

**Lab 5: Hooke's Law: Springs and Karate Boards**

Name: \_\_\_\_\_

Lab Partner(s): \_\_\_\_\_

**Honor Code Statement:** I affirm that I have carried out my academic endeavors with full academic honesty. \_\_\_\_\_

*Please neatly answer all of the questions in the lab packet. Make sure you attach any graphs generated, Excel files you produced, and any calculations/derivations you did. This lab packet is due one week from the completion of the lab.*

**Introduction**

Elastic materials are a class of materials that deform under a force and when the force is removed, the material returns to its original shape. Springs provide a simple but accurate model of how structural forces respond to deformations and are a great example of an elastic material. The expression for the spring force is given by Hooke's Law,  $\text{stress} = E \cdot \text{strain}$ , where the stress  $\sigma = \frac{F}{A}$ , for a material of cross-sectional area  $A$ . The strain of the material under the force  $F$ , is given by  $\frac{\Delta L}{L}$ , where  $\Delta L$  is the change in length of the system of length  $L$ , and  $E$  is the elastic (or Young's) modulus of the material. Rearranging the above we can cast Hooke's law in a more useful form,  $F = -kx$ , where  $x = \Delta L$ , the stretch or compression from equilibrium,  $k = \frac{EA}{L}$ , is the stiffness or spring constant, and the negative sign is due to the spring force being a restoring force, tending to restore the system to its equilibrium position. In the first part of the lab, we will investigate the relationship between the restoring spring force and the stretch of a spring. In the second part of the lab, we will look at a seemingly unspring like system, a pine karate board.

## Procedure

### Part I: Springs

1. Using Excel, make the following columns: mass, spring stretch  $x$ , and the magnitude of the spring force.
2. Hang a spring from the hook. Measure the position of the end of the spring and record this as the spring's *relaxed position*,  $L_0$ .
3. Hang a small mass on the end of the spring, allow the mass to come to rest, and measure and record the new position of the end of the spring (*not the end of the mass*),  $L$ . Determine the stretch of the spring,  $x = \Delta L = L - L_0$ .
4. Determine the spring force for this mass.
5. Hang successfully larger masses on the spring, measure the new position of the end of the spring  $L$ , and calculate  $x$  the spring force.
6. Repeat for a number of masses (use at least five masses, total).

### Analysis:

1. For each spring, plot the spring force vs. the stretch of the spring. Does your spring obey Hooke's Law? What aspects of the plot supports your claim? Consider the expression of Hooke's law and whether your plot fits a proportionality (a linear relation through the origin).
2. Fit a straight line to your curve (and write the equation of the curve fit for  $F$  below) and use the fit to calculate the spring stiffness parameter  $k$ . Should you demand that the straight line go through the origin? Explain.

$$F =$$

$$k =$$

3. Perform a linear regression analysis to get the uncertainties in the fit parameters. Does a zero y-intercept agree with your fit? Explain.

$$\Delta m =$$

$$\Delta b =$$

4. What is the stiffness of your spring with uncertainty? Does your value seem reasonable or not? Explain.

$$k_{spring} = k \pm \Delta k$$

## Part II: Restoring Force of Wooden Pine Karate Board

When a force is applied to a wood board, the board must exert a force in return to hold itself together. But there is a limit; if the external force does enough work on the board, it will break.

1. Measure and record in a *new Excel table* the mass of five different bricks and calculate the average. Calculate the standard error in your inferred mass of an individual brick.

$$m_{avg} =$$

$$\delta m =$$

2. Measure the mass of the apparatus' tray and its uncertainty and record these in the Excel table and below.

$$m_{tray} =$$

$$\delta m_{tray} =$$

3. Carefully place the piece of wood on the cross bars and place the gauge's needle at the center of the board. Read the initial setting of the gauge,  $L_0$ . Record this number in Excel and label as the relaxed position for the board. (Record your estimated uncertainty in this number also.)

$$L_0 =$$

$$\delta L_0 =$$

4. Input the equations to calculate the total force hanging on the board due to the tray and the deflection of board from the relaxed position,  $|L - L_0|$ . The gauge's reading will *decrease* as the deflection of the board increases and the gauge may go around a few times. You need to keep careful track of the gauge readings.
5. Carefully hang the tray on the board and then and record the new gauge reading,  $L$ . Excel should calculate the stretch from equilibrium as you enter the data.
6. Now, carefully and methodically add one brick at a time onto the middle of the. Read and record the new gauge setting  $L$  in the appropriate box.
7. Continue adding bricks, and entering the data until the board breaks. (Note: be careful to keep your feet away from the area below tray, in case the board breaks while you're there.)

## Analysis

1. Plot the weight added versus the displacement of the board from equilibrium. What is the relationship between the weight added and the displacement of the board from equilibrium? Does the wood board's restoring force obey Hooke's Law? Is Hooke's Law appropriate for modeling structural forces? Explain.

2. From your plot, what is the stiffness parameter  $k_{board}$ , with uncertainty  $\Delta k_{board}$ , for the board?

$$k = k_{board} \pm \Delta k_{board} =$$

3. Derive an expression for the work done by the force of gravity with the addition of each brick? In the column for work done on the board enter an equation that correctly calculates the amount of work done in each individual step.

$$W =$$

4. How much *total* work done in breaking the board from Excel.

$$W_{total} =$$

5. Since  $W = \int \vec{F} \cdot d\vec{r}$ , or  $\int F dy$  for the one-dimensional case here, the work done should also equal the area under the curve of the force vs. distance. Note the shape of this plot and calculate the area under the curve. Do you get the same answer as in step 4?

$$W_{total} =$$

6. For the total work done, you need to determine an uncertainty in the work done on the board, but unlike all of the calculations you've done so far, you have only one calculation, not many. Therefore, you now need to do "propagation of uncertainty" calculation. Your calculation of work depends on the mass of each brick, the number of bricks, and the deflections of the board. You should already have uncertainties in the mass of each brick and the deflection of the board. You now need to propagate these 3 uncertainties to get an uncertainty in the total work. To propagate the uncertainties, we add the uncertainties in quadrature according to:

$$\delta W = \sqrt{\left(\frac{\partial W}{\partial m}\right)^2 \Delta m^2 + \left(\frac{\partial W}{\partial N}\right)^2 \Delta N^2 + \left(\frac{\partial W}{\partial y}\right)^2 \Delta y^2}.$$

In the expression above each term has an expression  $\frac{\partial W}{\partial z}$ . These terms represent the partial derivative of the function ( $W$ ) taken with respect to a given variable (say mass  $m$ ) with the other variables held constant (number of bricks  $N$  and deflection  $y$ ). These terms could be negative; hence we square them to make them positive. Also, we need the uncertainty in the work done, so dimensionally, we need to multiply each square partial derivative term by an appropriate dimensional term in either mass  $\Delta m$ , number of bricks  $\Delta N$ , or deflection  $\Delta y$ .

Derive an expression (and then evaluate it) for the uncertainty in the work done and enter it below.

$\delta W =$

7. The work that the weight of the bricks did on the board did not change the board's kinetic energy. Where did this work done, or energy, go? Explain.
8. Consider breaking the board by dropping a hard object onto the board, such as your hand. As the object falls, gravity does work to give it kinetic energy. When the object hits the board, the force of the board does negative work on the object, your hand, to bring it to rest, and deflecting the board by an amount  $\Delta y$  at which point the board breaks. Using the Work-Kinetic energy theorem calculate the minimum height from which a 1-kg mass must be dropped in order for it to gain enough kinetic energy to break the board. Use propagation of uncertainty to get an uncertainty in this number.

$$h_{min} = h \pm \delta h =$$



9. Determine a method to calculate the mass of a human fist, with its uncertainty. Starting from the definition of work, calculate the speed that your hand must moving just before it strikes the board in order to break a board. Propagate your uncertainties again. Do you think this speed is reasonable and can be accomplished by you? Explain.

$$v_{min} = v \pm \delta v =$$