Name____

Physics 110 Quiz #3, January 24, 2020

Please show all work, thoughts and/or reasoning in order to receive partial credit. The quiz is worth 10 points total.

I affirm that I have carried out my academic endeavors with full academic honesty.

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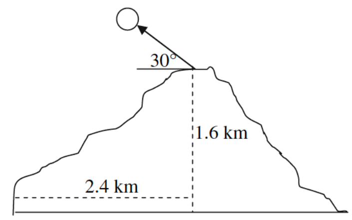
The islands of Hawaii are a beautiful place to live and a great vacation spot, boasting lush tropical forests, pristine beaches, and an active volcano. In 2018 the volcano Kilauea was actively erupting on the big island of Hawaii. Some residents were trying to defend their homes from active lava flows and from lava bombs. In fact, a man made the news in May 2018 after he was hit by a lava bomb trying to save his home. Lava bombs are solid chunks of rock ejected from a volcano. Consider a schematic of a volcano, shown on the right, with a lava bomb being ejected.



a. What would be the magnitude of the initial velocity, at the top of the volcano, have to be in order for a lava bomb to land at the base of the volcano?

$\vec{r}_i = \langle x_i, y_i, z_i \rangle = \langle 0,0,0 \rangle$

$\vec{r}_f = \langle x_f, y_f, z_f \rangle = \langle x_i + v_{ix}t + \frac{1}{2}a_x t^2, y_i + v_{iy}t + \frac{1}{2}a_y t^2, z_i + v_{iz}t + \frac{1}{2}a_z t^2 \rangle = \langle v_{ix}t, v_{iy}t - \frac{1}{2}gt^2, 0 \rangle$

$\vec{v}_i = \langle v_{ix}, v_{iy}, v_{iz} \rangle = \langle -v_i \cos\theta, v_i \sin\theta, 0 \rangle$

$\vec{v}_f = \langle v_{fx}, v_{fy}, v_{fz} \rangle = \langle v_{ix} + a_x t, v_{iy} + a_y t, v_{iz} + a_z t \rangle = \langle v_{ix}, v_{iy} - gt, 0 \rangle$

$\vec{a} = \langle a_x, a_x, a_x \rangle = \langle 0, -g, 0 \rangle$

$$x_f = v_{ix}t \rightarrow t = \frac{x_f}{v_{ix}} \text{ and } y_f = v_{iy}t - \frac{1}{2}gt^2 = x_f \tan\theta - \frac{g x_f^2}{2 v_i^2 \cos^2\theta}$$

Which we can solve for the initial velocity. $\frac{1}{v_i^2} = \frac{-2(y_f - x_f \tan\theta)\cos^2\theta}{g x_f^2} =$

$\frac{-2(-1600m - 2400m \tan 30)\cos^2 30}{9.8\frac{m}{s^2}(2400m)^2} \rightarrow v_i = 112.3\frac{m}{s}$

b. What would be the time of flight of this lava bomb from the top of the volcano to its base?

$$t = \frac{x_f}{v_{ix}} = \frac{-2400m}{-112.3\frac{m}{s}\cos 30} = 24.7s$$

c. What is the final velocity of the lava bomb, expressed as a vector and as a magnitude and direction, just before it hits the ground at the base?

$$\vec{v}_f = \langle v_{fx}, v_{fy}, v_{fz} \rangle = \langle v_{ix}, v_{iy} - gt, 0 \rangle = \langle -112.3\frac{m}{s}\cos 30, 112.3\frac{m}{s}\sin 30, 0 \rangle = \langle -97.3, -185.9, 0 \rangle \frac{m}{s}$$

$$\left|\vec{v}_f\right| = \sqrt{v_x^2 + v_y^2 + v_z^2} = \sqrt{\left(-97.3\frac{m}{s}\right)^2 + \left(-185.9\frac{m}{s}\right)^2 + \left(0\frac{m}{s}\right)^2} = 209.8\frac{m}{s}$$

$\tan \emptyset = \frac{v_{fy}}{v_{fx}} \rightarrow \emptyset = \tan^{-1}\left(\frac{-185.9\frac{m}{s}}{-97.3\frac{m}{s}}\right) = 62.3^0$ below the horizontal.

d. What is the acceleration of the lava bomb just before it hits the ground at the base?

$$\vec{a} = \langle a_x, a_x, a_x \rangle = \langle 0, -g, 0 \rangle$$

Physics 120 Formulas

Motion

$\Delta x = x_f - x_i$
$v_{avg} = \frac{\Delta x}{\Delta t}$
$a_{avg} = \frac{\Delta v}{\Delta t}$

Equations of Motion

displacement: $\begin{cases} x_f = x_i + v_{ix}t + \frac{1}{2}a_x t^2 \\ y_f = y_i + v_{iy}t + \frac{1}{2}a_y t^2 \end{cases}$

velocity: $\begin{cases} v_{fx} = v_{ix} + a_x t \\ v_{fy} = v_{iy} + a_y t \end{cases}$

time-independent: $\begin{cases} v_{fx}^2 = v_{ix}^2 + 2a_x \Delta x \\ v_{fy}^2 = v_{iy}^2 + 2a_y \Delta y \end{cases}$

Uniform Circular Motion

$F_r = ma_r = m\frac{v^2}{r}; \quad a_r = \frac{v^2}{r}$

$v = \frac{2\pi r}{T}$

$F_G = G\frac{m_1 m_2}{r^2}$

Geometry /Algebra

Circles	*Triangles*	*Spheres*
$C = 2\pi r$	$A = \frac{1}{2}bh$	$A = 4\pi r^2$
$A = \pi r^2$		$V = \frac{4}{3}\pi r^3$

Quadratic equation : $ax^2 + bx + c = 0$,

whose solutions are given by : $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Vectors

magnitude of a vector: $v = |\vec{v}| = \sqrt{v_x{}^2 + v^2{}_y}$

direction of a vector: $\phi = \tan^{-1}\left(\frac{v_y}{v_x}\right)$

Useful Constants

$g = 9.8\,{}^{m}/_{s^2} \qquad G = 6.67\times10^{-11}\,{}^{Nm^2}/_{kg^2}$

$N_A = 6.02\times10^{23}\,{}^{atoms}/_{mole} \qquad k_B = 1.38\times10^{-23}\,{}^{J}/_{K}$

$\sigma = 5.67\times10^{-8}\,{}^{W}/_{m^2K^4} \qquad v_{sound} = 343\,{}^{m}/_{s}$

Linear Momentum/Forces

$\vec{p} = m\vec{v}$

$\vec{p}_f = \vec{p}_i + \vec{F}\Delta t$

$\vec{F} = m\vec{a}$

$\vec{F}_s = -k\vec{x}$

$F_f = \mu F_N$

Work/Energy

$K_t = \frac{1}{2}mv^2$

$K_r = \frac{1}{2}I\omega^2$

$U_g = mgh$

$U_S = \frac{1}{2}kx^2$

$W_T = FdCos\theta = \Delta E_T$

$W_R = \tau\theta = \Delta E_R$

$W_{net} = W_R + W_T = \Delta E_R + \Delta E_T$

$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = 0$

$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = -\Delta E_{diss}$

Rotational Motion

$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$

$\omega_f = \omega_i + \alpha t$

$\omega^2{}_f = \omega^2{}_i + 2\alpha\Delta\theta$

$\tau = I\alpha = rF$

$L = I\omega$

$L_f = L_i + \tau\Delta t$

$\Delta s = r\Delta\theta : \; v = r\omega : \; a_t = r\alpha$

$a_r = r\omega^2$

Simple Harmonic Motion/Waves

$\omega = 2\pi f = \frac{2\pi}{T}$

$T_S = 2\pi\sqrt{\frac{m}{k}}$

$T_P = 2\pi\sqrt{\frac{l}{g}}$

$v = \pm\sqrt{\frac{k}{m}}A\left(1 - \frac{x^2}{A^2}\right)^{\frac{1}{2}}$

$x(t) = A\sin\left(\frac{2\pi t}{T}\right)$

$v(t) = A\sqrt{\frac{k}{m}}\cos\left(\frac{2\pi t}{T}\right)$

$a(t) = -A\frac{k}{m}\sin\left(\frac{2\pi t}{T}\right)$

$v = f\lambda = \sqrt{\frac{F_T}{\mu}}$

$f_n = nf_1 = n\frac{v}{2L}$

$I = 2\pi^2 f^2 \rho v A^2$