Name____

Physics 110 Quiz #4, February 7, 2020

Please show all work, thoughts and/or reasoning in order to receive partial credit. The quiz is worth 10 points total.

I affirm that I have carried out my academic endeavors with full academic honesty.

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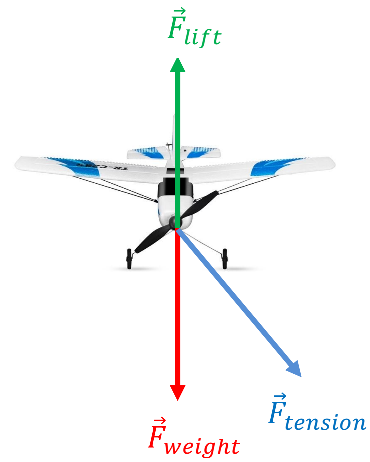
The Earth revolves about the Sun in a nearly circular orbit with radius $r_{ES} = 1.5 \times 10^{11} m$, due to the gravitational interaction between the Earth and the Sun. (The sun also undergoes a small circular orbit due to same gravitational interaction but the orbit of the Sun is within the Sun itself.) The gravitational interaction is given by $\left|\vec{F}_G\right| = G\frac{M_E M_S}{r_{ES}^2}$, where $G = 6.67 \times 10^{-11}\frac{Nm^2}{kg^2}$.

a. What is the (assumed) constant speed of the Earth in orbit, if the period of Earth's orbit about the Sun is $T = 365.25\, days = 31.5 \times 10^6 s$?

$$v = \frac{2\pi r_{ES}}{T} = \frac{2\pi(1.5 \times 10^{11} m)}{31.5 \times 10^6 s} = 3 \times 10^4 \frac{m}{s}$$

b. What is the mass of the Sun?

$$\left|\vec{F}_G\right| = G\frac{M_E M_S}{r_{ES}^2} = M_E\frac{v^2}{r_{ES}} \rightarrow M_S = \frac{v^2 r_{ES}}{G} = \frac{\left(3 \times 10^4 \frac{m}{s}\right)^2 \times 1.5 \times 10^{11} m}{6.67 \times 10^{-11}\frac{Nm^2}{kg^2}} = 2 \times 10^{30} kg$$



A little closer to home suppose that you are flying a remote-controlled airplane that has a mass of $m = 1.5kg$ attached to a $2m$ long string (you're holding the other end) and the plane is flown in a horizontal circle at a constant speed. The string makes a 20^0angle with respect to the vertical and the plane takes $1.5s$ to complete one circular orbit. The airflow over the wings of the plane generates a lifting force $\vec{F}_{lift}$ that is always perpendicular to the wings of the plane.

c. What is the magnitude of the lifting force $|\vec{F}_{lift}|$?

From the geometry in the problem we can calculate the radius of the circular motion.

$\sin\theta = \frac{r}{L} \rightarrow r = L\sin\theta$

The speed of the constant speed of the plane is given by $v = \frac{2\pi r}{T} = \frac{2\pi L\sin\theta}{T}$

$$\vec{F}_{net} = \vec{F}_{lift} + \vec{F}_{tension} + \vec{F}_{weight} = \langle 0, F_{lift}, 0\rangle + \langle F_T\sin\theta\,, -F_T\cos\theta\,, 0\rangle + \langle 0, -mg, 0\rangle = m\vec{a}$$

x-direction

$$F_T\sin\theta = ma_x = m\frac{v^2}{r} \rightarrow F_T = \frac{mv^2}{r\sin\theta} = \frac{4\pi^2 mL^2\sin^2\theta}{LT^2\sin^2\theta} = \frac{4\pi^2 mL}{T^2}$$

y-direction

$$F_{lift} - F_T\cos\theta - mg = ma_y = 0 \rightarrow F_{lift} = F_T\cos\theta + mg = \frac{4\pi^2 mL}{T^2}\cos\theta + mg$$

$$F_{lift} = \frac{4\pi^2 mL}{T^2}\cos\theta + mg = \frac{4\pi^2 \times 1.5kg \times 2m}{(1.5s)^2}\cos 20 + \left(1.5kg \times 9.8\frac{m}{s^2}\right) = 64.1N$$

d. What is the magnitude of the tension force $|\vec{F}_T|$ in the string?

$$F_T = \frac{4\pi^2 mL}{T^2} = \frac{4\pi^2 \times 1.5kg \times 2m}{(1.5s)^2} = 52.6N$$

Photo: https://www.toprace.com/product/top-race-3-channel-remote-control-airplane-built-in-6-axis-gyro-system-super-easy-to-fly-rtf-tr-c285

Physics 120 Formula Sheet

General Definitions of Motion

$d\vec{r} = \langle dx, dy, dz\rangle = \langle x_f - x_i, y_f - y, z_f - z_i\rangle$

$\vec{v} = \langle v_x, v_y, v_z\rangle = \frac{d\vec{r}}{dt} = \langle \frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\rangle$

$\vec{a} = \langle a_x, a_y, a_z\rangle = \frac{d\vec{v}}{dt} = \langle \frac{dv_x}{dt}, \frac{dv_y}{dt}, \frac{dv_z}{dt}\rangle$

Geometry

$C = 2\pi r$ $A_{circle} = \pi r^2$; $A_{rect} = LW$

$A_{triangle} = \frac{1}{2}bh$; $A_{sphere} = 4\pi r^2$

$V_{sphere} = \frac{4}{3}\pi r^3$; $V_{cyl} = \pi r^2 h$; $V_{cone} = \frac{1}{3}\pi r^2 h$

Motion with constant acceleration

$\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2 \rightarrow \langle x_f, y, z_f\rangle = \langle x_i + v_{ixt} + \frac{1}{2}a_x t^2, y_i + v_{iy}t + \frac{1}{2}a_y t^2, z + v_{iz}t + \frac{1}{2}a_z t^2\rangle$

$\vec{v}_f = \vec{v}_i + \vec{a}t \rightarrow \langle v_{fx}, v_{fy}, v_{fz}\rangle = \langle v_{ix} + a_x t, v_{iy} + a_y t, v_{iz} + a_z t\rangle$

Forces

$\vec{p} = m\vec{v}$

$\vec{F}_{net} = \frac{d\vec{p}}{dt} = m\vec{a}$

$\vec{p}_f - \vec{p}_i = \int d\vec{p} = \int \vec{F}_{net} dt$

$\vec{F}_{net} = \vec{F}_{\parallel} + \vec{F}_{\perp} = \frac{dp}{dt}\hat{p} + p\frac{d\hat{p}}{dt} = m\vec{a}_{\parallel} + m\vec{a}_{\perp}$

$|\vec{F}_{\perp}| = m|\vec{a}_{\perp}| = m\frac{v^2}{r}$

$\vec{F}_G = G\frac{M_1 M_2}{r_{12}^2}\hat{r}_{12} \rightarrow |\vec{F}_G| = G\frac{M_1 M_2}{r_{12}^2}$

$\vec{F}_G = m\vec{g}$; $\vec{g} = G\frac{M_{cb}}{(R_{cb} + h)}\hat{r}$

$|\vec{F}_{fr}| = \mu|\vec{F}_N|$

$\vec{F}_s = -k\Delta\vec{r}$

Constants

$g = 9.8\frac{m}{s^2}$; $G = 6.67 \times 10^{-11}\frac{Nm^2}{kg^2}$

$v_{sound} = 343\frac{m}{s}$; $v_{light} = c = 3 \times 10^8 \frac{m}{s}$

$N_A = 6.02 \times 10^{23}$

Vectors

$\vec{C} = \vec{A} + \vec{B} \rightarrow \langle C_x, C_y, C_z\rangle = \langle A_x + B_x, A_y + B_y, A_z + B_z\rangle + \langle C_x, C_y, C_z\rangle$; $|\vec{C}| = \sqrt{C_x^2 + C_y^2 + C_z^2}$

$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}|\cos\theta = A_x B_x + A_y B_y + A_z B_z$

$\vec{A} \times \vec{B} = |\vec{A}||\vec{B}|\sin\theta = \langle , , \rangle$

Work and Energy

$W_T = \int dW_T = \int \vec{F} \cdot d\vec{r} = \Delta K_T = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

$W_R = \int dW_{TR} = \int \vec{\tau} \cdot d\vec{\theta} = \Delta K_R = \frac{1}{2}I\omega_f^2 - \frac{1}{2}Iv\omega_i^2$

$W_{net} = W_T + W_R = \Delta E_{sys} = \begin{cases} 0 \\ -W_{fr} \end{cases}$

$W_{net} = -\sum \Delta U = \Delta K_T + \Delta K_R$

$U_g = mgy$

$U_s = \frac{1}{2}kx^2$