Physics 120

Exam #2

February 13, 2026

Name___________________________________

Please read and follow these instructions carefully:

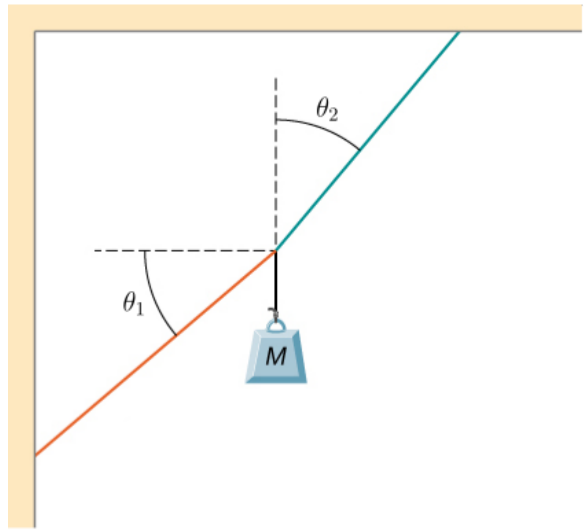
- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So, erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example, $|\vec{p}| \approx m|\vec{v}| = (5kg)\times(2\frac{m}{s}) = 10\frac{kg\cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or reasonable value for the quantity you cannot calculate (explain that you are doing this), and use it to do the rest of the problem.
- Each free-response part is worth 6 points

Problem #1	/18
Problem #2	/18
Problem #3	/18
Problem #4	/18
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

____

1. A $M = 6.75kg$ mass is suspended from the ceiling and the left wall by two light ropes that make angles $\theta_1 = 37^0$ and $\theta_2 = 21^0$ as shown on the right. Let the tension in the upper rope be $F_{T,upper}$ and $F_{T,lower}$ the tension in the lower rope.



a. Starting with the vector expression for Newton's Laws of motion in full vector form, what is the magnitude of the tension force in the upper rope, $F_{T,upper}$? Be sure to specify to your choice of coordinate system for the problem.

Assuming the positive x-direction is to the right and the positive y-direction is vertically up we have:

$$\vec{F}_{net} = \langle F_{Tu} \sin\theta_2 - F_{Tl} \cos\theta_1 , F_{Tu} \cos\theta_2 - F_{Tl} \sin\theta_1 - F_W, 0\rangle$$
$$\vec{F}_{net} = m\vec{a} = m\langle a_x, a_y, a_z\rangle = \langle 0,0,0\rangle$$

In the x-direction:

$$F_{Tu} \sin\theta_2 - F_{Tl} \cos\theta_1 = ma_x = 0 \rightarrow F_{Tl} = \left(\frac{\sin\theta_2}{\cos\theta_1}\right) F_{Tu}$$

In the y-direction:

$$F_{Tu} \cos\theta_2 - F_{Tl} \sin\theta_1 - F_W = ma_y = 0 \rightarrow F_{Tu} \cos\theta_2 - \left(\frac{\sin\theta_2}{\cos\theta_1}\right) F_{Tu} \sin\theta_1 = F_W$$

$$F_{Tu} = \frac{F_W}{\cos\theta_2 - \left(\frac{\sin\theta_2}{\cos\theta_1}\right)\sin\theta_1} = \frac{mg}{\cos\theta_2 - \left(\frac{\sin\theta_2}{\cos\theta_1}\right)\sin\theta_1} = \frac{6.75kg \times 9.8\frac{m}{s^2}}{\cos 21 - \left(\frac{\sin 21}{\cos 37}\right)\sin 37} = 99.7N$$

b. Starting with the vector expression for Newton's Laws of motion or continuing with the results of part a, what is the magnitude of the tension force in the lower rope, $F_{T,lower}$?

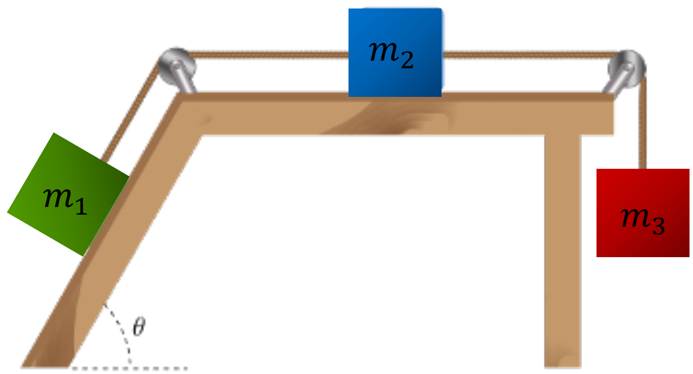
From part a,

$$F_{Tl} = \left(\frac{\sin \theta_2}{\cos \theta_1}\right) F_{Tu} = \left(\frac{\sin 21}{\cos 37}\right) \times 99.7N = 44.7N$$

c. Based on your results from parts a and b, which tension force $F_{T,upper}$ or $F_{T,lower}$ is larger and is this reasonable? To earn full credit, please be sure to fully explain your answer and why you think it is or is not reasonable.

The tension force in the upper rope is larger. The x-components cancel as there is no horizontal motion so there is nothing to say here. This upper tension force being larger is reasonable because the tension force in the positive y-direction in the upper rope has to support the weight and the downward pull in the negative y-direction from the tension in the lower rope. Since the y-component needs to be larger to support the net downward force, it makes sense that the upper tension force is larger.

2. Consider the arrangement of masses shown below, where a block of mass $m_2 = 3m$ is sitting on the horizontal surface. To this mass a block of mass $m_3 = 10m$ is connected on the right side of the $3m$ block and is suspended vertically from a massless pulley by a light string. To the left of the $3m$ block a block of mass $m_1 = 4m$ connected by a light string that passes over a massless pulley and is on an incline, inclined at angle $\theta = 45^0$measured with respect to the horizontal. You may assume that the surfaces are all frictionless and that all of the blocks start from rest.



a. Starting from Newton's Laws of motion, write the *full vector equations* that govern each block. There should be three equations in total, one for each block and be sure to specify to your choice of coordinate system for each block.

Assuming a tilted coordinate system for mass m_1, where the positive x-direction is up the ramp and the positive y-direction is perpendicular to the ramp we have:

$$\vec{F}_{net,1} = \langle F_{TL} - F_{W1x}, F_{N_1} - F_{W_{1y}}, 0\rangle = m_1\vec{a}_1 = m_1\langle a, 0,0\rangle$$

Assuming the positive x-direction is to the right and the positive y-direction is vertically up we have:

$$\vec{F}_{net,2} = \langle F_{TR} - F_{TL}, F_{N_2} - F_{W2}, 0\rangle = m_2\vec{a}_2 = m_2\langle a, 0,0\rangle$$

Assuming the positive y-direction is vertically up we have:

$$\vec{F}_{net,3} = \langle 0, F_{TR} - F_{W3}, 0\rangle = m_3\vec{a}_3 = m_3\langle 0, -a, 0\rangle$$

In each of the above, we've assumed that the acceleration of the system is in the chosen positive x-direction for blocks of mass m_1, and m_2, while it is in the negative y-direction for block m_3.

b. If the block of mass $3m$ is released from rest, determine the expression for the magnitude of the acceleration of the system in terms of the acceleration due to gravity g. That is, your answer should be in of the form $a = C \cdot g$, where C is a number. Do not actually evaluate the actual acceleration at this moment.

Again, we'll assume that the acceleration is up the ramp for m_1, to the right for m_2, and vertically down for m_3. If these are the correct directions then when we calculate the acceleration, it will be a positive number. If we picked the direction incorrectly, it will come with a negative. The magnitude will be the same.

From the forces on m_1 parallel to the ramp we have:

$$F_{TL} - F_{W1x} = F_{TL} - m_1 g \sin\theta = m_1 a \rightarrow F_{TL} = m_1 a + m_1 g \sin\theta$$

From the forces on m_3 in the vertical direction we have:

$$F_{TR} - F_{W3} = F_{TR} - m_3 g = -m_3 a \rightarrow F_{TR} = m_3 g - m_3 a$$

From the forces on m_3 parallel to the surface we have:

$$F_{TR} - F_{TL} = m_2 a \rightarrow m_3 g - m_3 a - (m_1 a + m_1 g \sin\theta) = m_2 a$$

$$a = \left(\frac{m_3 - m_1 \sin\theta}{m_1 + m_2 + m_3}\right) g = \left(\frac{10m - 4m \sin 45}{4m + 3m + 10m}\right) g = \left(\frac{7.2m}{17m}\right) g = 0.42g$$

Since this is a positive number, we chose our direction for the acceleration correctly.

c. What is the speed of the block of mass $3m$ if the block of mass $10m$ falls a distance $0.75m$ from rest?

Since all three blocks are connected by the rope, they will all have the same speed. Thus let $v_{1f} = v_{2f} = v_{3f} = v_f$. Since the acceleration is constant, we can use one of our equations of motion for constant acceleration, and we have:

$$v_f^2 = v_i^2 + 2ad \rightarrow v_f = \sqrt{2ad} = \sqrt{2 \times 0.42 \times 9.8\tfrac{m}{s^2} \times 0.75m} = 2.5\tfrac{m}{s}$$

As a side note, although this problem wanted you to use forces to solve the problem, you could have done this problem by using the work kinetic energy theorem. There are three work terms, one for m_1, one for m_2, and one for m_3 and the total work done changes the kinetic energy of each of the blocks. And since they are all connected by the rope, the achieve the same final speed.

$$W_1 = \int \vec{F}_{net,1} \cdot d\vec{r}_1 = \int \langle F_{TL} - F_{Wx1}, F_{N1} - F_{Wy1}, 0\rangle \cdot \langle dx, 0,0\rangle = \int_0^d (F_{TL} - F_{Wx1})dx$$
$$W_1 = F_{TL}d - m_1 g \sin\theta$$

$$W_2 = \int \vec{F}_{net,2} \cdot d\vec{r}_2 = \int \langle F_{TR} - F_{TL}, F_{N2} - F_{W2}, 0\rangle \cdot \langle dx, 0,0\rangle = \int_0^d (F_{TR} - F_{TL})dx$$
$$W_2 = F_{TR}d - F_{TL}d$$

$$W_3 = \int \vec{F}_{net,3} \cdot d\vec{r}_3 = \int \langle 0, F_{TR} - F_{W3}, 0\rangle \cdot \langle 0, dy, 0\rangle = \int_0^{-d} (F_{TR} - F_{W3})dy$$
$$W_3 = -F_{TR}d + m_3 g d$$

The total work and thus the change in kinetic energy is given by:

$$W_{net} = W_1 + W_2 + W_3 = \Delta K = \Delta K_1 + \Delta K_2 + \Delta K_3 = \tfrac{1}{2}(m_1 + m_2 + m_3)v_f^2 = \tfrac{17}{2}mv_f^2$$

$$W_{net} = F_{TL}d - m_1 g \sin\theta + F_{TR}d - F_{TL}d - F_{TR}d + m_3 g d = m_3 g d - m_1 g \sin\theta$$

$$W_{net} = m_3 g d - m_1 g \sin\theta = (10 - 4\sin 45)mgd = \tfrac{17}{2}mv_f^2$$

$$\rightarrow v_f = \sqrt{2\left(\frac{10 - 4\sin 45}{17}\right)gd} = \sqrt{2(0.42g)d} = \sqrt{0.82gd}$$

The first and second terms on the right in the parenthesis are the answer to part b and the last is the answer to part c. Notice that these expressions are the same that you get by using forces to solve the problem.

3. We've said in class that roadways and racetracks are banked so that cars can negotiate the curves without having to rely on friction to take the turn, but instead rely on a component of the normal force from the track/roadway itself. Suppose that a racetrack, shown below, is banked at an angle $\theta = 31^0$. This racetrack is designed to allow the racecar to take the curve at a speed $v = 200\frac{mi}{hr} = 89\frac{m}{s}$.

https://www.ajc.com/sports/racing/rains-delay-end-daytona-500-until-monday/Ctvg7ozshchtRxGVsfjpYK/

a. Starting from Newton's laws of motion in full vector form, what is the design radius of the track? That is, what is the radius of the horizontal circle that the racecar would travel in around the curve? Be sure to specify your coordinate system.

Assuming a standard cartesian coordinate system where the positive x-direction is to the right, the positive y-direction is vertically up and the positive z-direction is out of the page, we have:

$$\vec{F}_{net} = \langle F_{Nx}, F_{Ny} - F_W, F_{Engine} - F_{air} \rangle = m\vec{a} = m\langle a_x, a_y, a_z \rangle = m\left\langle \frac{v^2}{R}, 0,0 \right\rangle$$

From the forces in the z-direction:

$$F_{Engine} - F_{air} = ma_z = 0 \rightarrow F_{Engine} = F_{air}$$

From the forces in the y-direction:

$$F_{Ny} - F_W = ma_y = 0 \rightarrow F_N \cos\theta - mg = 0 \rightarrow F_N = \frac{mg}{\cos\theta}$$

From the forces in the x-direction:

$$F_{Nx} = ma_x \rightarrow F_N \sin\theta = \left(\frac{mg}{\cos\theta}\right)\sin\theta = mg\tan\theta = m\frac{v^2}{R} \rightarrow R = \frac{v^2}{g\tan\theta}$$

$$R = \frac{v^2}{g\tan\theta} = \frac{\left(89\frac{m}{s}\right)^2}{9.8\frac{m}{s^2}\tan 31} = 1345m$$

b. Suppose that the racecar driver wanted to take the curve at a higher speed than the track was designed. If the coefficient of friction between the tires and the track is $\mu = 0.4$, what is the racecar's allowable *maximum speed* in order to take the curve at the radius found in part a?

Assuming the same coordinate system as in part a, and noting that if the car tries to negotiate the curve to fast, they will slide to the outside of the curve (or up the ramp). This makes friction point down the ramp.

$$\vec{F}_{net} = \langle F_{Nx} + F_{fr,x}, F_{Ny} - F_{fr,y} - F_W, F_{Engine} - F_{air} \rangle = m\vec{a} = m\langle \frac{v^2}{R}, 0,0 \rangle$$

From the forces in the y-direction we have:

$$F_{Ny} - F_{fr,y} - F_W = ma_y = 0 \rightarrow F_N \cos\theta - \mu F_N \sin\theta = mg$$

$$\rightarrow F_N = \frac{mg}{\cos\theta - \mu \sin\theta}$$

From the forces in the x-direction we have:

$$F_{Nx} + F_{fr,x} = ma_x = m\frac{v^2}{R} \rightarrow F_N \sin\theta + \mu F_N \cos\theta = F_N(\sin\theta + \mu\cos\theta) = m\frac{v_{max}^2}{R}$$

$$\rightarrow mg\left(\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}\right) = m\frac{v_{max}^2}{R} \rightarrow v_{max} = \sqrt{\left(\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}\right)Rg}$$

$$v_{max} = \sqrt{\left(\frac{\sin 31 + 0.4\cos 31}{\cos 31 - 0.4\sin 31}\right) \times 1345m \times 9.8\frac{m}{s^2}} = 131\frac{m}{s}$$

c. Suppose that the racecar driver wanted to take the curve at a lower speed than the track was designed. If the coefficient of friction between the tires and the track is $\mu = 0.4$, what is the racecar's allowable *minimum speed* in order to take the curve at the radius found in part a?

Assuming the same coordinate system as in part a, and noting that if the car tries to negotiate the curve to slow, they will down to the inside of the curve (or down the ramp). This makes friction point up the ramp.

$$\vec{F}_{net} = \langle F_{Nx} - F_{fr,x}, F_{Ny} + F_{fr,y} - F_W, F_{Engine} - F_{air} \rangle = m\vec{a} = m\left\langle \frac{v^2}{R}, 0, 0 \right\rangle$$

From the forces in the y-direction we have:

$$F_{Ny} + F_{fr,y} - F_W = ma_y = 0 \rightarrow F_N \cos\theta + \mu F_N \sin\theta = mg$$

$$\rightarrow F_N = \frac{mg}{\cos\theta + \mu \sin\theta}$$

From the forces in the x-direction we have:

$$F_{Nx} - F_{fr,x} = ma_x = m\frac{v^2}{R} \rightarrow F_N \sin\theta - \mu F_N \cos\theta = F_N(\sin\theta - \mu\cos\theta) = m\frac{v_{min}^2}{R}$$

$$\rightarrow mg\left(\frac{\sin\theta - \mu\cos\theta}{\cos\theta + \mu\sin\theta}\right) = m\frac{v_{min}^2}{R} \rightarrow v_{min} = \sqrt{\left(\frac{\sin\theta - \mu\cos\theta}{\cos\theta + \mu\sin\theta}\right)Rg}$$

$$v_{min} = \sqrt{\left(\frac{\sin 31 - 0.4\cos 31}{\cos 31 + 0.4\sin 31}\right) \times 1345m \times 9.8\frac{m}{s^2}} = 46\frac{m}{s}$$

As a side note we've said that the answer is coordinate system independent. In the problem I didn't tilt my axes but used a standard cartesian coordinate system. If you want to do the problem by tilting your coordinate axes so that the x-axis points parallel to the incline and the y-axis is perpendicular to the incline the acceleration has components in both the x- and y-directions, that's fine. The solution to problem #2 assuming a tilted set of axes is below.

For part a:

$$\vec{F}_{net} = \langle F_{Wx}, F_N - F_{Wy}, F_{Engine} - F_{air}\rangle = m\vec{a} = m\,\langle \frac{v^2}{R}\cos\theta\,, \frac{v^2}{R}\sin\theta\,, 0\rangle$$

In the y-direction:

$$F_N - F_{Wy} = F_N - mg\cos\theta = m\frac{v^2}{R}\sin\theta \rightarrow F_N = mg\cos\theta + m\frac{v^2}{R}\sin\theta$$

In the x-direction:

$$F_{Wx} = mg\sin\theta = m\frac{v^2}{R}\cos\theta \rightarrow R = \frac{v^2}{g\tan\theta}$$

This is the exact same expression we got without tilting the coordinate axes.

For parts b and c:

To determine the maximum speed, friction points down the ramp as we'd tend to move to the outside of the track. The net force including friction is:

$$\vec{F}_{net} = \langle F_{Wx} + F_{fr}, F_N - F_{Wy}, F_{Engine} - F_{air}\rangle = m\vec{a} = m\,\langle \frac{v^2}{R}\cos\theta\,, \frac{v^2}{R}\sin\theta\,, 0\rangle$$

Where the frictional force is determined from the forces in the y-direction:

$$F_{fr} = \mu F_N = \mu\left(mg\cos\theta + m\frac{v^2}{R}\sin\theta\right) = \mu mg\left(\cos\theta - \frac{v^2}{Rg}\sin\theta\right)$$

In the x-direction:

$$F_{Wx} + F_{fr} = mg\sin\theta + \mu F_N = mg\sin\theta + \mu mg\left(\cos\theta - \frac{v_{max}^2}{Rg}\sin\theta\right) = m\frac{v_{max}^2}{R}\cos\theta$$

$$\rightarrow v_{max} = \sqrt{\left(\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}\right)Rg}$$

Which is the solution we got in part b without tilting the axes.

To determine the minimum speed, friction points up the ramp as we'd tend to move to the inside of the tract. The net force including friction (where the normal force is the same as above since there is no friction in the y-direction):

$$\vec{F}_{net} = \langle F_{Wx} - F_{fr}, F_N - F_{Wy}, F_{Engine} - F_{air} \rangle = m\vec{a} = m \langle \frac{v^2}{R}\cos\theta, \frac{v^2}{R}\sin\theta, 0 \rangle$$

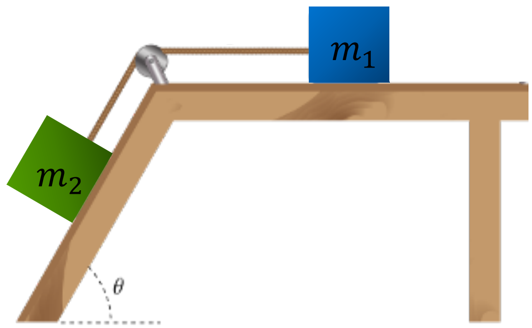
In the x-direction:

$$F_{Wx} - F_{fr} = mg\sin\theta - \mu F_N = mg\sin\theta - \mu mg\left(\cos\theta - \frac{v_{min}^2}{Rg}\sin\theta\right) = m\frac{v_{min}^2}{R}\cos\theta$$

$$\rightarrow v_{min} = \sqrt{\left(\frac{\sin\theta - \mu\cos\theta}{\cos\theta + \mu\sin\theta}\right)Rg}$$

Which is the solution we got in part c without tilting the axes.

4. Consider the arrangement of masses shown below. A block of mass m_1 sits at rest on the horizontal surface while a block of mass m_2 sits at rest on the incline which is oriented at an angle θ measured with respect to the horizontal. Friction exists on all surfaces with coefficient of friction μ.



a. Starting from the general definition of work, what are the expressions for the work done on masses m_1 and m_2 if both masses are released from rest? There should be two expressions, one for mass m_1 and one for mass m_2.

For mass m_1 assuming to the right is the positive x-direction and vertically up is the positive y-direction we have:

$$W_1 = \int \vec{F}_{net1} \cdot d\vec{r} = \int \langle -F_T + F_{fr1}, F_{N1} - F_{W1}, 0 \rangle \cdot \langle dx, dy, dz \rangle$$

$$W_1 = \int_0^{-d} \left(-F_T + F_{fr1}\right) dx + \int_0^{0} \left(F_{N1} - F_{W1}\right) dy$$

$$W_1 = -\int_0^{-d} F_T dx + \int_0^{-d} \mu m_1 g dx = F_T d - \mu m_1 g d$$

For mass m_2 assuming to the down the ramp is the positive x-direction and perpendicular to the ramp and away is the positive y-direction we have:

$$W_2 = \int \vec{F}_{net2} \cdot d\vec{r} = \int \langle -F_T - F_{fr2} + F_{W2x}, F_{N2} - F_{W2y}, 0 \rangle \cdot \langle dx, dy, dz \rangle$$

$$W_2 = \int_0^{d} \left(-F_T - F_{fr2} - F_{W2x}\right) dx + \int_0^{0} \left(F_{N2} - F_{W2y}\right) dy$$

$$W_2 = -\int_0^{d} F_T dx - \int_0^{d} \mu m_2 g \cos\theta \, dx + \int_0^{d} m_2 g \sin\theta \, dx$$

$$W_2 = -F_T d - \mu m_2 g d \cos\theta + m_2 g d \sin\theta$$

b. Starting with the work-kinetic energy theorem and using your results from part a, what is the expression for the speed of the bock of mass m_1, if both blocks are released from rest?

$$W_{net} = W_1 + W_2 = \Delta K_1 + \Delta K_2 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2 = \frac{1}{2}(m_1 + m_2)v_f^2$$

$$W_{net} = F_T d - \mu m_1 g d - F_T d - \mu m_2 g d \cos\theta + m_2 g d \sin\theta = \frac{1}{2}(m_1 + m_2)v_f^2$$

$$\rightarrow -\mu m_1 g d - \mu m_2 g d \cos\theta + m_2 g d \sin\theta = \frac{1}{2}(m_1 + m_2)v_f^2$$

$$v_f = \sqrt{2\left(\frac{m_2 \sin\theta - \mu(m_1 + m_2 \cos\theta)}{m_1 + m_2}\right) g d}$$

c. How much work was done on the system of masses m_1 and m_2 by gravity and how does this relate to the change in gravitational potential energy? To earn full credit, either show how the change in gravitational potential energy is related to the work done using some mathematical equations, or fully explain the result.

The change in gravitational potential energy is related to the change in arrangement of the masses in the gravitational field. The only mass that changes arrangement, or height with respect to the Earth is mass m_2 moving down the ramp. The component of the weight parallel to the ramp does work moving the block down the ramp. The work done by this component of gravity parallel to the ramp lowers the potential energy of the block of mass m_2 and in turn speeds the system of masses m_1 and m_2 up since energy is conserved.

$$W_{g2} = -\Delta U_g = \int_0^d F_{W2x} dx = \int_0^d m_2 g \sin\theta \, dx = m_2 g d \sin\theta$$

$$\Delta U_g = -W_{g2} = -m_2 g d \sin\theta = -m_2 g y$$

The work done here is positive (leading to an increase in speed) and thus the change in gravitational potential energy must be negative, reflective of the loss of height in block of mass m_2.