Name____

Physics 120 Quiz #2, January 21, 2022

Please show all work, thoughts and/or reasoning in order to receive partial credit. The quiz is worth 10 points total.

I affirm that I have carried out my academic endeavors with full academic honesty.

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Suppose that a projectile was launched from the ground at a point $\vec{r}_i = \langle 0,0,0\rangle m$ with an initial velocity $\vec{v}_i = \langle 8.7,5.0,0.0\rangle \frac{m}{s}$. For the questions below, ignore air resistance and assume a standard cartesian coordinate system with the positive x-direction to the right and the positive y-direction vertically up.

1. At what initial angle θ measured with respect to the positive x-axis was the projectile launched and what was the launch speed, $|\vec{v}_i|$?

$$\vec{v}_i = \langle v_{ix}, v_{iy}, v_{iz}\rangle = \langle 8.7,10.0,0\rangle \frac{m}{s}$$

$$v_i = |\vec{v}_i| = \sqrt{v_{ix}^2 + v_{iy}^2 + v_{iz}^2} = \sqrt{\left(8.7\frac{m}{s}\right)^2 + \left(5.0\frac{m}{s}\right)^2 + \left(0\frac{m}{s}\right)^0} = 10\frac{m}{s}$$

$$\tan\theta = \frac{v_{fy}}{v_{fx}} \rightarrow \theta = \tan^{-1}\frac{v_{fy}}{v_{fx}} = \tan^{-1}\frac{5.0\frac{m}{s}}{8.7\frac{m}{s}} = 29.9^0$$

2. At what time(s) will the projectile be at a vertical height of $y = 1m$?

$$\vec{r}_f = \vec{r}_i + \vec{v}_i t + \tfrac{1}{2}\vec{a}t^2 \rightarrow \langle x_f, y_f, z_f\rangle = \langle x_i + v_{ix}t + \tfrac{1}{2}a_x t^2, y_i + v_{iy}t + \tfrac{1}{2}a_y t^2, 0\rangle$$

In the y-direction:

$$y_f = y_i + v_{iy}t + \tfrac{1}{2}a_y t^2 \rightarrow 1 = 5t - \tfrac{1}{2}(9.8)t^2 \rightarrow -4.9t^2 + 5t - 1 = 0$$

By the quadratic formula: $t = \begin{cases} 0.27s \\ 0.75s \end{cases}$

Both times are acceptable. The time $t = 0.27s$ is for the projectile rising and $t = 0.75s$ is for the projectile falling.

3. What is the time of flight of the projectile? Assume that the projectile starts and ends on the ground.

$$\vec{r}_f = \vec{r}_i + \vec{v}_i t + \tfrac{1}{2}\vec{a}t^2 \rightarrow \langle x_f, y_f, z_f \rangle = \langle x_i + v_{ix}t + \tfrac{1}{2}a_x t^2, y_i + v_{iy}t + \tfrac{1}{2}a_y t^2, 0 \rangle$$

In the y-direction: $y_f = y_i + v_{iy}t + \frac{1}{2}a_y t^2 \rightarrow 0 = 0 + v_{iy}t - \frac{1}{2}gt^2 \rightarrow \begin{cases} t = 0 \\ t = \frac{2v_{iy}}{g} \end{cases}$

$$t = \frac{2v_{iy}}{g} = \frac{2 \times 5\frac{m}{s}}{9.8\frac{m}{s^2}} = 1.02s$$

4. How far horizontally does the projectile travel across the ground from its launch point to its landing point?

In the x-direction: $x_f = x_i + v_{ix}t + \frac{1}{2}a_x t^2 = v_{ix}t = 8.7\frac{m}{s} \times 1.02s = 8.9m$

5. What is the impact velocity $\vec{v}_f$ of the projectile just before striking the ground? That is, what are the components of the final velocity in $\vec{v}_f = \langle v_{fx}, v_{fy}, v_{fz} \rangle$.

$$\vec{v}_f = \vec{v}_i + \vec{a}t \rightarrow \langle v_{fx}, v_{fy}, v_{fz} \rangle = \langle v_{ix} + a_x t, v_{iy} + a_y t, 0 \rangle$$

In the x-direction: $v_{fx} = v_{ix} = 8.7\frac{m}{s}$

In the y-direction: $v_{fy} = v_{iy} - gt = 5\frac{m}{s} - 9.8\frac{m}{s^2} \times 1.02s = -5\frac{m}{s}$

$$\vec{v}_f = \langle v_{fx}, v_{fy}, v_{fz} \rangle = \langle 8.7, -5, 0 \rangle \frac{m}{s}$$

Physics 120 Formula Sheet

General Definitions of Motion

$\Delta\vec{r} = \langle \Delta x, \Delta y, \Delta z\rangle = \langle x_f - x_i, y_f - y, z_f - z_i\rangle$

$\vec{v} = \frac{\Delta\vec{r}}{\Delta t} = \langle \frac{\Delta x}{\Delta t}, \frac{\Delta y}{\Delta t}, \frac{\Delta z}{\Delta t}\rangle$

$\vec{a} = \frac{\Delta\vec{v}}{\Delta t} = \langle \frac{\Delta v_x}{\Delta t}, \frac{\Delta v_y}{\Delta t}, \frac{\Delta v_z}{\Delta t}\rangle$

$d\vec{r} = \langle dx, dy, dz\rangle$

$$\vec{v} = \langle v_x, v_y, v_z\rangle = \frac{d\vec{r}}{dt} = \langle \frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\rangle$$

$$\vec{a} = \langle a_x, a_y, a_z\rangle = \frac{d\vec{v}}{dt} = \langle \frac{dv_x}{dt}, \frac{dv_y}{dt}, \frac{dv_z}{dt}\rangle$$

Geometry

$C = 2\pi r$ $A_{circle} = \pi r^2$; $A_{rect} = LW$

$A_{triangle} = \frac{1}{2}bh$; $A_{sphere} = 4\pi r^2$

$V_{sphere} = \frac{4}{3}\pi r^3$; $V_{cyl} = \pi r^2 h$; $V_{cone} = \frac{1}{3}\pi r^2 h$

Motion with constant acceleration

$\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2 \rightarrow \langle x_f, y, z_f\rangle = \langle x_i + v_{ix}t + \frac{1}{2}a_x t^2, y_i + v_{iy}t + \frac{1}{2}a_y t^2, z + v_{iz}t + \frac{1}{2}a_z t^2\rangle$

$\vec{v}_f = \vec{v}_i + \vec{a}t \rightarrow \langle v_{fx}, v_{fy}, v_{fz}\rangle = \langle v_{ix} + a_x t, v_{iy} + a_y t, v_{iz} + a_z t\rangle$

Forces/Momentum

$\vec{p} = m\vec{v}$

$\vec{F}_{net} = \frac{d\vec{p}}{dt} = m\vec{a}$

$\vec{p}_f - \vec{p}_i = \int d\vec{p} = \int \vec{F}_{net} dt$

$$\vec{J} = \int \vec{F}_{net} dt$$

$$\vec{F}_{net} = \vec{F}_{\parallel} + \vec{F}_{\perp} = \frac{d\vec{p}}{dt}\hat{p} + \vec{p}\frac{d\hat{p}}{dt} = m\vec{a}_{\parallel} + m\vec{a}_{\perp}$$

$$|\vec{F}_{\perp}| = m|\vec{a}_{\perp}| = m\frac{v^2}{r}$$

$$\vec{F}_G = G\frac{M_1 M_2}{r_{12}^2}\hat{r}_{12} \rightarrow |\vec{F}_G| = G\frac{M_1 M_2}{r_{12}^2}$$

$$\vec{F}_G = m\vec{g}; \quad \vec{g} = G\frac{M_{cb}}{(R_{cb} + h)}\hat{r}$$

$|\vec{F}_{fr}| = \mu|\vec{F}_N|$

$\vec{F}_s = -k\Delta\vec{r}$

Constants

$g = 9.8\frac{m}{s^2}$; $G = 6.67 \times 10^{-11}\frac{Nm^2}{kg^2}$

$v_{sound} = 343\frac{m}{s}$; $v_{light} = c = 3 \times 10^8 \frac{m}{s}$

$N_A = 6.02 \times 10^{23}$

Vectors

$\vec{C} = \vec{A} + \vec{B} \rightarrow \langle C_x, C_y, C_z\rangle = \langle A_x + B_x, A_y + B_y, A_z + B_z\rangle + \langle C_x, C_y, C_z\rangle$; $\overrightarrow{|C|} = \sqrt{C_x^2 + C_y^2 + C_z^2}$

$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}|\cos\theta = A_x B_x + A_y B_y + A_z B_z$

$\vec{A} \times \vec{B} = |\vec{A}||\vec{B}|\sin\theta = \langle a_y b_z - b_y a_z, a_z b_x - b_z a_x, a_x b_y - b_x a_y\rangle$

Work and Energy

$$W_T = \int dW_T = \int \vec{F} \cdot d\vec{r} = \Delta K_T = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 = \frac{p_f^2}{2m} - \frac{p_i^2}{2m}$$

$$W_R = \int dW_R = \int \vec{\tau} \cdot d\vec{\theta} = \Delta K_R = \tfrac{1}{2}I\omega_f^2 - \tfrac{1}{2}I\omega_i^2 = \frac{L_f^2}{2I} - \frac{L_i^2}{2I}$$

$$W_{net} = W_T + W_R = \Delta E_{sys} = \begin{cases} 0 \\ W_{fr} \end{cases}$$

$$W_{net} = -\sum \Delta U = \Delta K_T + \Delta K_R$$

$$U_g = mgy$$

$$U_s = \tfrac{1}{2}kx^2$$

Rotational Motion

$$s = r\theta \rightarrow ds = rd\theta$$

$$\frac{ds}{dt} = r\frac{d\theta}{dt} \rightarrow v = r\omega; \quad \omega = \frac{d\theta}{dt}$$

$$a = \frac{dv}{dt} = r\frac{d\omega}{dt} = r\alpha; \quad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$\theta_f = \theta_i + \omega_i t + \tfrac{1}{2}\alpha t^2$$

$$\omega_f = \omega_i + \alpha t$$

$$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$$

Rotational Forces/Momentum

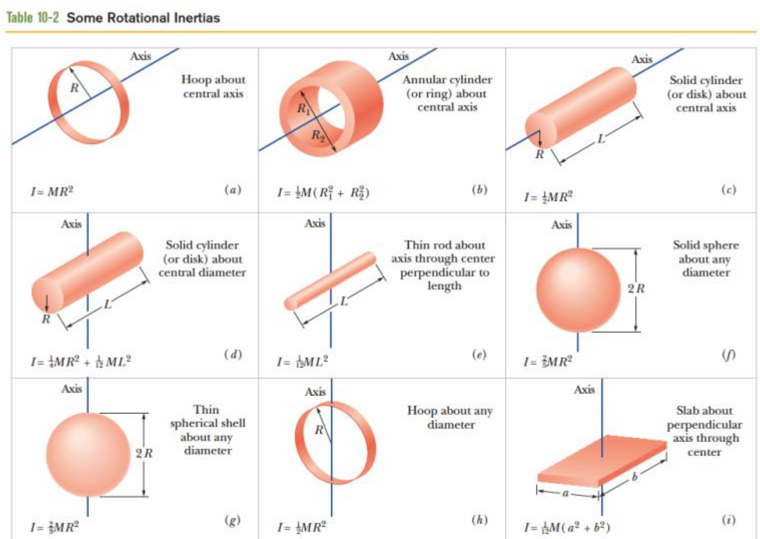
$$\vec{\tau} = \vec{r} \times \vec{F} = \frac{d\vec{L}}{dt} = I\vec{\alpha}$$

$$|\vec{\tau}| = rF\sin\theta = r_\perp F = rF_\perp$$

$$\vec{L} = I\vec{\omega}$$

$$I = \int r^2 dm$$

$$\vec{L}_f = \vec{L}_i + \int \vec{\tau}_{net} dt$$



Some moments of inertia from Halliday, Resnick, & Walker, 10th edition.