Physics 120

Exam #3

May 27, 2011

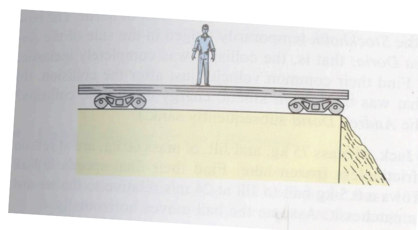
Name____________________________________

Multiple Choice	/16
Problem #1	/28
Problem #2	/28
Problem #3	/28
Total	/100

Part I: Multiple-Choice: *Circle the best answer to each question. Any other marks will not be given credit. Each multiple-choice question is worth 4 points for a total of 16 points.*

1. Suppose that a mass is dropped from a tall building. At the top of the building the mass has a gravitational potential energy U_g while at the base of the building (just before it collides with the ground) the mass has a kinetic energy *KE*. Taking into account air resistance we have that the

   a. $KE < U_g$.
   b. $KE = U_g$.
   c. $KE > U_g$.
   d. relation between *KE* and U_g changes in a way that we cannot predict.



2. Super spy James Bond find himself caught in a trap set by *SPECTRE* in which Bond finds himself at the center of a railway car that has been placed at the edge of a cliff. Which way should Bond walk to minimize the danger of falling off of the edge of the cliff? (He cannot jump forward or backwards off of the car or be rescued by anyone from above.)

   a. To the left. b. To the right. c. There is no way to minimize the danger.

3. Suppose that the momentum of a particle is *p* and this particle has a kinetic energy *KE*. If the momentum of the particle doubles, the kinetic energy
   a. decreases by a factor of 2.
   b. increases by a factor of 2.
   c. increases by a factor of 4.
   d. cannot be determined since there is no relationship between momentum and kinetic energy.

4. If it takes 4J of work to stretch a spring by 10cm from its equilibrium length, how much extra work does it take to stretch the same spring by an additional 10cm?
   a. 4J b. 8J c. 12 J d.16 J

Part II: Free Response Problems: *The three problems below are worth 84 points total and each subpart is worth 7 points each. Please show all work in order to receive partial credit. If your solutions are illegible or illogical no credit will be given. A number with no work shown (even if correct) will be given no credit. Please use the back of the page if necessary, but number the problem you are working on.*

1. Two identical springs of stiffness $k = 100\ N/m$ lie horizontally on a frictionless track and are attached to either side of a $2kg$ mass. The mass is initially pulled to the right and the right spring is compressed by an amount $A = 0.25m$ from the right spring's equilibrium length while the left spring is stretched by the same amount $A = 0.25m$ from that spring's equilibrium length. The mass is released from rest at a time $t_i = 0s$.

   a. Using energy methods, what is the expression for and the speed of the mass as it passes through the equilibrium point?

$$\Delta E = 0 = \Delta KE + \Delta U_g + \Delta U_s = \left(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2\right) + \left(\frac{1}{2}kx_{f,L}^2 - \frac{1}{2}kx_{i,L}^2\right) + \left(\frac{1}{2}kx_{f,R}^2 - \frac{1}{2}kx_{i,R}^2\right)$$

$$0 = \frac{1}{2}mv_f^2 - \frac{1}{2}kx_{i,L}^2 - \frac{1}{2}kx_{i,R}^2 = \frac{1}{2}mv_f^2 - \frac{1}{2}k(A)^2 - \frac{1}{2}k(-A)^2$$

$$v_f = \sqrt{\frac{2kA^2}{m}} = \sqrt{\frac{k_{eff}}{m}}A = \omega A = \sqrt{\frac{2\times 100\,\frac{N}{m}}{2kg}} \times 0.25m = 2.5\,\tfrac{m}{s}$$

   b. Using energy methods, what is the expression for and the speed of the mass when it is at a location $x_f = \dfrac{A}{2}$ moving to the left?

$$\Delta E = 0 = \Delta KE + \Delta U_g + \Delta U_s = \left(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2\right) + \left(\frac{1}{2}kx_{f,L}^2 - \frac{1}{2}kx_{i,L}^2\right) + \left(\frac{1}{2}kx_{f,R}^2 - \frac{1}{2}kx_{i,R}^2\right)$$

$$0 = \frac{1}{2}mv_f^2 + \left(\frac{1}{2}k\left(\frac{A}{2}\right)^2 - \frac{1}{2}k(A)^2\right) + \left(\frac{1}{2}k\left(\frac{-A}{2}\right)^2 - \frac{1}{2}k(-A)^2\right) = \frac{1}{2}mv_f^2 - \frac{3kA^2}{4}$$

$$v_f = \sqrt{\frac{6kA^2}{4m}} = \sqrt{\frac{6\times 100\,\frac{N}{m}}{4\times 2kg}} \times 0.25m = 2.17\,\tfrac{m}{s}$$

   c. What is the initial acceleration of the mass?

$$\vec{F}_{net} = \langle -kA - kA, 0, 0\rangle = \frac{d\vec{p}}{dt} = m\frac{d\vec{v}}{dt} = m\vec{a}$$

$$\vec{a} = \frac{\langle -2kA, 0, 0\rangle}{m} = \left\langle \frac{-2\times 100\,\frac{N}{m}\times 0.25m}{2kg}, 0, 0\right\rangle = \langle -25, 0, 0\rangle\,\tfrac{m}{s^2}$$

   d. What is the period of the masses oscillation?

$$T = 2\pi\sqrt{\frac{m}{k_{eff}}} = 2\pi\sqrt{\frac{m}{2k}} = 2\pi\sqrt{\frac{2kg}{2\times 100\,\frac{N}{m}}} = 0.63s$$

2. A karate expert strikes downward with her hand of mass $m_{fist} = 0.7kg$ breaking a $0.14kg$ board. The stiffness of the board is $4.1x10^4$ N/m and the board breaks at a deflection $y_f = 16mm$ from a starting position of $y_i = 0mm$. You should note that this is not exactly the same situation as you encountered in lab. Here we are going to model the collision between your hand and the board as an inelastic collision and immediately after the collision your hand and the board ($v_{hand + board}$) will be moving with the same speed. The board will do work on your hand bringing it to rest after the board has been deflected by the distance of *16mm*.

   a. Apply conservation of energy to the situation of your hand striking the board and determine how fast are *the board and your hand* ($v_{hand + board}$) are moving just after you strike the board.

$$\Delta E = 0 = \Delta KE + \Delta U_g + \Delta U_s = \left(\frac{1}{2}m_{h+b}v_{h+b,f}^2 - \frac{1}{2}m_{h+b}v_{h+b,i}^2\right) + \left(m_{h+b}gy_f - m_{h+b}gy_i\right) + \left(\frac{1}{2}ky_f^2 - \frac{1}{2}ky_i^2\right)$$

$$0 = -\frac{1}{2}m_{h+b}v_{h+b,i}^2 + m_{h+b}gy_f + \frac{1}{2}ky_f^2 = -\frac{1}{2}m_{h+b}v_{h+b,i}^2 - m_{h+b}gd + \frac{1}{2}k(-d)^2$$

$$v_{h+b} = \sqrt{\frac{2\left(-m_{h+b}gd + \frac{1}{2}k(-d)^2\right)}{m_{h+b}}} = \sqrt{\frac{2\left(\left(-0.84kg \times 9.8\tfrac{m}{s^2} \times 16\times10^{-3}m\right) + \left(\frac{1}{2}\times 4.1\times10^4\tfrac{N}{m}\left(16\times10^{-3}m\right)^2\right)\right)}{0.84kg}}$$

$$v_{h+b} = 3.55\tfrac{m}{s}$$

   b. What is the minimum speed that your hand (v_{hand}) must be moving before it collides with the board so that you can break this karate board?

$$\frac{d\vec{p}}{dt} = \vec{F}_{net,ext} = 0 \rightarrow \Delta\vec{p} = 0 \rightarrow \vec{p}_i = \vec{p}_f$$

$$\langle 0, -m_h v_h, 0\rangle = \langle 0, -m_{h+b}v_{h+b}, 0\rangle$$

$$v_h = \frac{m_{h+b}v_{h+b}}{m_h} = \frac{0.84kg \times 3.55\tfrac{m}{s}}{0.7kg} = 4.26\tfrac{m}{s}$$

c. If instead of a breaking a board, you wanted to break a cement block of mass *3.2kg*. What minimum speed would your hand, v_{hand}, need to be moving to break a cement block? Take $k_{block} = 2.6x10^6 N/m$ and suppose that a deflection of *1mm* will break the block.

$$\Delta E = 0 = \Delta KE + \Delta U_g + \Delta U_s = \left(\frac{1}{2}m_{h+b}v_{h+b,f}^2 - \frac{1}{2}m_{h+b}v_{h+b,i}^2\right) + \left(m_{h+b}gy_f - m_{h+b}gy_i\right) + \left(\frac{1}{2}ky_f^2 - \frac{1}{2}ky_i^2\right)$$

$$0 = -\frac{1}{2}m_{h+b}v_{h+b,i}^2 + m_{h+b}gy_f + \frac{1}{2}ky_f^2 = -\frac{1}{2}m_{h+b}v_{h+b,i}^2 - m_{h+b}gd + \frac{1}{2}k(-d)^2$$

$$v_{h+b} = \sqrt{\frac{2\left(-m_{h+b}gd + \frac{1}{2}k(-d)^2\right)}{m_{h+b}}} = \sqrt{\frac{2\left(\left(-3.9kg \times 9.8\tfrac{m}{s^2} \times 1\times10^{-3}m\right) + \left(\frac{1}{2}\times 2.6\times10^6\tfrac{N}{m}\left(1\times10^{-3}m\right)^2\right)\right)}{3.9kg}}$$

$$v_{h+b} = 0.80\tfrac{m}{s}$$

$$\frac{d\vec{p}}{dt} = \vec{F}_{net,ext} = 0 \rightarrow \Delta\vec{p} = 0 \rightarrow \vec{p}_i = \vec{p}_f$$

$$\langle 0, -m_h v_h, 0\rangle = \langle 0, -m_{h+b}v_{h+b}, 0\rangle$$

$$v_h = \frac{m_{h+b}v_{h+b}}{m_h} = \frac{3.9kg \times 0.80\tfrac{m}{s}}{0.7kg} = 4.48\tfrac{m}{s}$$

Which coincidentally is not much faster than you'd need to break the board.

d. How much work was done on your hand by the board in performing this "feat of strength?" by the block?

$$W_{board} = \Delta KE_{hand} = \frac{1}{2}m_h v_{f,after}^2 - \frac{1}{2}m_h v_{i,after}^2 = -\frac{1}{2}m_h v_{i,after}^2 = -\frac{1}{2}(0.7kg)\left(3.55\tfrac{m}{s}\right)^2 = -4.41J$$

$$W_{block} = \Delta KE_{hand} = \frac{1}{2}m_h v_{f,after}^2 - \frac{1}{2}m_h v_{i,after}^2 = -\frac{1}{2}m_h v_{i,after}^2 = -\frac{1}{2}(0.7kg)\left(0.8\tfrac{m}{s}\right)^2 = -0.22J$$

3. You are driving your car ($m_1 = 1000kg$) down a straight road in the middle of winter and the road you are driving on is icy (with a coefficient of kinetic friction between your tires and the ice of $\mu_k = 0.4$.) In front of you is another car (of mass $m_2 = 1400kg$). The traffic light ahead on the road that both your car and the one in front of you are traveling on turns from green to red and both of you apply the brakes in your respective cars. You both slide along the ice and the car in front of you stops at the light, but you do not quite make it and unfortunately hit the stopped car in front of you.

   a. After the collision at the light you stop a distance of *7.3m* from the point of impact with the car in front of you. How fast were you going after the collision?

$$W = -F_{fr}d = \Delta KE = \frac{1}{2}mv_{f,after}^2 - \frac{1}{2}mv_{i,after}^2$$

$$v_{i,after} = \sqrt{\frac{2F_{fr}d}{m}} = \sqrt{\frac{2\mu_k mgd}{m}} = \sqrt{2\mu_k gd} = \sqrt{2\times 0.4\times 9.8\tfrac{m}{s^2}\times 7.3m}$$

$$v_{i,after} = 7.6\tfrac{m}{s}$$

   b. If the car you hit ended up at a distance of *10.4m* ahead of the point of impact, with what initial speed did you hit the car in front of you with? Compare this speed with the posted speed on the road (*30mph ~ 13m/s*.) Would a cop have issued you a ticket? Justify your answer with calculations.

$$\frac{d\vec{p}}{dt} = \vec{F}_{net,ext} = 0 \rightarrow \Delta\vec{p} = 0 \rightarrow \vec{p}_i = \vec{p}_f$$

$$\langle m_y v_{y,before}, 0, 0\rangle = \langle m_y v_{y,after} + m_o v_{o,after}, 0, 0\rangle$$

$$v_{y,before} = \frac{m_y v_{y,after} + m_o v_{o,after}}{m_y} = \frac{1100kg\times 7.6\tfrac{m}{s} + 1400kg\times 9.0\tfrac{m}{s}}{1100kg} = 19.1\tfrac{m}{s}$$,where

$$v_{o,after} = \sqrt{\frac{2F_{fr}d}{m}} = \sqrt{\frac{2\mu_k mgd}{m}} = \sqrt{2\mu_k gd} = \sqrt{2\times 0.4\times 9.8\tfrac{m}{s^2}\times 10.4m}$$

$$v_{o,after} = 9.0\tfrac{m}{s}$$

Not only would you have probably got a ticket for speeding (19.1m/s ~ 43mph) but you would also probably have been issued one for failure to drive according to the road conditions, which means that you should have probably been doing less than 30mph on the ice anyway.

c. How much work did friction do in bringing you to rest after you hit the car in front of you?

$$W_{fr} = -F_{fr}d = -\mu_k mgd = -0.4 \times 1100kg \times 9.8\tfrac{m}{s^2} \times 7.3m = -3.2 \times 10^4 J$$

$$W_{fr} = \Delta KE = \frac{1}{2}m_y v^2_{y,f,after} - \frac{1}{2}m_y v^2_{y,i,after} = -\frac{1}{2} \times 1100kg \times \left(7.6\tfrac{m}{s}\right)^2 = -3.2 \times 10^4 J$$

d. If the collision lasted for a time *Δt = 0.8s*, what was the impulse force on your car due to the collision with the car in front of you?

$$\frac{d\vec{p}_y}{dt} = \frac{\left\langle m_y v_{y,i,after} - m_y v_{y,before}, 0, 0\right\rangle}{dt} = \vec{F}_{net,ext,y}$$

$$\vec{F}_{net,ext,y} = \frac{1100kg \times \left\langle 7.6\tfrac{m}{s} - 19.1\tfrac{m}{s}, 0, 0\right\rangle}{0.8s} = \left\langle -1.6 \times 10^4, 0, 0\right\rangle N$$

Useful formulas:

$\vec{p} = \gamma m\vec{v}$

$k_{eff,parallel} = n_{parallel} k_{individual}$

$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

$k_{eff,series} = \frac{k_{individual}}{n_{series}}$

$\vec{v}_{avg} = \frac{\vec{v}_i + \vec{v}_f}{2}$

$stress = Ystrain \rightarrow \frac{F}{A} = Y\frac{\Delta L}{L}$

$\vec{F}_g = m\vec{g}$

$\vec{F}_{gravity} = \frac{GM_1M_2}{r_{12}^2}\hat{r}_{12}$

$T = \frac{1}{f} = 2\pi\sqrt{\frac{m}{k_{eff}}}$

$\vec{F}_{spring} = -k\vec{s};\quad \vec{s} = (L - L_o)\hat{s}$

$W = \int \vec{F}\cdot d\vec{r} = \Delta KE = -\Delta U$

$U_g = -\frac{GM_1M_2}{r}$

$U_g = mgy$

$U_s = \frac{1}{2}ks^2$

$KE = \frac{1}{2}mv^2$

$KE = (\gamma - 1)mc^2$

$E_{rest} = mc^2$

Constants

$g = 9.8\,\frac{m}{s^2}$

$G = 6.67\times10^{-11}\,\frac{Nm^2}{kg^2}$

$1e = 1.6\times10^{-19}C$

$k = \frac{1}{4\pi\varepsilon_o} = 9\times10^9\,\frac{C^2}{Nm^2}$

$\varepsilon_o = 8.85\times10^{-12}\,\frac{Nm^2}{C^2}$

$1eV = 1.6\times10^{-19}J$

$\mu_o = 4\pi\times10^{-7}\,\frac{Tm}{A}$

$c = 3\times10^8\,\frac{m}{s}$

$h = 6.63\times10^{-34}Js$

$m_e = 9.11\times10^{-31}kg = \frac{0.511MeV}{c^2}$

$m_p = 1.67\times10^{-27}kg = \frac{937.1MeV}{c^2}$

$m_n = 1.69\times10^{-27}kg = \frac{948.3MeV}{c^2}$

$1amu = 1.66\times10^{-27}kg = \frac{931.5MeV}{c^2}$

$N_A = 6.02\times10^{23}$

$Ax^2 + Bx + C = 0 \rightarrow x = \frac{-B\pm\sqrt{B^2-4AC}}{2A}$

Momentum Principle:

$\vec{p}_f = \vec{p}_i + \vec{F}_{net}\Delta t$

$\frac{d\vec{p}}{dt} = \vec{F}_{net}$

Position-update:

$\vec{r}_f = \vec{r}_i + \vec{v}_{avg}\Delta t = \vec{r}_i + \frac{\vec{p}}{m\sqrt{1+\frac{p^2}{m^2c^2}}}\Delta t$

Energy Principle:

$\Delta E = W = \Delta U_g + \Delta U_s + \Delta KE$

$W = \int \vec{F}\cdot d\vec{r}$

Vectors

magnitude of a vector: $|\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$

writing a vector: $\vec{a} = \langle a_x, a_y, a_z\rangle = |\vec{a}|\hat{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

dot product: $\vec{A}\cdot\vec{B} = AB\cos\theta$

magnitude of the cross product: $\vec{A}\times\vec{B} = AB\sin\theta$

Geometry

Circles: $C = 2\pi r = \pi D \qquad A = \pi r^2$

Triangles: $A = \frac{1}{2}bh$

Spheres: $A = 4\pi r^2 \qquad V = \frac{4}{3}\pi r^3$