{0.7kg(6.8\frac{m}{s}-1.2\frac{m}{s})}{2.6\times10^6\frac{N}{m}\times0.0015m} = 0.001s = 1ms$$

2. A block of mass $m = 0.5kg$ is released from rest from a height $h = 1m$ above a horizontal table. The block slides down the ramp, inclined at an angle $\theta = 70^0$ measured with respect to the horizontal.



a. Using energy ideas, and assuming that the acceleration of the block is constant down the ramp, what is the speed of the block at the bottom of the ramp if friction exists along the ramp with coefficient of friction $\mu = 0.2$?

By Forces: Assuming a tilted coordinate system with the positive y-direction perpendicular to the ramp and up away from the ground and the positive x-axis parallel to the ramp and toward the ground.

Vertical forces: $\sum F_y\text{: } F_N - F_{wy} = ma_y = 0 \rightarrow F_N = F_w \cos\theta = mg\cos\theta$

Horizontal forces: $\sum F_x\text{: } -F_{fr} + F_{wx} = ma_x \rightarrow a = \frac{F_w \sin\theta - \mu F_N}{m}$

$a = \frac{mg\sin\theta - \mu mg\cos\theta}{m} = (\sin\theta - \mu\cos\theta)g = (\sin 70 - 0.2\cos 70) \times 9.8\frac{m}{s^2}$

$a = 8.5\frac{m}{s^2}$

$v_{fx}^2 = v_{ix}^2 + 2a_x\Delta x = 2a_x\Delta x \rightarrow v_{fx} = \sqrt{2a_x\Delta x} = \sqrt{2 \times 8.5\frac{m}{s^2} \times 1.06m}$

$v_{fx} = 4.3\frac{m}{s}$

Where the distance down the ramp is given by:

$$\sin\theta = \frac{h}{\Delta x} \rightarrow \Delta x = \frac{h}{\sin\theta} = \frac{1m}{\sin 70} = 1.06m$$

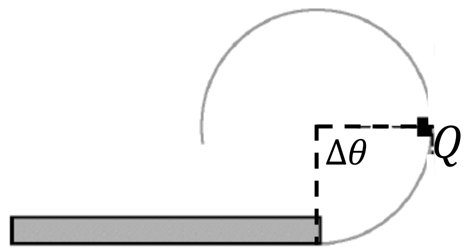
Or by Work-Energy:

$$\Delta E = W_{fr} = -\mu mg\cos\theta\left(\frac{h}{\sin\theta}\right) = \Delta K + \Delta U_g + \Delta U_s$$

$$-\mu mg\cos\theta\left(\frac{h}{\sin\theta}\right) = \left(\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2\right) + \left(mgy_f - mgy_i\right) = \tfrac{1}{2}mv_f^2 - mgy_i$$

$$v_f = \sqrt{2gh - \mu g\cos\theta\left(\frac{h}{\sin\theta}\right)} = \sqrt{2 \times 9.8\frac{m}{s^2} \times 1m - 2 \times 0.2 \times 9.8\frac{m}{s^2}\frac{1m}{\tan 70}} = 4.3\frac{m}{s}$$

b. When the block reaches the bottom of the incline it slides along the horizontal surface of the table guided in the circular path by a circular wall of radius $R = 0.5m$. Three ***is*** friction between the block and the wall ($\mu = 0.2$) but ***not*** between the block and the floor. Since there is friction between the block and the wall the block slows down as it moves along the table due to the block's interaction with the wall. What is the speed of the block when it is at point Q. Assume that the block has moved through an angle $\Delta\theta = \frac{\pi}{2}\ radians$. The displacement of the block along the wall is given by $\Delta x = R\Delta\theta$, for $\Delta\theta$measured in radians. Also, assume that the acceleration of the block along the wall is constant.



Top View

The distance traveled by the block is: $\Delta x = 0.5m \times \frac{\pi}{2} = \frac{\pi}{4}m$

$$\Delta E = W_{fr} = \Delta K \rightarrow \mu F_N \Delta x \cos\phi = \mu F_N \Delta x \cos 180 = \frac{1}{2}mv_Q^2 - \frac{1}{2}mv_i^2$$

Here, the normal force is from the wall, and this gives rise to the centripetal acceleration of the block.

$$-\mu F_N \Delta x = -\mu\left(\frac{mv_Q^2}{R}\right)\Delta x = \frac{1}{2}mv_Q^2 - \frac{1}{2}mv_i^2$$

$$v_Q = \sqrt{\frac{v_i^2}{1 + \frac{2\mu}{R}\Delta x}} = \sqrt{\frac{\left(4.3\frac{m}{s}\right)^2}{1 + \frac{2 \times 0.2 \times \frac{\pi}{4}m}{0.5m}}} = 3.4\frac{m}{s}$$

c. What is the magnitude of the normal force from the wall on the block at point Q?

$$F_N = m\frac{v_Q^2}{R} = 0.5kg \times \frac{\left(3.4\frac{m}{s}\right)^2}{0.5m} = 11.3N$$

d. What is the magnitude and direction of the net acceleration of the block at point Q?

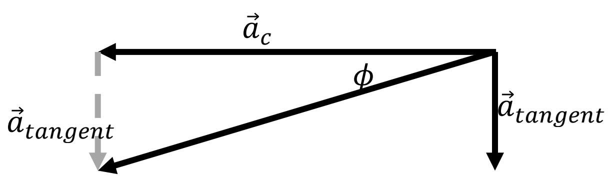
$$\vec{a}_{net} = \vec{a}_c + \vec{a}_{tangent}$$

$$a_c = |\vec{a}_c| = \frac{F_N}{m} = \frac{11.3N}{0.5kg} = 22.1\frac{m}{s^2}$$

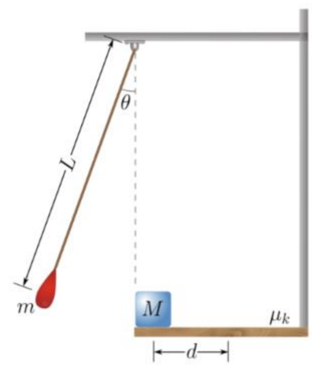
$$a_{tangent} = \left|\vec{a}_{tangent}\right| = \frac{F_{fr}}{m} = \frac{\mu F_N}{m} = \frac{\mu m g}{m} = \mu g = 0.2 \times 9.8\frac{m}{s^2} = 1.96\frac{m}{s^2}$$

$$a_{tangent} = \left|\vec{a}_{tangent}\right| = \sqrt{a_c^2 + a_{tangent}^2} = \sqrt{\left(22.1\frac{m}{s^2}\right)^2 + \left(1.96\frac{m}{s^2}\right)^2} = 22.2\frac{m}{s^2}$$

$\phi = \tan^{-1}\frac{a_{tangent}}{a_c} = \tan^{-1}\frac{1.96\frac{m}{s^2}}{22.1\frac{m}{s^2}} = 4.9^0$below the negative x-axis.



3. A laboratory experiment is conducted to measure coefficients of friction. A small balloon containing sand with a combined mass of $m = 95g$ is suspended from a string of length $L = 81cm$. When released from rest the pendulum swings through an angle of $\theta = 46^0$ and collides with a block of mass $M = 350g$ located where the pendulum string becomes vertical. After the collision with the balloon, the block slides along the horizontal surface coming to rest after a distance $d = 5cm$ as shown on the right.



a. What is the speed v_{im} of the balloon filled with sand just before the collision with the block?

$$\Delta E_{sys} = \Delta K + \Delta U_g + \Delta U_s = 0$$
$$0 = \left(\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2\right) + \left(mgy_f - mgy_i\right) = \tfrac{1}{2}mv_f^2 + mg(L - L\cos\theta)$$
$$v = \sqrt{2gL(1-\cos\theta)} = \sqrt{2 \times 9.8\tfrac{m}{s^2} \times 0.81m(1 - \cos 46)} = 2.2\tfrac{m}{s}$$

b. Suppose immediately after the collision with the balloon filled with sand, the block is observed to be moving with a speed $v_M = 0.8\frac{m}{s}$. What is the coefficient of friction μ_k between the block and the horizontal surface? Hint: The balloon filled with sand does not stick to the block.

$$\Delta E_{sys} = \Delta K + \Delta U_g + \Delta U_s = W_{fr} \rightarrow \Delta K = W_{fr}$$
$$\left(\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2\right) = F_{fr}d\cos\phi = \mu F_N d\cos 180 = -\mu F_N d = -\mu mgd$$
$$-\tfrac{1}{2}mv_i^2 = -\mu mgd \rightarrow \mu = \frac{v_i^2}{2gd} = \frac{\left(0.8\frac{m}{s}\right)^2}{2 \times 9.8\frac{m}{s^2} \times 0.05m} = 0.65$$

c. What is the momentum (magnitude and direction) of the balloon filled with sand immediately after the collision?

$\Delta\vec{p} = 0 \rightarrow \vec{p}_f - \vec{p}_i = 0 \rightarrow \vec{p}_i = \vec{p}_f$
$\langle mv_{im}, 0,0 \rangle = \langle mv_{fm}, 0,0 \rangle + \langle Mv_{fM}, 0,0 \rangle$

$mv_{im} = mv_{fm} + Mv_{fM} \rightarrow v_{fm} = \frac{mv_{im} - Mv_{fM}}{m} = \frac{0.095kg \times 2.2\frac{m}{s} - 0.35kg \times 0.8\frac{m}{s}}{0.095kg}$

$v_{fm} = -0.75\frac{m}{s}$

d. What is the change in kinetic energy during the collision and from this, what type of collision occurred between the balloon filled with sand and the bock?

$K_i = \frac{1}{2}mv_{im}^2 = \frac{1}{2} \times 0.095kg \times \left(2.2\frac{m}{s}\right)^2 = 0.2299J$

$K_f = \frac{1}{2}mv_{fm}^2 + \frac{1}{2}Mv_{fM}^2 = \frac{1}{2} \times 0.095kg \times \left(-0.75\frac{m}{s}\right)^2 + \frac{1}{2} \times 0.350kg \times \left(0.8\frac{m}{s}\right)^2$
$K_f = 0.1390J$

$\Delta K = K_f - K_i = 0.1390J - 0.2299J = -0.091J$
Since the change in the energy of motion is not zero, the collision must be inelastic.

Physics 120 Formula Sheet

General Definitions of Motion

$\Delta\vec{r} = \langle \Delta x, \Delta y, \Delta z \rangle = \langle x_f - x_i, y_f - y, z_f - z_i \rangle$

$\vec{v} = \frac{\Delta\vec{r}}{\Delta t} = \langle \frac{\Delta x}{\Delta t}, \frac{\Delta y}{\Delta t}, \frac{\Delta z}{\Delta t} \rangle$

$\vec{a} = \frac{\Delta\vec{v}}{\Delta t} = \langle \frac{\Delta v_x}{\Delta t}, \frac{\Delta v_y}{\Delta t}, \frac{\Delta v_z}{\Delta t} \rangle$

$d\vec{r} = \langle dx, dy, dz \rangle$

$\vec{v} = \langle v_x, v_y, v_z \rangle = \frac{d\vec{r}}{dt} = \langle \frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt} \rangle$

$\vec{a} = \langle a_x, a_y, a_z \rangle = \frac{d\vec{v}}{dt} = \langle \frac{dv_x}{dt}, \frac{dv_y}{dt}, \frac{dv_z}{dt} \rangle$

Geometry

$C = 2\pi r \;\; A_{circle} = \pi r^2; \;\; A_{rect} = LW$

$A_{triangle} = \frac{1}{2}bh; \;\; A_{sphere} = 4\pi r^2$

$V_{sphere} = \frac{4}{3}\pi r^3; \;\; V_{cyl} = \pi r^2 h; \;\; V_{cone} = \frac{1}{3}\pi r^2 h$

Motion with constant acceleration

$\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2 \rightarrow \langle x_f, y, z_f \rangle$

$= \langle x_i + v_{ix}t + \frac{1}{2}a_x t^2, y_i + v_{iy}t + \frac{1}{2}a_y t^2, z + v_{iz}t + \frac{1}{2}a_z t^2 \rangle$

$\vec{v}_f = \vec{v}_i + \vec{a}t \rightarrow \langle v_{fx}, v_{fy}, v_{fz} \rangle = \langle v_{ix} + a_x t, v_{iy} + a_y t, v_{iz} + a_z t \rangle$

Forces/Momentum

$\vec{p} = m\vec{v}$

$\vec{F}_{net} = \frac{d\vec{p}}{dt} = m\vec{a}$

$\vec{p}_f - \vec{p}_i = \int d\vec{p} = \int \vec{F}_{net} dt$

$\vec{J} = \int \vec{F}_{net} dt$

$\vec{F}_{net} = \vec{F}_\parallel + \vec{F}_\perp = \frac{d\vec{p}}{dt}\hat{p} + \vec{p}\frac{d\hat{p}}{dt} = m\vec{a}_\parallel + m\vec{a}_\perp$

$|\vec{F}_\perp| = m|\vec{a}_\perp| = m\frac{v^2}{r}$

$\vec{F}_G = G\frac{M_1 M_2}{r_{12}^2}\hat{r}_{12} \rightarrow |\vec{F}_G| = G\frac{M_1 M_2}{r_{12}^2}$

$\vec{F}_G = m\vec{g}; \quad \vec{g} = G\frac{M_{cb}}{(R_{cb} + h)}\hat{r}$

$|\vec{F}_{fr}| = \mu|\vec{F}_N|$

$\vec{F}_s = -k\Delta\vec{r}$

Constants

$g = 9.8\frac{m}{s^2}; \;\; G = 6.67 \times 10^{-11}\frac{Nm^2}{kg^2}$

$v_{sound} = 343\frac{m}{s}; \;\; v_{light} = c = 3 \times 10^8 \frac{m}{s}$

$N_A = 6.02 \times 10^{23}$

Vectors

$\vec{C} = \vec{A} + \vec{B} \rightarrow \langle C_x, C_y, C_z \rangle = \langle A_x + B_x, A_y + B_y, A_z + B_z \rangle + \langle C_x, C_y, C_z \rangle; \quad \overrightarrow{|C|} = \sqrt{C_x^2 + C_y^2 + C_z^2}$

$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}|\cos\theta = A_x B_x + A_y B_y + A_z B_z$

$\vec{A} \times \vec{B} = |\vec{A}||\vec{B}|\sin\theta = \langle a_y b_z - b_y a_z, a_z b_x - b_z a_x, a_x b_y - b_x a_y \rangle$

Work and Energy

$$W_T = \int dW_T = \int \vec{F} \cdot d\vec{r} = \Delta K_T = \tfrac{1}{2} m v_f^2 - \tfrac{1}{2} m v_i^2 = \frac{p_f^2}{2m} - \frac{p_i^2}{2m}$$

$$W_R = \int dW_R = \int \vec{\tau} \cdot d\vec{\theta} = \Delta K_R = \tfrac{1}{2} I \omega_f^2 - \tfrac{1}{2} I \omega_i^2 = \frac{L_f^2}{2I} - \frac{L_i^2}{2I}$$

$$W_{net} = W_T + W_R = \Delta E_{sys} = \begin{cases} 0 \\ W_{fr} \end{cases}$$

$$W_{net} = -\sum \Delta U = \Delta K_T + \Delta K_R$$

$$U_g = mgy$$

$$U_s = \tfrac{1}{2} k x^2$$

$$\Delta E_{sys} = \Delta K + \Delta U_g + \Delta U_s = \begin{cases} 0 \\ W_{fr} \end{cases}$$

Rotational Motion

$$s = r\theta \rightarrow ds = r d\theta$$

$$\frac{ds}{dt} = r \frac{d\theta}{dt} \rightarrow v = r\omega; \quad \omega = \frac{d\theta}{dt}$$

$$a = \frac{dv}{dt} = r \frac{d\omega}{dt} = r\alpha; \quad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$\theta_f = \theta_i + \omega_i t + \tfrac{1}{2} \alpha t^2$$

$$\omega_f = \omega_i + \alpha t$$

$$\omega_f^2 = \omega_i^2 + 2\alpha \Delta\theta$$

Rotational Forces/Momentum

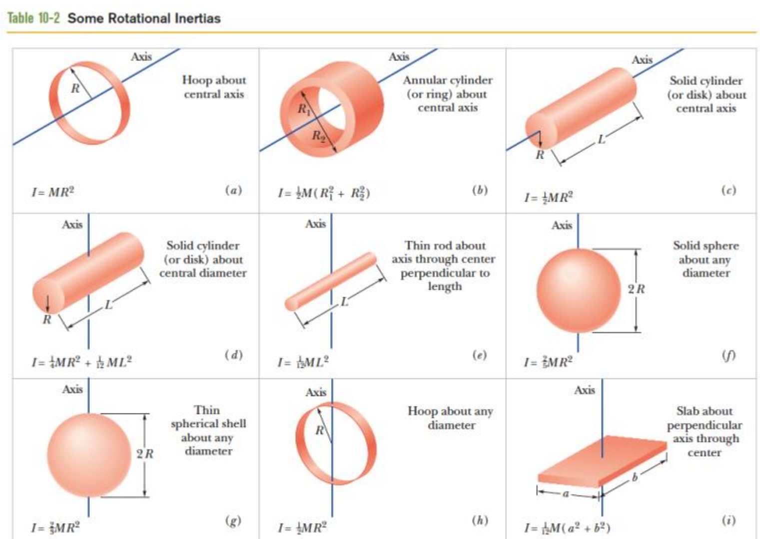
$$\vec{\tau} = \vec{r} \times \vec{F} = \frac{d\vec{L}}{dt} = I\vec{\alpha}$$

$$|\vec{\tau}| = rF \sin\theta = r_\perp F = r F_\perp$$

$$\vec{L} = I\vec{\omega}$$

$$I = \int r^2 dm$$

$$\vec{L}_f = \vec{L}_i + \int \vec{\tau}_{net} dt$$



Some moments of inertia from Halliday, Resnick, & Walker, 10th edition.