Physics 120 Mid-Term Exam 2

February 22, 2012 – Winter 2012

Name Solutions   Section 01 & 02

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example $|\vec{p}| \approx m|\vec{v}| = (5kg)\times(2\frac{m}{s}) = 10\frac{kg\cdot m}{s}$
- Give standard SI units with your results.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.

Exam Section	Points Possible	Points Earned
Short Answer – Multiple-Choice	36	
Problem #1	10	
Problem #2	10	
Problem #3	10	
Problem #4	9	
Exam Total	*75*	

In keeping with the Union College policy on academic honesty, it is assumed that you will neither accept nor provide unauthorized assistance in the completion of this work.

Part 1: *Multiple-choice and short-answer questions. Please circle the best answer to each multiple-choice question and write a brief answer to each short-answer question.*

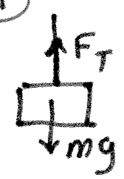
1. (4 points) A climber whose mass is 55 kg hangs motionless from a rope. Later, a different climber whose mass is 75 kg hangs motionless from the same rope. Circle the letter of all true statements below.

Like T/F
1pt each

   a. Since the same rope is used, the tension in the rope is the same in both cases.
   b. The tension in the rope is bigger for the 55 kg man than the 75 kg man.
   c. The tension in the rope is bigger for the 75 kg man than the 55 kg man.
   d. The rope is slightly longer when it is supporting the heavier climber than when it is supporting the lighter climber. (2) Stress = Y Strain $\rightarrow \frac{mg}{A} = Y\frac{\Delta L}{L}$

as $m\uparrow$ $\Delta L\uparrow$

(1)



$\frac{d\vec{p}}{dt} = \langle 0,0,0\rangle = \vec{F}_{net} = \langle 0,\ F_T - mg,\ 0\rangle$

y-dir

$0 = F_T - mg \Rightarrow \boxed{F_T = mg}$ as $m\uparrow$ $F_T\uparrow$

2. (4 points) If it takes *4 J* of work to stretch a spring by *10cm* from its equilibrium length, ***how much extra work*** does it take to stretch the same spring by an additional *10cm*?
   a. *4 J*
   b. *8 J*
   c. *12 J*
   d. *16 J*

$W_1 = \frac{1}{2}kx^2 \rightarrow 4J = \frac{1}{2}kx_f^2 - \cancel{\frac{1}{2}kx_i^2} = \frac{1}{2}k(0.1m)^2$ ($x_i = 0$)

$\hookrightarrow \underline{k = 800 N/m}$

$W_{Total} = 4J + \text{Extra} = \frac{1}{2}kx_{ff}^2 - \cancel{\frac{1}{2}kx_i^2}$ ($x_i = 0$)

$W_{total} = 4J + \text{Extra} = \frac{1}{2}(800 N/m)(0.2m)^2 = 16J$

$\boxed{\text{Extra work} = 12J}$

3. (3 points) For a vertical spring-mass oscillator, which of the following statements are true? Circle all that apply.
   a. At the lowest point in the oscillation, the momentum is zero. – mass stops moving
   b. At the lowest point in the oscillation, the rate of change of the momentum is zero. $\hookrightarrow F_{net} = 0$
   c. At the lowest point in the oscillation, $mg = k_s|s|$. – mass wouldn't move
   d. At the lowest point in the oscillation, $mg > k_s|s|$. – mass moves $\downarrow$
   e. At the lowest point in the oscillation, $mg < k_s|s|$. – mass moves $\uparrow$.

4. (5 points) A proton of rest mass $m_p = 940\frac{MeV}{c^2}$ initially at rest at a location $\vec{r}_1 = \langle 0,0,0 \rangle m$ is acted on by a force $\vec{F} = \langle 0, 1.6\times10^{-13}, 0\rangle N$. After some time, the proton is found at a location $\vec{r}_2 = \langle 0,2,0\rangle m$. What is the kinetic energy of the proton in eV?

$E_{rest} = m_p c^2 \rightarrow E_{f,rest} = E_{i,rest}$

$KE_i = (\gamma_i - 1) m_p c^2 = 0$ (at rest)

$\therefore KE_f = KE_i + W_{ext} = KE_i + \int \vec{F}\cdot d\vec{r}$

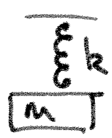
$= 0 + \langle 0, 1.6\times10^{-13}, 0\rangle N \cdot \int \langle dx, dy, dz\rangle$

$KE_f = 1.6\times10^{-13} N \cdot 2m = 3.2\times10^{-13} J \times \frac{1eV}{1.6\times10^{-19} J}$

$\boxed{KE_f = 2\times10^{6} eV = 2MeV}$

5. (4 points) You hang a 10 kg block from a spring, and the spring stretches by 8 cm. You then suspend the same mass from two side-by-side springs that are identical to the first. What happens?
   a. Each wire stretches by 4 cm.
   b. Each wire stretches by 8 cm.
   c. Each wire stretches by 16 cm.

(1)



$\frac{d\vec{p}}{dt} = \vec{F}_{net}$

$\langle 0,0,0\rangle = \langle 0, ky - mg, 0\rangle$

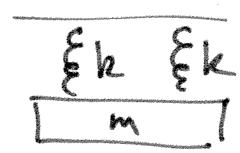
y-dir

$0 = ky - mg$

$k = \frac{mg}{y} = \frac{(10kg)(9.8m/s^2)}{0.08m}$

$\boxed{k = 1225 N/m}$

(2)



$\frac{d\vec{p}}{dt} = \vec{F}_{net}$

$\langle 0,0,0\rangle = \langle 0, ky' + ky' - mg, 0\rangle$

y-dir

$0 = 2ky' - mg$

$y' = \frac{mg}{2k} = \frac{(10kg)(9.8m/s^2)}{2(1225N/m)}$

$y' = 0.04m$

$\boxed{y' = 4cm}$

6. (4 points) A large crate has a mass of 120 kg and sits on a horizontal floor. You push on the crate with a horizontal force of 550 N, but the crate does not move. The coefficient of static friction between the crate and the floor is 0.6.
   a. What is the magnitude of the static frictional force acting on the crate?

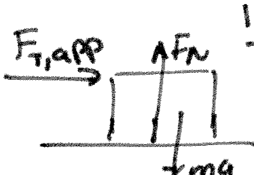
$$F_{fr,s} = \mu_s F_N \;;\quad F_N - mg = 0 \rightarrow F_N = mg$$

2pts $\quad F_{fr,s} = (0.6)(120\,kg)(9.8\,m/s^2)$

$$\boxed{F_{fr,s} = 705.6\,N}$$

   b. Which of the following will help you move the crate?
      i. Pull it from the front instead of pushing it from the back. Doesn't matter
      ii. Have a friend join you and apply a force of 300 N in the same direction as your force.
      iii. Have a friend join you and apply a force of 300 N vertically upward.
      iv. Lay the crate down on a different side whose surface area is half that of the original surface of contact. F_s indep of Area

$\frac{1}{2}$pt each = 2pts



ii: if $F_{T,app} > 705.6\,N$ block slides

$F_{T,app} = 550\,N + 300\,N = 850\,N$ block will slide

iii: if $F_N \downarrow$ $F_{fr,s} \downarrow$; $F_N + 300 - mg = 0 \rightarrow F_N = (120\,kg)(9.8\,m/s^2) - 300$

$F_N = 876\,N$

$F_{fr,s} = (0.6)(876\,N)$
$= 525.6\,N$

∴ block will slide w/ 550 N applied force

7. (6 points) A "free" neutron (that is, one outside of a nucleus) is unstable and decays into a proton, an electron, and an antineutrino. In this decay reaction, the sum of the kinetic energies of the proton, electron, and antineutrino has been measured to be *1.25×10^{-13} J*. The mass of the proton is *1.6726×10^{-27} kg*, the mass of the electron is *9.11×10^{-31} kg*, and the mass of the antineutrino is much smaller than the electron mass. Calculate the mass of the neutron in *kg to 3 significant digits and be sure to state any assumptions that you make.*

Assume neutron at rest
$m_{\bar{\nu}_e} = 0$

$${}^1_0n \rightarrow {}^1_1p + {}^{\;0}_{-1}e + {}^0_0\bar{\nu}_e$$

$$M_n c^2 \rightarrow M_p c^2 + M_e c^2 + KE_p + KE_e + KE_{\bar{\nu}_e}$$

$$m_N = (M_p + M_e) + \frac{KE_T}{c^2}$$

$$M_N = (1.6726 \times 10^{-27}\,kg + 9.11 \times 10^{-31}\,kg) + \frac{1.25 \times 10^{-13}\,J}{(3 \times 10^8\,m/s)^2}$$

$$\boxed{M_N = 1.6749 \times 10^{-27}\,kg}$$

8. (4 points) You are pushing a box of mass m = 1kg a distance of 2 meters across the ground, taken to be the x-axis, at a constant speed. The force you apply is parallel to the ground and is given by $\vec{F} = \langle 10,0,0 \rangle N$.

   a. What is the work done by the applied force?

$$W = \int dW = \int \vec{F} \cdot d\vec{r} = \vec{F} \int \cdot d\vec{r} = \vec{F} \cdot \Delta\vec{r}$$

$$W_{you} = F_{you}\,\Delta x = 10N \cdot 2m = \boxed{20J = W_{you}}$$

   b. What is the work done by the normal force?

$$\boxed{W_{you} = F_N\,\Delta y = 0N}$$

   c. What is the work done by the force of gravity?

$$\boxed{W_g = -F_w\,\Delta y = 0N}$$

   d. What is the work done by the force of friction?

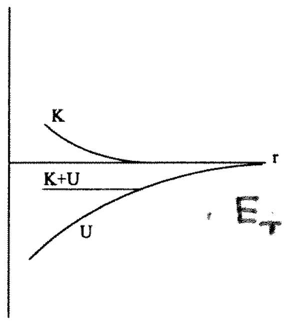
$$W_{friction} + W_{you} = 0 = \Delta E$$

block moves at constant speed.

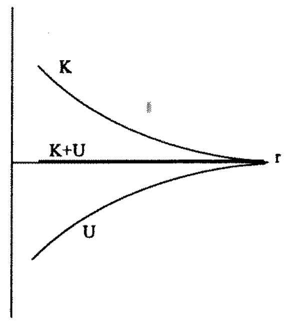
$$\therefore\ W_{friction} = -W_{you}$$

$$\boxed{W_{friction} = -20N}$$

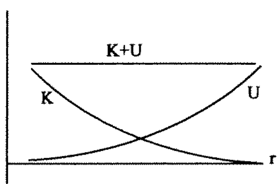
9. (2 points) You blast off from Mars, and you turn off the rockets when you are *a distance D* from the center of Mars, well above its thin atmosphere and headed away from the planet. You intend to leave Mars for good, and by the time you get very far away you want to be coasting at a speed of $V > 0$. Which of the following energy graphs best describes the Mars-spacecraft system? ***Circle the number next to the correct graph.***

1


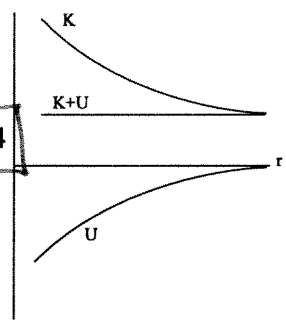
$E_T < 0$

2


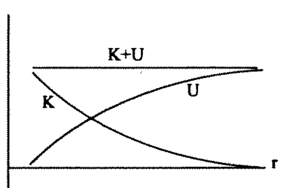
$E_T = 0$

3


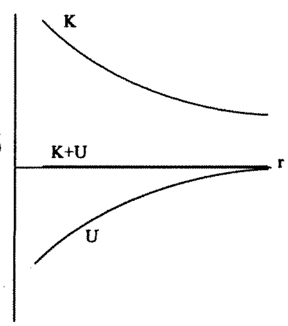
U_G incorrect

$\boxed{4}$


$E_T > 0$

5


U_g incorrect.

6


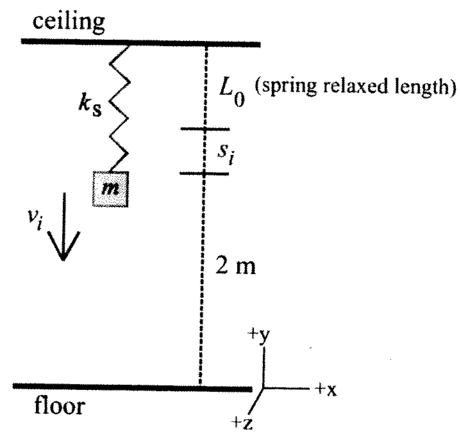
$E_T = 0$

Part 2: *Free-response problems.*

1. (10 points) A block with a mass of 0.045 kg is attached to a low mass spring, hanging vertically from the ceiling. The spring has a stiffness of 15.0 N/m. At time t = 0, the spring is stretched from its relaxed length by s_i = 0.08 m, and the block is 2.0 m above the floor (measured from the top of the block) and moving downward at a speed of v_i = 2.0 m/s. The block moves down and reaches a point where the spring has a stretch of s_f = 0.14 m.

   a. How much work is done by the spring on the mass from t = 0 to this new configuration? **Show all work.**

6pts



$$W_{spring} = \int \vec{F}_s \cdot d\vec{s}$$

$$= -\int_{s_i}^{s_f} ks \cdot ds$$

$$= -\left[\frac{1}{2} k s_f^2 - \frac{1}{2} k s_i^2\right]$$

$$W_{spring} = -\frac{1}{2}(15 N/m)\left[(0.14m)^2 - (0.08m)^2\right]$$

$$\boxed{W_{spring} = -0.099 J \doteq -0.1 J}$$

   b. What is the period of oscillation of the mass?

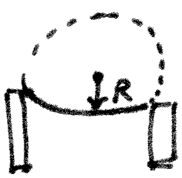
4pts

$$T = 2\pi\sqrt{\frac{M}{K}} = 2\pi\sqrt{\frac{0.045 kg}{15 N/m}} = \underline{0.344 s}$$

$$\boxed{T = 0.344 s}$$

2. (10 points) A suspended bridge is supported by its two end points across a canyon, and has a curved, bowl-like shape with radius of curvature R = 90m at its central, lowest point. A 10,000kg truck ignores a strange speed limit sign that resembles a physics equation, and drives down the slope, reaching a speed of 20 m/s at the lowest point.

   a. Assume for now that the bridge supports the truck. What is the acceleration of the truck at the lowest point?



4points

$$\vec{F}_{net} = \frac{d\vec{p}}{dt} \Rightarrow \langle 0, F_N - mg, 0 \rangle = \langle 0, \frac{mv^2}{R}, 0 \rangle$$

$$acc = \frac{v^2}{R} = \frac{(20\,m/s)^2}{90\,m} = \boxed{4.44\,m/s^2 = acc}$$

$$\boxed{\vec{a} = \langle 0, 4.44, 0 \rangle m/s^2}$$

   b. What is the normal force of the bridge supporting the truck at the same location?

4points

y-dir

$$F_N - mg = \frac{mv^2}{R} \rightarrow F_N = mg + \frac{mv^2}{R} = m(g + \frac{v^2}{R})$$

$$F_N = 10{,}000\,kg\,[9.8\,m/s^2 + 4.44\,m/s^2]$$

$$\boxed{F_N = 1.424 \times 10^5\,N} \Rightarrow \boxed{\vec{F}_N = \langle 0, 1.424 \times 10^5, 0 \rangle N}$$

   c. If the bridge can provide a maximum upward force of 1.2×10^5N before breaking will the truck make it across?

2points

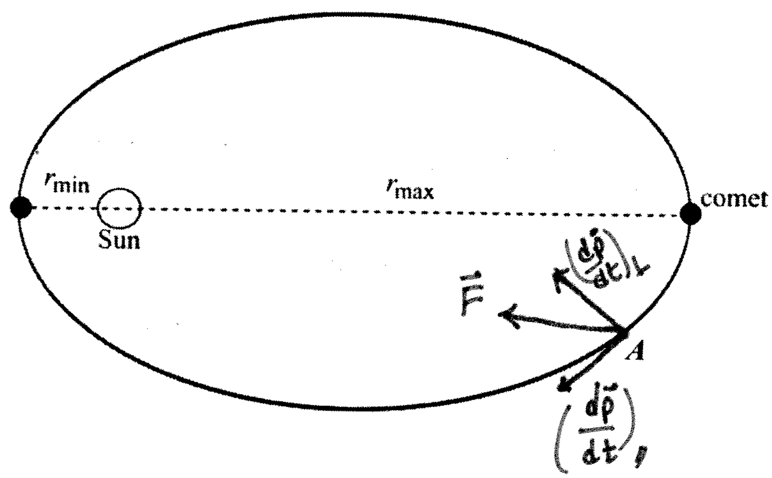
No Since $|\vec{F}_N| > |\vec{F}_{N,max}|$

3. (10 points) A comet is in an elliptical orbit around a star, as shown in the diagram.

   a. On the diagram, draw an arrow representing the net force on the comet when it is at location A. Label the arrow $\vec{F}$. 1 Point

   b. On the diagram, draw an arrow representing the component of $\frac{d\vec{p}}{dt}$ parallel to the comet's momentum. Label the arrow $\left(\frac{d\vec{p}}{dt}\right)_{//}$. This arrow must be consistent in magnitude and direction with your answer to (a) for full credit. 1 Point

   c. On the diagram, draw an arrow representing the component of $\frac{d\vec{p}}{dt}$ perpendicular to the comet's momentum. Label the arrow $\left(\frac{d\vec{p}}{dt}\right)_{\perp}$. This arrow must be consistent in magnitude and direction with your answers to (a) and (b) for full credit. 1 point



   d. The comet's closest approach to the star is a distance of $r_{\min} = 7\times10^{10}\,m$, at which point its speed is $7.5\times10^{4}\,\frac{m}{s}$. Its farthest distance from the star is $r_{\max} = 1.94\times10^{11}\,m$. The mass of the star is $M_{star} = 4\times10^{30}\,kg$, and the mass of the comet is $M_{comet} = 3\times10^{14}\,kg$. What is the speed of the comet when it is farthest from the star? ***Start from a fundamental principle and show all steps in your work.*** System: Star + Comet

$$\Delta E = \Delta W = 0 \rightarrow E_f = E_i \rightarrow KE_f + U_{G_f} = KE_i + U_{G_i}$$

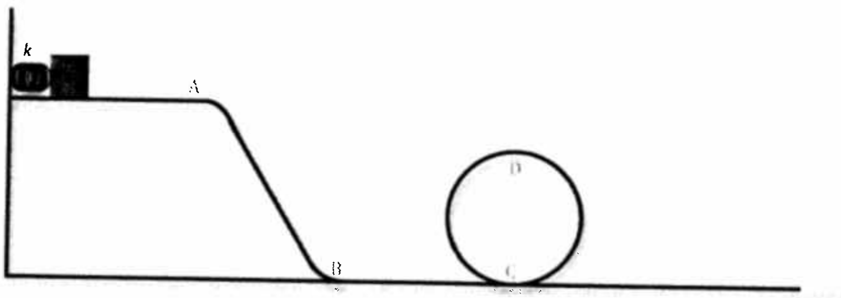
$$\frac{1}{2}\cancel{m_c}V_f^2 - \frac{G\cancel{M_c}M_s}{r_{max}} = \frac{1}{2}\cancel{m_c}V_i^2 - \frac{G\cancel{M_c}M_s}{r_{min}}$$

$$V_f^2 = V_i^2 - \frac{2GM_s}{r_{min}} + \frac{2GM_s}{r_{max}}$$

$$V_f^2 = (7.5\times10^4\,m/s)^2 + 2(6.67\times10^{-11}\,\tfrac{Nm^2}{kg^2})(4\times10^{30}\,kg)\left[\frac{-1}{7\times10^{10}\,m} + \frac{1}{1.94\times10^{11}\,m}\right]$$

$$V_f = \sqrt{7.527\times10^8\,m^2/s^2} = \boxed{2.74\times10^4\,m/s = V_f}$$

4. (9 points) Consider the amusement park thrill ride shown below. The cart has a mass of $m_{cart} = 300kg$ and 10 people at a time can ride and assume that the average mass of a rider is $m_{rider} = 70kg$. The ride starts out horizontal and is $20m$ above the ground and the cart and riders go down over the hill and around the loop-the-loop (of radius $R = 10m$). Unless otherwise stated, assume that the entire track is frictionless.



a. What is the speed of the cart and riders at point B if the speed at point A is $v_A = 2m/s$?

System: cart + riders + Earth

4 points

$$\Delta E = \Delta W = 0 \rightarrow \Delta KE + \Delta U_g + \cancel{\Delta U_s}^{\,0} + \cancel{\Delta E_{est}}^{\,0} = 0$$

Choose $U_g = 0$ @ $y = 0$; $\left(\frac{1}{2} m v_B^2 - \frac{1}{2} m v_A^2\right) + (mgy_B - mgy_A) = 0$

$$v_B = \sqrt{v_a^2 + 2g(y_A - y_B)} = \sqrt{(2 m/s)^2 + 2(9.8 m/s^2)\cdot(20m)}$$

$$\boxed{V_B = 19.9 m/s}$$

b. How much work is done by gravity as the cart and riders move between points C and D?

4 points

$$W_{grav} = \int \vec{F} \cdot d\vec{r} = \vec{F} \cdot \int d\vec{r} = \vec{F} \cdot \Delta\vec{r}$$

$$W_{grav} = \langle 0, -mg, 0 \rangle \cdot \langle \Delta x, \Delta y, \Delta z \rangle$$

$$= -mg\Delta y = -(1000kg)(9.8 m/s^2)(20m)$$

$$\boxed{W_{grav} = -1.96 \times 10^5 J}$$

c. Suppose that the sum of the kinetic and gravitational potential energies of the cart and riders at point A is E_{top}, while at point B is E_{bottom}. Taking into account air resistance between points A and B, we have that the

1 point

I. $E_{top} < E_{bottom}$.

II. $E_{top} = E_{bottom}$.

$\boxed{III.}$ $E_{top} > E_{bottom}$.

IV. relation between E_{top} and E_{bottom} changes in a way that we cannot predict.

$$\Delta E = W_{ext}$$

$$E_f = E_i - W_{friction}$$

$$\therefore E_f < E_i$$

$$\boxed{E_B < E_{Top}}$$

Equations and Constants

$$\vec{v}_{avg} = \frac{\Delta \vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i} \qquad \vec{r}_f = \vec{r}_i + \vec{v}_{avg}\Delta t$$

$$\vec{p} = \gamma m \vec{v} \qquad \gamma = \frac{1}{\sqrt{1 - \left(\frac{|\vec{v}|}{c}\right)^2}},\ where\ c = 3 \times 10^8\ m/s$$

$$\Delta \vec{p} = \vec{F}_{net}\Delta t \qquad \vec{p}_f = \vec{p}_i + \vec{F}_{net}\Delta t$$

$$\Delta \vec{p}_{total} = \vec{F}_{net,ext}\Delta t \qquad \Delta \vec{p}_{system} + \Delta \vec{p}_{surroundings} = 0$$

$$\vec{F}_{grav\,on\,2\,by\,1} = -G\frac{m_1 m_2}{|\vec{r}|^2}\hat{r},\ where\ \vec{r} = \vec{r}_2 - \vec{r}_1\ and\ G = 6.7 \times 10^{-11} N \cdot m^2/kg^2$$

$$\left|\vec{F}_{grav}\right| = mg,\ where\ g = 9.8\,N/kg \qquad \left|\vec{F}_{spring}\right| = k_s\,|s|\,,\ where\ s = L - L_0$$

$$\vec{F}_{elec\,on\,2\,by\,1} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{|\vec{r}|^2}\hat{r},\ \vec{r} = \vec{r}_2 - \vec{r}_1\ and\ \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 N \cdot m^2/C^2$$

$$\frac{d\vec{p}}{dt} = \vec{F}_{net} \qquad \left(\frac{d\,|\vec{p}|}{dt}\right)\hat{p} = \vec{F}_{\parallel}\ and\ |\vec{p}|\frac{d\hat{p}}{dt} = |\vec{p}|\frac{|\vec{v}|}{R}\hat{n} = \vec{F}_{\perp}$$

$$x = A\cos(\omega t)\,,\ with\ \omega = \sqrt{\frac{k_s}{m}} \qquad T = \frac{2\pi}{\omega} \qquad f = \frac{1}{T} = \frac{\omega}{2\pi}$$

$$f_s \leq \mu_s F_N \qquad f_k = \mu_k F_N$$

$$E_{particle} \equiv \gamma mc^2 = mc^2 + K \qquad E^2 - (pc)^2 = \left(mc^2\right)^2$$

$$K \approx \frac{1}{2}mv^2 = \frac{p^2}{2m}\ (if\ v \ll c) \qquad \Delta E_{sys} = W_{surr}$$

$$W_F = \vec{F} \bullet \Delta\vec{r} = F_x\Delta x + F_y\Delta y + F_z\Delta z = F\Delta r\cos\theta$$

$$W_F = \sum \vec{F} \bullet \Delta\vec{r}\ or\ W_F = \int_i^f \vec{F} \bullet d\vec{r} \qquad \Delta E_{sys} + \Delta E_{surr} = 0$$

$$E_{sys} \equiv \left(m_1c^2 + m_2c^2 + \ldots\right) + (K_1 + K_2 + \ldots) + (U_{12} + \ldots)$$

$$\Delta U = -W_{int} \qquad F_x = -\frac{dU}{dx} \qquad U = -G\frac{m_1 m_2}{r}$$

$$\Delta U = \Delta\,(mgy) \qquad U_{el} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$$