7Name________________________________

Physics 120 Quiz #3, April 22, 2011

Please show all work, thoughts and/or reasoning in order to receive partial credit. The quiz is worth 10 points total.

1. Suppose that a block of mass $m = 0.25kg$ is attached to a horizontally oriented spring, with stiffness constant $k = 8N/m$. Connecting the left end of the spring to a wall, taken as the origin, the spring has a natural length of $L_o = 0.3m$ and the block, attached to the right end of the spring, is pulled out to a length $L = 0.35m$ and held motionless.

   a. If the block is released from rest at time $t = 0$, what will the final position of the block be after 4 time steps, each of size $dt = 0.069s$? How does the final position of the block compare to L_o?

   Time Step #1:

   $$\vec{v}_f = \vec{v}_i + \frac{\vec{F}_{net}}{m}dt = \langle 0,0,0\rangle \tfrac{m}{s} + \langle -8(0.35-0.3)m,0,0\rangle \frac{0.069s}{0.25kg} = \langle -0.110,0,0\rangle \tfrac{m}{s}$$

   $$\vec{r}_f = \vec{r}_i + \vec{v}_f dt = \langle 0.35,0,0\rangle m + \langle -0.110,0,0\rangle \tfrac{m}{s} \times 0.069s = \langle 0.342,0,0\rangle m$$

   Time Step #2:

   $$\vec{v}_f = \vec{v}_i + \frac{\vec{F}_{net}}{m}dt = \langle -0.110,0,0\rangle \tfrac{m}{s} + \langle -8(0.342-0.3)m,0,0\rangle \frac{0.069s}{0.25kg} = \langle -0.203,0,0\rangle \tfrac{m}{s}$$

   $$\vec{r}_f = \vec{r}_i + \vec{v}_f dt = \langle 0.342,0,0\rangle m + \langle -0.203,0,0\rangle \tfrac{m}{s} \times 0.069s = \langle 0.328,0,0\rangle m$$

   Time Step #3:

   $$\vec{v}_f = \vec{v}_i + \frac{\vec{F}_{net}}{m}dt = \langle -0.203,0,0\rangle \tfrac{m}{s} + \langle -8(0.328-0.3)m,0,0\rangle \frac{0.069s}{0.25kg} = \langle -0.265,0,0\rangle \tfrac{m}{s}$$

   $$\vec{r}_f = \vec{r}_i + \vec{v}_f dt = \langle 0.328,0,0\rangle m + \langle -0.265,0,0\rangle \tfrac{m}{s} \times 0.069s = \langle 0.310,0,0\rangle m$$

   Time Step #4:

   $$\vec{v}_f = \vec{v}_i + \frac{\vec{F}_{net}}{m}dt = \langle -0.265,0,0\rangle \tfrac{m}{s} + \langle -8(0.310-0.3)m,0,0\rangle \frac{0.069s}{0.25kg} = \langle -0.287,0,0\rangle \tfrac{m}{s}$$

   $$\vec{r}_f = \vec{r}_i + \vec{v}_f dt = \langle 0.310,0,0\rangle m + \langle -0.287,0,0\rangle \tfrac{m}{s} \times 0.069s = \langle 0.290,0,0\rangle m$$

   The final position is within *3%* of L_o. Which if the time step was smaller, then the block would probably be at L_o.

   b. Assuming that the motion of the block is symmetric about L_o, meaning that there is no friction between the block and the surface it's riding on, how much time does it take for the block to return to its starting position at $L = 0.35m$? This time is called the period, T, of the block's motion.

   In the 4 time steps above the block is at L_o, and since the motion is symmetric we have a total of 16 time steps each of which is *0.069s* long. Thus the period is

   $$T = 16 \times 0.069s = 1.10s$$

c. Calculations, like those you did in *part a* are tedious when trying to describe the trajectory of the block-spring system. In class we'll come up with an analytic solution to this problem and we'll write the trajectory of the block on the spring, and we'll also determine the period of the block's motion. For now, I'll just say without proof that the period is given by the following formula: $T = 2\pi\sqrt{\frac{m}{k}}$. How does the period calculated in *part b* compare to the period calculated using this formula?

The period is $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.25kg}{8\frac{N}{m}}} = 1.11s$ which is approximately 0.9% from the actual period. So not too bad.

Useful formulas:

$$\vec{p} = \gamma m\vec{v}$$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\vec{v}_{avg} = \frac{\vec{v}_i + \vec{v}_f}{2}$$

$$\vec{F}_g = m\vec{g}$$

$$\vec{F}_{gravity} = \frac{GM_1M_2}{r_{12}^2}\hat{r}_{12}$$

$$\vec{F}_{spring} = -k\vec{s}; \quad \vec{s} = (L - L_o)\hat{s}$$

Momentum Principle:

$$\vec{p}_f = \vec{p}_i + \vec{F}_{net}\Delta t; \quad \Delta t = \text{large}$$

$$\vec{p}_f = \vec{p}_i + \vec{F}_{net}dt; \quad dt = \frac{\Delta t}{n} = \text{small}$$

Position-update:

$$\vec{r}_f = \vec{r}_i + \vec{v}_{avg}\Delta t = \vec{r}_i + \frac{\vec{p}}{m\sqrt{1+\frac{p^2}{m^2c^2}}}\Delta t; \quad \Delta t = \text{large}$$

$$\vec{r}_f = \vec{r}_i + \vec{v}_f dt; \quad dt = \frac{\Delta t}{n} = \text{small}$$

Geometry /Algebra

Circles	*Triangles*	*Spheres*
$C = 2\pi r$	$A = \frac{1}{2}bh$	$A = 4\pi r^2$
$A = \pi r^2$		$V = \frac{4}{3}\pi r^3$

Quadratic equation: $ax^2 + bx + c = 0$,

whose solutions are given by: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Vectors

magnitude of a vector: $|\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$

writing a vector: $\vec{a} = \langle a_x, a_y, a_z \rangle = |\vec{a}|\hat{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$