

Physics 120

Exam #3

March 6, 2026

Name _____

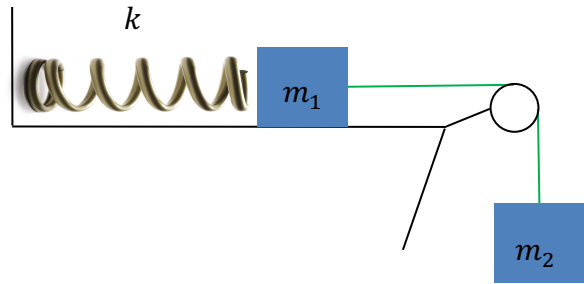
Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So, erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example,
 $|\vec{p}| \approx m|\vec{v}| = (5\text{kg}) \times (2\frac{\text{m}}{\text{s}}) = 10\frac{\text{kg}\cdot\text{m}}{\text{s}}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or reasonable value for the quantity you cannot calculate (explain that you are doing this), and use it to do the rest of the problem.
- Each free-response part is worth 6 points

Problem #1	/18
Problem #2	/18
Problem #3	/18
Problem #4	/18
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

1. Suppose you have the setup shown on the right in which two masses $m_1 = 1kg$ and $m_2 = 3kg$ are connected by a light rope passed over a massless pulley. To the block of mass m_1 , a spring of stiffness $k = 100\frac{N}{m}$ is attached.



- a. Define your system and using energy ideas, what is the maximum extension y_{max} of the spring when mass m_2 is released from rest. Assume that the spring starts from its equilibrium position.

Considering the system to be the two masses, the spring and the world, we have:

$$\Delta E_{system} = 0 = \Delta K_1 + \Delta K_2 + \Delta U_{g1} + \Delta U_{g2} + \Delta U_s$$

At maximum extension $\Delta K_1 = \Delta K_2 = 0$, mass m_1 never changes height so $\Delta U_{g1} = 0$, and mass m_2 falls by y_{max} stretching the spring by y_{max} .

$$\Delta E_{system} = 0 = \Delta U_{g2} + \Delta U_s = (m_2 g y_f - m_2 g y_i) + \left(\frac{1}{2} k y_f^2 - \frac{1}{2} k y_i^2 \right)$$

$$0 = -m_2 g y + \frac{1}{2} k y^2 = \left(-m_2 g + \frac{1}{2} k y \right) y$$

$$\rightarrow \begin{cases} y_{min} = 0 \\ y_{max} = \frac{2m_2 g}{k} = \frac{2 \times 3kg \times 9.8\frac{m}{s^2}}{100\frac{N}{m}} = 0.59m \end{cases}$$

- b. Define your system and using energy ideas, what are the speeds of the blocks when the spring has been stretched by an amount $y = 0.3y_{max}$ from equilibrium?

Considering the system to be the two masses, the spring and the world, we have:

$$\Delta E_{system} = 0 = \Delta K_1 + \Delta K_2 + \Delta U_{g1} + \Delta U_{g2} + \Delta U_s$$

Mass m_1 rises by $0.3y_{max}$ and acquires a speed v_{1f} . Mass m_2 falls by $0.3y_{max}$ and acquires a speed v_{2f} , where $v_{1f} = v_{2f} = v_f$ and the spring stretches by $0.3y_{max}$.

$$0 = \frac{1}{2}m_1v_f^2 + \frac{1}{2}m_2v_f^2 + m_1gy_f - m_2gy_f + \frac{1}{2}ky_f^2$$

$$\rightarrow v_f = \sqrt{\frac{2(m_2 - m_1)gy_f - ky_f^2}{m_1 + m_2}}$$

$$v_f = \sqrt{\frac{2(3kg - 1kg) \times 9.8 \frac{m}{s^2} \times 0.3 \times 0.59m - 100 \frac{N}{m} (0.3 \times 0.59m)^2}{1kg + 3kg}} = 0.98 \frac{m}{s}$$

- c. Suppose that the surface that the block of mass m_1 slides across were not frictionless, but that friction existed with coefficient of friction $\mu = 0.4$. In this case, what would be the maximum extension of the spring? How does this compare to the case without friction? Does the answer make sense? Explain why it does or does not.

Considering the system to be the two masses, the spring and the world, we have:

$$\Delta E_{system} = W_{fr} = \Delta K_1 + \Delta K_2 + \Delta U_{g1} + \Delta U_{g2} + \Delta U_s$$

At maximum extension $\Delta K_1 = \Delta K_2 = 0$, mass m_1 never changes height so $\Delta U_{g1} = 0$, and mass m_2 falls by y'_{max} stretching the spring by y'_{max} .

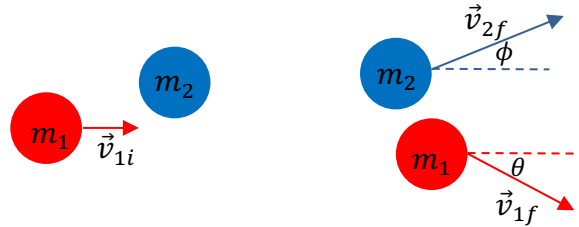
$$\Delta E_{system} = F_{Fr}y \cos 180 = \Delta U_{g2} + \Delta U_s = (m_2gy_f - m_2gy_i) + \left(\frac{1}{2}ky_f^2 - \frac{1}{2}ky_i^2\right)$$

$$-\mu F_{N1}y = -\mu m_1gy = -m_2gy + \frac{1}{2}ky^2 = \left(-m_2g + \frac{1}{2}ky\right)y$$

$$\rightarrow \begin{cases} y_{min} = 0 \\ y_{max} = \frac{2(m_2 - \mu m_1)g}{k} = \frac{2 \times (3kg - 0.4 \times 1kg) \times 9.8 \frac{m}{s^2}}{100 \frac{N}{m}} = 0.51m \end{cases}$$

Yes, this does makes sense. Friction does work on the system and the loss of gravitational potential energy in mass m_2 does not all get converted into elastic potential energy in the spring, as some energy is lost to friction. Thus, the spring doesn't stretch as far as it would in the case without friction.

2. Two balls of masses $m_1 = m$ and $m_2 = 3m$ shown below undergo a glancing collision in the plane of the page. Ball of mass m_1 has an initial velocity $\vec{v}_i = \langle 4, 0, 0 \rangle \frac{m}{s}$ and ball of mass m_2 is at rest. Because of the glancing collision the ball of mass m_2 is scattered through an angle $\phi = 21^\circ$ while the ball of mass m_1 is scattered through an angle $\theta = 38^\circ$.



- a. Starting with the *full vector equations* that govern the momenta of each ball before and after the collision, what is the **velocity** of the ball of mass m_2 after the collision?

Conservation of momentum:

$$\vec{p}_{i,system} = \vec{p}_{f,system} \rightarrow \vec{p}_{i1} + \vec{p}_{i2} = \vec{p}_{f1} + \vec{p}_{f2}$$

$$m_1 \langle v_{1i}, 0, 0 \rangle = m_1 \langle v_{1f} \cos \theta, -v_{1f} \sin \theta, 0 \rangle + m_2 \langle v_{2f} \cos \phi, v_{2f} \sin \phi, 0 \rangle$$

y-direction:

$$0 = -m_1 v_{1f} \sin \theta + m_2 v_{2f} \sin \phi$$

$$\rightarrow v_{1f} = \left(\frac{m_2 \sin \phi}{m_1 \sin \theta} \right) v_{2f} = \left(\frac{3m \sin 21}{m \sin 38} \right) v_{2f} = 1.75 v_{2f}$$

x-direction:

$$m_1 v_{1i} = m_1 v_{1f} \cos \theta + m_2 v_{2f} \cos \phi$$

$$m v_{1i} = m (1.75 v_{2f}) \cos \theta + 3m v_{2f} \cos \phi$$

$$v_{2f} = \frac{v_{1i}}{1.75 \cos \theta + 3 \cos \phi} = \frac{4 \frac{m}{s}}{1.75 \cos 38 + 3 \cos 21} = 0.96 \frac{m}{s}$$

$$\vec{v}_{2f} = \langle 0.90, 0.34, 0 \rangle \frac{m}{s} \text{ or } 0.96 \frac{m}{s} \text{ at } \phi = 21^\circ \text{ above the positive x-axis.}$$

- b. Using the *full vector equations* that you wrote in part a, what is the **velocity** of the ball of mass m_1 after the collision?

See part a for the work.

$$\rightarrow v_{1f} = 1.75 v_{2f} = 1.75 \times 0.96 \frac{m}{s} = 1.67 \frac{m}{s}$$

$$\vec{v}_{1f} = \langle 1.31, -1.03, 0 \rangle \frac{m}{s} \text{ or } 1.67 \frac{m}{s} \text{ at } \phi = 38^\circ \text{ below the positive x-axis.}$$

- c. Explain what it means for a collision to be elastic and inelastic and using your explanation, was the collision above elastic or inelastic? To answer the second part, you will need to perform a calculation and assume $m = 1kg$.

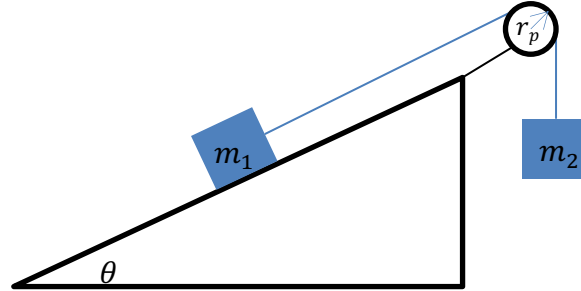
To be elastic $\Delta K = 0$. If $\Delta K \neq 0$ then the collision is inelastic. To determine which, we calculate $\Delta K = K_f - K_i$.

$$\Delta K = K_f - K_i = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2 - \frac{1}{2}m_1v_{1i}^2$$

$$\Delta K = \frac{1kg}{2} \left(\left(1.67\frac{m}{s}\right)^2 - \left(4\frac{m}{s}\right)^2 \right) + \frac{3 \times 1kg}{2} \left(0.96\frac{m}{s}\right)^2 = -5.52J$$

Since $\Delta K \neq 0$ this collision is inelastic.

3. Consider the system of masses shown below where a block of mass m_1 is on a ramp inclined at angle θ measured with respect to the horizontal. Friction exists between this block and the ramp with coefficient of friction μ . To this mass, a block of mass m_2 is suspended by a light rope that passes over a pulley with mass m_p and radius r_p .



- a. Starting from Newton's laws of motion in *full vector form for the forces and torques*, what is the magnitude of the acceleration of the system if the system is released from rest? Be sure to specify your coordinate system for each mass and for the direction for the torques.

For mass m_1 , we assume a tilted coordinate system with positive x-axis up the ramp and the positive y-axis perpendicular to the incline while for mass m_2 , we assume a standard cartesian coordinate system with positive y-vertically up. For the pulley, we assume counterclockwise rotations are positive and clockwise rotations are negative.

For mass m_1 :

$$\vec{F}_{net1} = \langle F_{TL} - F_{fr} - F_{W1x}, F_N - F_{W1y}, 0 \rangle = m_1 \vec{a} = \langle m_1 a, 0, 0 \rangle$$

$$\rightarrow F_{TL} = F_W \sin \theta + \mu F_{N1} + m_1 a = m_1 g \sin \theta + \mu m_1 g \cos \theta + m_1 a$$

For mass m_2 :

$$\vec{F}_{net2} = \langle 0, F_{TR} - F_{W2}, 0 \rangle = m_2 \vec{a} = \langle 0, -m_2 a, 0 \rangle$$

$$\rightarrow F_{TR} = F_{W2} - m_2 a = m_2 g - m_2 a$$

For the pulley:

$$\vec{\tau}_{net} = \langle 0, 0, r_p F_{TL} - r_p F_{TR} \rangle = I \vec{\alpha} = \langle 0, 0, -I \alpha \rangle$$

$$F_{TL} - F_{TR} = -I \frac{\alpha}{r_p}$$

$$\rightarrow m_1 g \sin \theta + \mu m_1 g \cos \theta + m_1 a - m_2 g + m_2 a = -\left(\frac{1}{2} m_p r_p^2\right) \frac{a}{r_p^2}$$

$$a = \left(\frac{m_2 - m_1 \sin \theta - \mu m_1 \cos \theta}{m_1 + m_2 + \frac{1}{2} m_p} \right) g$$

- b. Using the result from part a, if the block of mass m_1 slides a distance d up the incline, what are the final speeds of the blocks of mass m_1 and m_2 ?

$v_{1f} = v_2 = v_f$ since they are connected by the rope.

$$v_f^2 = v_i^2 + 2ad \rightarrow v_f = \sqrt{2ad} = \sqrt{2 \left(\frac{m_2 - m_1 \sin \theta - \mu m_1 \cos \theta}{m_1 + m_2 + \frac{1}{2}m_p} \right) gd}$$

- c. Defining your system and using energy ideas, what is the final speed of the blocks of masses m_1 and m_2 if the block of mass m_1 slides a distance d up the ramp if the system is released from rest? What can you say about the speed of the block determined in part b and the speed determined here in part c? Are they the same? Should they be why or why not? Explain.

Let the system be m_1, m_2 , the pulley and the world.

$$\Delta E = W_{fr} = \Delta K_1 + \Delta U_{g1} + \Delta K_2 + \Delta U_{g2} + \Delta K_R$$

$$W_{fr} = F_{fr1} d \cos \phi = \mu m_1 g \cos \theta d \cos 180 = -\mu m_1 g d \cos \theta$$

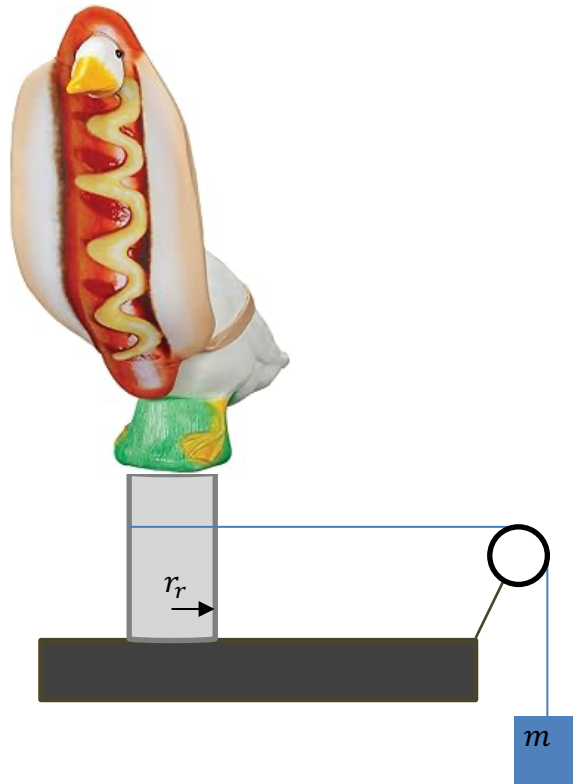
$$-\mu m_1 g d \cos \theta = \frac{1}{2} m_1 v_f^2 + \frac{1}{2} m_2 v_f^2 + \frac{1}{2} I \omega_f^2 + m_1 g \sin \theta - m_2 g$$

$$-\mu m_1 g d \cos \theta = \frac{1}{2} m_1 v_f^2 + \frac{1}{2} m_2 v_f^2 + \frac{1}{2} \left(\frac{1}{2} m_p r_p^2 \right) \frac{v_f^2}{r_p^2} + m_1 g \sin \theta - m_2 g$$

$$v_f = \sqrt{2 \left(\frac{m_2 - m_1 \sin \theta - \mu m_1 \cos \theta}{m_1 + m_2 + \frac{1}{2}m_p} \right) gd}$$

Yes, these two expressions are exactly the same, as they should be. There is no difference between looking at the forces and torques to calculate the acceleration and then the speed of the system, or applying conservation of energy. Both should yield the same answer.

4. Consider the system shown below in which a priceless piece of art is attached to a rod of radius r_r . To the rod a light string is wound and the other end of the string passes over a light pulley and a mass m is connected as shown below.



- a. Starting with the full vector expression for the forces and torques, what is the expression for the moment of inertia of the piece of art in terms of the acceleration created by the falling mass m if the mass m is released from rest?

For mass m assuming vertically up is the positive y-direction:

$$\vec{F}_{net} = \langle 0, F_T - F_W, 0 \rangle = m\vec{a} = \langle 0, -ma, 0 \rangle$$

$$\rightarrow F_T = F_W - ma = mg - ma$$

For the pulley assuming counter clockwise rotations are positive:

$$\vec{\tau}_{net} = \langle 0, 0, r_r F_T \rangle = I\vec{\alpha} = \langle 0, 0, I\alpha \rangle$$

$$F_T = I \frac{\alpha}{r_r} \rightarrow mg - ma = I \left(\frac{a}{r_r^2} \right)$$

$$\rightarrow I = \left(\frac{g}{a} - 1 \right) m r_r^2$$

- b. Using energy ideas, what is the what is the expression for the moment of inertia of the piece of art in terms of the speed of the falling mass m if the mass m is released from rest and falls through a height y ?

Assuming that the system is the mass, the piece of art and the world, we have from conservation of energy:

$$\Delta E_{system} = 0 = \Delta K_m + \Delta K_R + \Delta U_g = \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2 - mgy$$

$$0 = \frac{1}{2}mv_f^2 + \frac{1}{2}I\left(\frac{v_f}{r_r}\right)^2 - mgy \rightarrow 2gy = v_f^2\left(1 + \frac{I}{mr_r^2}\right)$$

$$\rightarrow I = mr_r^2\left(\frac{2gy}{v_f^2} - 1\right)$$

- c. You have two expressions for the moment of inertia that you derived in parts a and b. One was determined by looking at the forces and torques involved and the other by conservation of energy. Explain in full detail, which of the two methods, if either, would yield a better measurement of the moment of inertia for the piece of art? If both would yield an acceptable moment of inertia, explain why they both would.

From parts a and b, we have two expressions for the moment of inertia of the piece of art. Equating these two expressions we have:

$$I = mr_r^2\left(\frac{2gy}{v_f^2} - 1\right) = \left(\frac{g}{a} - 1\right)mr_r^2$$

These two expressions give me one of my equations of motion: $v_f^2 = 2ay$. Since I get one of my equations of motion back, this says that both methods are equivalent as I would expect. The methods to solve problems, either forces/torques or energies should yield the same outcome, independent of the method.