Name____

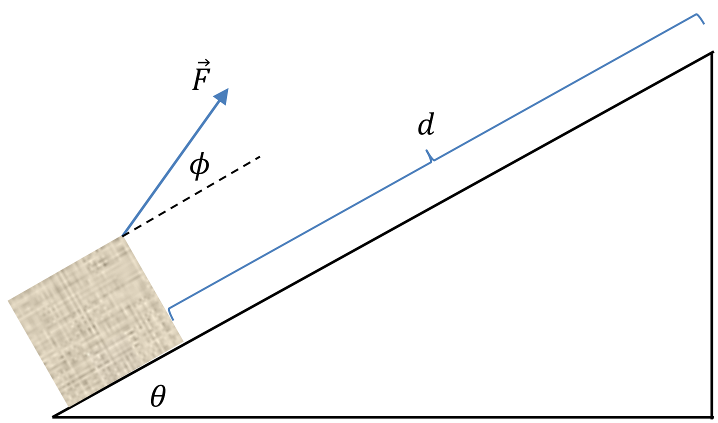
Physics 120 Quiz #4, February 11, 2022

Please show all work, thoughts and/or reasoning in order to receive partial credit. The quiz is worth 10 points total.

I affirm that I have carried out my academic endeavors with full academic honesty.

____

A crate of mass $m_{crate} = 10kg$ is pulled up a rough ramp ($\theta = 30^0$) with an initial speed of $v_i = 1.5\frac{m}{s}$. The pulling force up the ramp has a magnitude of $F = 100N$ and is oriented at an angle of $\phi = 20^0$ measured with respect to the ramp. The coefficient of friction is $\mu = 0.4$and the crate is pulled a distance of $d = 5m$ along the ramp.



1. How much work was done by the gravitational force?

$$W_{gravity} = \int F_{Wx} dx \cos 180 = -mgd \sin\theta = -10kg \times 9.8\frac{m}{s^2} \times 5m \times \sin 30 = -245J$$

2. How much energy is lost (or work was done) by friction?

$$W_{fr} = \int F_{fr} dx \cos 180 = -F_{fr} d = -\mu F_N d$$

Where, in the vertical direction: $F_N - mg\cos\theta + F\sin\phi = ma_y = 0$

$F_N = mg\cos\theta - F\sin\phi$

$$W_{fr} = -\mu(mg\cos\theta - F\sin\phi)d$$

$$W_{fr} = -0.4\left(10kg \times 9.8\frac{m}{s^2}\cos 30 - 100N \sin 20\right) \times 5m = -101.3J$$

3. How much work was done by the pulling force?

$$W_F = \int F dx \cos\phi = Fd\cos\phi = 100N \times 5m \times \cos 20 = 469.8J$$

4. What is the change in kinetic energy of the crate?

$$W_{net} = W_{fr} + W_{gravity} + W_{F_N} + W_F = \Delta K$$
$$W_{net} = \Delta K = -101.3J - 245J + 469.8J = 123.6J$$

5. What tis the final speed of the crate?

$$\Delta K = K_f - K_i = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 \rightarrow v_f = \sqrt{v_i^2 + \frac{2\Delta K}{m}}$$

$$v_f = \sqrt{v_i^2 + \frac{2\Delta K}{m}} = \sqrt{\left(1.5\tfrac{m}{s}\right)^2 + \frac{2 \times 123.6J}{10kg}} = 5.2\tfrac{m}{s}$$

Physics 120 Formula Sheet

General Definitions of Motion

$\Delta\vec{r} = \langle \Delta x, \Delta y, \Delta z\rangle = \langle x_f - x_i, y_f - y, z_f - z_i\rangle$

$\vec{v} = \frac{\Delta\vec{r}}{\Delta t} = \langle \frac{\Delta x}{\Delta t}, \frac{\Delta y}{\Delta t}, \frac{\Delta z}{\Delta t}\rangle$

$\vec{a} = \frac{\Delta\vec{v}}{\Delta t} = \langle \frac{\Delta v_x}{\Delta t}, \frac{\Delta v_y}{\Delta t}, \frac{\Delta v_z}{\Delta t}\rangle$

$d\vec{r} = \langle dx, dy, dz\rangle$

$$\vec{v} = \langle v_x, v_y, v_z\rangle = \frac{d\vec{r}}{dt} = \langle \frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\rangle$$

$$\vec{a} = \langle a_x, a_y, a_z\rangle = \frac{d\vec{v}}{dt} = \langle \frac{dv_x}{dt}, \frac{dv_y}{dt}, \frac{dv_z}{dt}\rangle$$

Geometry

$C = 2\pi r \;\; A_{circle} = \pi r^2;\;\; A_{rect} = LW$

$A_{triangle} = \frac{1}{2}bh;\;\; A_{sphere} = 4\pi r^2$

$V_{sphere} = \frac{4}{3}\pi r^3;\;\; V_{cyl} = \pi r^2 h;\;\; V_{cone} = \frac{1}{3}\pi r^2 h$

Motion with constant acceleration

$\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2 \rightarrow \langle x_f, y, z_f\rangle = \langle x_i + v_{ix}t + \frac{1}{2}a_x t^2, y_i + v_{iy}t + \frac{1}{2}a_y t^2, z + v_{iz}t + \frac{1}{2}a_z t^2\rangle$

$\vec{v}_f = \vec{v}_i + \vec{a}t \rightarrow \langle v_{fx}, v_{fy}, v_{fz}\rangle = \langle v_{ix} + a_x t, v_{iy} + a_y t, v_{iz} + a_z t\rangle$

Forces/Momentum

$\vec{p} = m\vec{v}$

$\vec{F}_{net} = \frac{d\vec{p}}{dt} = m\vec{a}$

$\vec{p}_f - \vec{p}_i = \int d\vec{p} = \int \vec{F}_{net} dt$

$$\vec{J} = \int \vec{F}_{net} dt$$

$$\vec{F}_{net} = \vec{F}_\parallel + \vec{F}_\perp = \frac{d\vec{p}}{dt}\hat{p} + \vec{p}\frac{d\hat{p}}{dt} = m\vec{a}_\parallel + m\vec{a}_\perp$$

$$|\vec{F}_\perp| = m|\vec{a}_\perp| = m\frac{v^2}{r}$$

$$\vec{F}_G = G\frac{M_1 M_2}{r_{12}^2}\hat{r}_{12} \rightarrow |\vec{F}_G| = G\frac{M_1 M_2}{r_{12}^2}$$

$$\vec{F}_G = m\vec{g};\quad \vec{g} = G\frac{M_{cb}}{(R_{cb} + h)}\hat{r}$$

$|\vec{F}_{fr}| = \mu|\vec{F}_N|$

$\vec{F}_s = -k\Delta\vec{r}$

Constants

$g = 9.8\frac{m}{s^2};\;\; G = 6.67 \times 10^{-11}\frac{Nm^2}{kg^2}$

$v_{sound} = 343\frac{m}{s};\;\; v_{light} = c = 3 \times 10^8\,\frac{m}{s}$

$N_A = 6.02 \times 10^{23}$

Vectors

$\vec{C} = \vec{A} + \vec{B} \rightarrow \langle C_x, C_y, C_z\rangle = \langle A_x + B_x, A_y + B_y, A_z + B_z\rangle + \langle C_x, C_y, C_z\rangle;\;\; |\vec{C}| = \sqrt{C_x^2 + C_y^2 + C_z^2}$

$\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta = A_x B_x + A_y B_y + A_z B_z$

$\vec{A}\times\vec{B} = |\vec{A}||\vec{B}|\sin\theta = \langle a_y b_z - b_y a_z, a_z b_x - b_z a_x, a_x b_y - b_x a_y\rangle$

Work and Energy

$$W_T = \int dW_T = \int \vec{F} \cdot d\vec{r} = \Delta K_T = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 = \frac{p_f^2}{2m} - \frac{p_i^2}{2m}$$

$$W_R = \int dW_R = \int \vec{\tau} \cdot d\vec{\theta} = \Delta K_R = \tfrac{1}{2}I\omega_f^2 - \tfrac{1}{2}I\omega_i^2 = \frac{L_f^2}{2I} - \frac{L_i^2}{2I}$$

$$W_{net} = W_T + W_R = \Delta E_{sys} = \begin{cases} 0 \\ W_{fr} \end{cases}$$

$$W_{net} = -\sum \Delta U = \Delta K_T + \Delta K_R$$

$$U_g = mgy$$

$$U_s = \tfrac{1}{2}kx^2$$

Rotational Motion

$$s = r\theta \rightarrow ds = rd\theta$$

$$\frac{ds}{dt} = r\frac{d\theta}{dt} \rightarrow v = r\omega; \quad \omega = \frac{d\theta}{dt}$$

$$a = \frac{dv}{dt} = r\frac{d\omega}{dt} = r\alpha; \quad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$\theta_f = \theta_i + \omega_i t + \tfrac{1}{2}\alpha t^2$$

$$\omega_f = \omega_i + \alpha t$$

$$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$$

Rotational Forces/Momentum

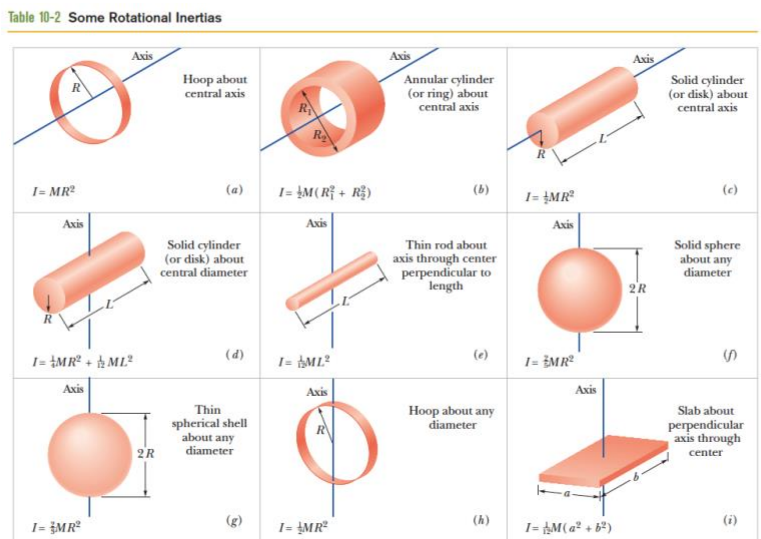
$$\vec{\tau} = \vec{r} \times \vec{F} = \frac{d\vec{L}}{dt} = I\vec{\alpha}$$

$$|\vec{\tau}| = rF\sin\theta = r_\perp F = rF_\perp$$

$$\vec{L} = I\vec{\omega}$$

$$I = \int r^2 dm$$

$$\vec{L}_f = \vec{L}_i + \int \vec{\tau}_{net} dt$$



Some moments of inertia from Halliday, Resnick, & Walker, 10th edition.