Physics 120

Exam #2

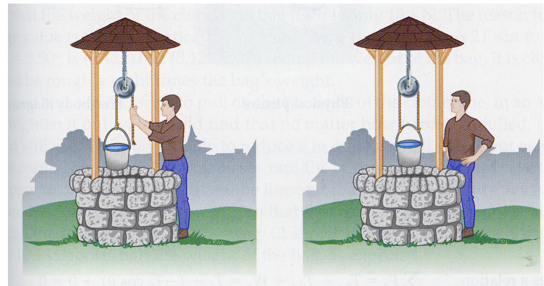
May 4, 2011

Name____________________________________

Multiple Choice	/16
Problem #1	/28
Problem #2	/28
Problem #3	/28
Total	/100

Part I: Multiple-Choice: *Circle the best answer to each question. Any other marks will not be given credit. Each multiple-choice question is worth 4 points for a total of 16 points.*

1. A person hoists a bucket of water from a well and holds the rope, keeping the bucket at rest as in the left photo. Call this situation *A*. A short time later the person ties the rope to the bucket so that the rope holds the bucket in place as in the right photo. Call this situation *B*.



   a. The tension in situation *B is greater than* in situation *A*.
   b. The tension in situation *B is equal to* that in situation *A*.
   c. The tension in situation *B is less than* that in situation *A*.
   d. Three is no way that the individual tensions in the ropes can be determined.

2. A 2.0×10^3 *kg* car travels at a constant speed of *12 m/s* around a circular curve of radius *30 meters*. As the car goes around the curve, the time rate of change of the car's momentum is directed

   a. toward the center of the circular curve.
   b. away from the circular curve.
   c. tangent to the curve in the direction of motion.
   d. tangent to the curve opposite the direction of motion.

3. If the Earth orbit the sun in a nearly circular orbit of radius $r = 1.5x10^{11}m$ and the orbital period of the Earth in its orbit is $T = 31.5x10^6 s$ (1 year), the mass of the sun is most nearly given by the expression

   a. $M_{Sun} = \frac{2\pi r^2}{GT^2}$ b. $M_{Sun} = \frac{4\pi^2 r^3}{GT^2}$ c. $M_{Sun} = \frac{2\pi^2 G r^3}{T^2}$ d. $M_{Sun} = \frac{4\pi r^3}{GT^2}$

4. A spring of stiffness *k* is suspended from the ceiling an allowed to hang vertically. The spring has a relaxed length (measured from the ceiling) of L_o. A mass *m* is suspended from the spring and the spring stretches by an amount *L* (measured from the ceiling.) The stiffness of the spring can be approximated using

   a. $k = \frac{m}{|L - L_o|}$ b. $k = \frac{|L - L_o|}{mg}$ c. $k = \frac{mg}{|L|}$ d. $k = \frac{mg}{|L - L_o|}$

Part II: Free Response Problems: *The three problems below are worth 84 points total and each subpart is worth 7 points each. Please show all work in order to receive partial credit. If your solutions are illegible or illogical no credit will be given. A number with no work shown (even if correct) will be given no credit. Please use the back of the page if necessary, but number the problem you are working on.*

1. Suppose that you are going to make a *1m* long tungsten wire that has a square cross-sectional area of $1cm^2$. The density of tungsten is $\rho = 19250\,\frac{kg}{m^3}$ and its molecular mass is $183.8\,\frac{g}{mole}$.

   a. What is the interatomic bond spacing, *d*?

   Assuming a cube with a $1m^3$ volume we have $m = \rho V = 19250\,\frac{kg}{m^3} \times 1m^3 = 19250kg$. The number of atoms in this volume is given by

   $\#_{volume} = 19250kg \times \frac{1mol}{0.1838kg} \times \frac{6.02\times10^{23}\,atoms}{1mol} = 6.3\times10^{28}$. The number of atoms on a side is

   $\#_{side} = \sqrt[3]{6.3\times10^{28}} = 3.98\times10^{9}$. Therefore in a side of length *1m*, we have

   $$d = \frac{1m}{\#_{side}} = \frac{1m}{3.98\times10^{9}} = 2.51\times10^{-10}m$$

   b. What is the interatomic bond stiffness, k_{iab}? (Hint: Assume that the wire has a stiffness $k_{wire} = 4.11x10^7\ N/m$.)

   In *1m* of wire there are $N_{series} = \frac{1m}{2.51\times10^{-10}m} = 3.98\times10^{9}$ springs in series. The stiffness of an interatiomic bond is $k_{iab} = N_{series}k_{chain}$ where $k_{chain} = \frac{k_{wire}}{N_{parallel}}$ and the number of parallel chains of atoms is $N_{parallel} = \frac{1cm^2 \times \left(\frac{1m^2}{(100cm)^2}\right)}{\left(2.51\times10^{-10}m\right)^2} = 1.58\times10^{15}$. Therefore,

   $$k_{iab} = N_{series}k_{chain} = \frac{N_{series}k_{wire}}{N_{parallel}} = \frac{3.98\times10^{9}}{1.58\times10^{15}} \times 4.11\times10^{7}\,\frac{N}{m} = 103.5\,\frac{N}{m}.$$

   c. What is Young's modulus for tungsten?

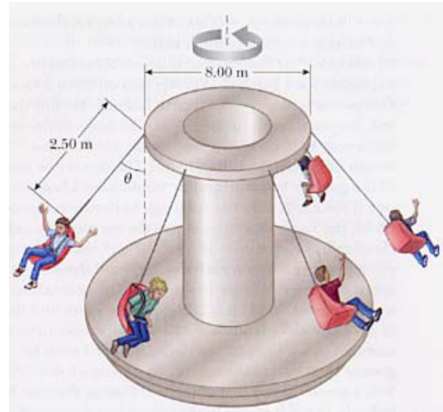
   $$Y = \frac{k_{iab}}{d} = \frac{103.5\,\frac{N}{m}}{2.51\times10^{-10}m} = 4.12\times10^{11}\,\frac{N}{m^2}$$

   d. If a *5kg* mass were hung from this wire when the wire is held vertical, by how much would the wire stretch?

   $$Stress = Y \times strain \rightarrow \frac{F}{A} = Y\frac{\Delta L}{L}$$

   $$\Delta L = \frac{FL}{YA} = \frac{5kg \times 9.8\,\frac{m}{s^2} \times 1m}{4.12\times10^{11}\,\frac{N}{m^2} \times \left(1cm^2 \times \frac{1m^2}{(100cm)^2}\right)} = 1.2\times10^{-6}m = 1.2\mu m$$

2. An amusement park ride shown below, consists of a rotating circular platform *8.0m* in diameter from which *10kg* seats are suspended from a chain with mass *4kg* and assume that the mass of the chain is located at the center of the chain. When the system rotates it does so at constant speed the chain makes and angle $\theta = 28^o$ with respect to the vertical and a *40kg* child is riding in a chair.



   a. Starting with the momentum principle, what is the speed of the chair when it is empty (*no child is in the seat)?*

$$\frac{dp}{dt} = \left\langle \frac{m_{total}v^2}{R}, 0, 0 \right\rangle = F_{net} = \left\langle F_T \sin\theta, F_T \cos\theta - m_{chair}g - m_{chain}g, 0 \right\rangle$$

$$y-dir: \quad F_T \cos\theta - m_{chair}g - m_{chain}g = 0 \Rightarrow F_T = \frac{(m_{chair} + m_{chain})g}{\cos\theta} = \frac{14kg \times 9.8\frac{m}{s^2}}{\cos 28} = 155.4N$$

$$x-dir: \quad \frac{m_{total}v^2}{R} = F_T \sin\theta \Rightarrow v = \sqrt{\frac{F_T R \sin\theta}{m_{total}}} = \sqrt{\frac{155.4N \times (4m + 2.5m \sin 28)\sin 28}{14kg}} = 5.2\frac{m}{s}$$

   b. What is the tension in the chain when there *is no child* in the chair?

$$F_T = 155.4N$$

   c. Again, using the momentum principle, what is the speed of the chair *when the child is in the seat*?

$$\frac{dp}{dt} = \left\langle \frac{m_{total}v^2}{R}, 0, 0 \right\rangle = F_{net} = \left\langle F_T \sin\theta, F_T \cos\theta - m_{chair}g - m_{chain}g - m_{child}g, 0 \right\rangle$$

$$y-dir: \quad F_T \cos\theta - m_{chair}g - m_{chain}g - m_{child}g = 0 \Rightarrow F_T = \frac{(m_{chair} + m_{chain} + m_{child})g}{\cos\theta} = \frac{54kg \times 9.8\frac{m}{s^2}}{\cos 28} = 599.4N$$

$$x-dir: \quad \frac{m_{total}v^2}{R} = F_T \sin\theta \Rightarrow v = \sqrt{\frac{F_T R \sin\theta}{m_{total}}} = \sqrt{\frac{599.4N \times (4m + 2.5m \sin 28)\sin 28}{54kg}} = 5.2\frac{m}{s}$$

   d. What is the tension in the chain *when a child is in the chair*?

$$F_T = 599.4N$$

3. A block of mass $m = 5kg$ is moving along a horizontal frictionless surface at a constant velocity of $\vec{v}_i = \langle 2,0,0 \rangle \frac{m}{s}$. At a location taken as the origin the block encounters a region in which there is friction present and the coefficient of kinetic friction is $\mu_k = 0.08$.

   a. Starting from the momentum principle $\frac{d\vec{p}}{dt} = \vec{F}_{net}$, what is the velocity of the block as a function of time? (Hint: You will need the integral $\int_{x_i}^{x_f} x^n dx = \left. \frac{x^{n+1}}{n+1} \right|_{x_i}^{x_f}$.)

$$\frac{d\vec{p}}{dt} = \left\langle \frac{dp_x}{dt},0,0 \right\rangle = \vec{F}_{net} = \langle -\mu_k F_N, F_N - mg, 0 \rangle$$

$$y-dir: \quad F_N - mg = 0 \Rightarrow F_N = mg$$

$$x-dir: \quad \frac{dp_x}{dt} = m\frac{dv_x}{dt} = -\mu_k mg \Rightarrow v_{fx} - v_{ix} = \int_{v_{ix}}^{v_{fx}} dv = -\mu_k g \int_0^t dt = -\mu_k g t$$

$$\vec{v}_f = \vec{v}_i + \langle -\mu_k g t, 0, 0 \rangle = \langle v_{ix} - \mu_k g t, 0, 0 \rangle$$

   b. How long will it take before the block comes to rest? (Hint: Take time $t_i = 0$ when the block first encounters the region with friction.)

$$v_{fx} = 0 = v_{ix} - \mu_k g t \to t = \frac{v_{ix}}{\mu_k g} = \frac{2\frac{m}{s}}{0.08 \times 9.8 \frac{m}{s^2}} = 2.55s$$

   c. What is the final location of the block when it comes to rest?

$$\vec{r}_f = \vec{r}_i + \vec{v}_{avg}\Delta t = \langle 0,0,0 \rangle m + \left[\frac{\langle 2,0,0 \rangle \frac{m}{s} + \langle 0,0,0 \rangle \frac{m}{s}}{2} \right] \times 2.55s = \langle 2.55,0,0 \rangle m$$

   d. Suppose that instead of friction bringing the block to rest, the block encounters a region of space (the beginning of which is taken again as the origin) over which the block is subject to a variable force given by $\vec{F}_{res} = \langle -\rho v^2, 0, 0 \rangle N$, where ρ is a constant with units of Ns^2/m^2. In this case, what is the velocity of the block as a function of time, taking $t_i = 0$ when the block first encounters this variable force?

$$\frac{d\vec{p}}{dt} = \left\langle \frac{dp_x}{dt},0,0 \right\rangle = \vec{F}_{net} = \langle -\rho v^2, F_N - mg, 0 \rangle$$

$$y-dir: \quad F_N - mg = 0 \Rightarrow F_N = mg$$

$$x-dir: \quad \frac{dp_x}{dt} = m\frac{dv_x}{dt} = -\rho v^2 \Rightarrow \int_{v_{ix}}^{v_{fx}} \frac{dv}{v^2} = -\left[\frac{1}{v_{fx}} - \frac{1}{v_{ix}} \right] - \frac{\rho}{m} \int_0^t dt = -\frac{\rho}{m} t$$

$$v_{fx} = \frac{m v_{ix}}{\rho v_{ix} t + m} \Rightarrow \vec{v}_f = \left\langle \frac{m v_{ix}}{\rho v_{ix} t + m}, 0, 0 \right\rangle$$

Useful formulas:

$\vec{p} = \gamma m\vec{v}$

$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

$\vec{v}_{avg} = \frac{\vec{v}_i + \vec{v}_f}{2}$

$\vec{F}_g = m\vec{g}$

$\vec{F}_{gravity} = \frac{GM_1M_2}{r_{12}^2}\hat{r}_{12}$

$\vec{F}_{spring} = -k\vec{s}; \quad \vec{s} = (L - L_o)\hat{s}$

$\rho = \frac{M}{V}$

$k_{eff,parallel} = n_{parallel}k_{individual}$

$k_{eff,series} = \frac{k_{individual}}{n_{series}}$

$stress = Ystrain \rightarrow \frac{F}{A} = Y\frac{\Delta L}{L}; \quad Y = \frac{k_{iab}}{d}$

Momentum Principle:

$$\vec{p}_f = \vec{p}_i + \vec{F}_{net}\Delta t$$

$$\frac{d\vec{p}}{dt} = \vec{F}_{net}$$

Position-update:

$$\vec{r}_f = \vec{r}_i + \vec{v}_{avg}\Delta t = \vec{r}_i + \frac{\vec{p}}{m\sqrt{1+\frac{p^2}{m^2c^2}}}\Delta t$$

Vectors

$magnitude\ of\ a\ vector: |\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$

$writing\ a\ vector: \quad \vec{a} = \langle a_x, a_y, a_z \rangle = |\vec{a}|\hat{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

Constants

$g = 9.8\,\frac{m}{s^2}$

$G = 6.67\times10^{-11}\,\frac{Nm^2}{kg^2}$

$1e = 1.6\times10^{-19}C$

$k = \frac{1}{4\pi\varepsilon_o} = 9\times10^9\,\frac{C^2}{Nm^2}$

$\varepsilon_o = 8.85\times10^{-12}\,\frac{Nm^2}{C^2}$

$1eV = 1.6\times10^{-19}J$

$\mu_o = 4\pi\times10^{-7}\,\frac{Tm}{A}$

$c = 3\times10^8\,\frac{m}{s}$

$h = 6.63\times10^{-34}Js$

$m_e = 9.11\times10^{-31}kg = \frac{0.511MeV}{c^2}$

$m_p = 1.67\times10^{-27}kg = \frac{937.1MeV}{c^2}$

$m_n = 1.69\times10^{-27}kg = \frac{948.3MeV}{c^2}$

$1amu = 1.66\times10^{-27}kg = \frac{931.5MeV}{c^2}$

$N_A = 6.02\times10^{23}$

$Ax^2 + Bx + C = 0 \rightarrow x = \frac{-B\pm\sqrt{B^2-4AC}}{2A}$

Geometry

$Circles: \quad C = 2\pi r = \pi D \qquad A = \pi r^2$

$Triangles: \quad A = \frac{1}{2}bh$

$Spheres: \quad A = 4\pi r^2 \qquad V = \frac{4}{3}\pi r^3$