Name____

Physics 120 Quiz #5, February 14, 2020

Please show all work, thoughts and/or reasoning in order to receive partial credit. The quiz is worth 10 points total.

I affirm that I have carried out my academic endeavors with full academic honesty.

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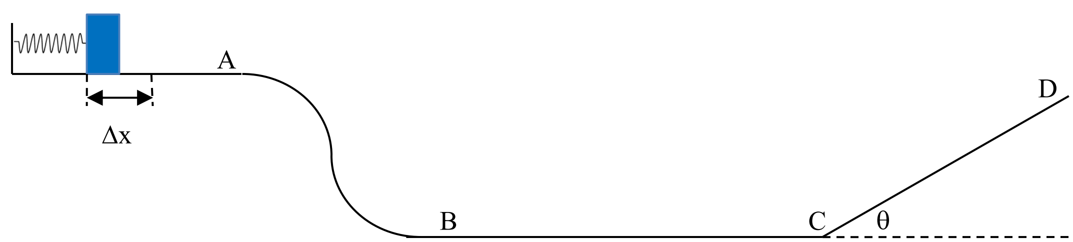
Consider the system shown below. A spring of stiffness $k = 1000\frac{N}{m}$ is compressed by an amount $|\Delta\vec{x}| = 0.25m$ from equilibrium. A mass $m = 10kg$ is placed against the spring and the mass is released from rest. All surfaces are frictionless except the region between points C and D.



a. Considering the work done by the spring on the mass and if the mass leaves the spring when the spring is at its equilibrium position, what is its speed of the mass at the top of the hill, labeled as point A?

$$W_s = \int \vec{F_s} \cdot d\vec{r} = \int \langle -kx, 0,0\rangle \cdot \langle dx, 0,0\rangle = \int_{-x}^{0} -kxdx = -\frac{1}{2}k\left(x_f^2 - x_i^2\right) = \frac{1}{2}kx_i^2 = \frac{1}{2}kx^2$$

$$W_s = \frac{1}{2}kx^2 = \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}mv_f^2$$

$$\frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2 \rightarrow v_A = \sqrt{\frac{k}{m}}x = \sqrt{\frac{1000\frac{N}{m}}{10kg}}(0.25m) = 2.5\frac{m}{s}$$

b. How much work is done by gravity on the mass between points A and B if $|\Delta\vec{y}| = 1.25m$?

$$W_g = \int \vec{F_g} \cdot d\vec{r} = \int \langle 0, -mg, 0\rangle \cdot \langle dx, dy, 0\rangle = \int_{y}^{0} -mgdy = -mg\left(y_f - y_i\right) = mgy$$

$$W_g = mgy = 10kg \times 9.8\frac{m}{s^2} \times 1.25m = 122.5J$$

c. What is the speed of the mass at point B?

$$W_g = mgy = \Delta K = \tfrac{1}{2}mv_B^2 - \tfrac{1}{2}mv_A^2 \rightarrow v_B = \sqrt{v_A^2 + 2gy}$$

$$v_B = \sqrt{\left(2.5\tfrac{m}{s}\right)^2 + \left(2 \times 9.8\tfrac{m}{s^2} \times 1.25m\right)} = 5.6\tfrac{m}{s}$$

d. The mass slides along the horizontal surface at the constant speed you found in point B. The block at point C encounters a ramp inclined at an angle θ measured with respect to the horizontal. There is friction between the block and the ramp (with coefficient of friction μ) between points C and D. Let the distance the block slides along the ramp be $\Delta x_{DC} = x$. Starting with the definition of work, derive an expression for the distance the block slides along the ramp $\Delta x_{DC} = x$ starting at point C with speed v_B and coming to rest at point D.

$$W = \Delta K = \tfrac{1}{2}mv_D^2 - \tfrac{1}{2}mv_C^2 = -\tfrac{1}{2}mv_C^2 = -\tfrac{1}{2}mv_B^2$$

$$W = \int \vec{F}_{net} \cdot d\vec{r} = \int \langle -mg\sin\theta - F_{fr}, F_N - mg\cos\theta, 0\rangle \cdot \langle dx, 0, 0\rangle = \int_{x_C}^{x_D} \left(-mg\sin\theta - F_{fr}\right)dx$$

$$W = -\tfrac{1}{2}mv_B^2 = \left(-mg\sin\theta - F_{fr}\right)(x_D - x_C) = -(mg\sin\theta + \mu F_N)x = -(mg\sin\theta + \mu mg\cos\theta)x$$

$$x = \frac{v_B^2}{2(g\sin\theta + \mu g\cos\theta)}$$

Where, $\vec{F}_{net} = \vec{F}_N + \vec{F}_w + \vec{F}_{fr} = \langle 0, F_N, 0\rangle + \langle -mg\sin\theta, -mg\cos\theta, 0\rangle + \langle -F_{fr}, 0, 0\rangle$

$\vec{F}_{net} = \langle -mg\sin\theta - F_{fr}, F_N - mg\cos\theta, 0\rangle$ and $d\vec{r} = \langle dx, 0, 0\rangle$

Physics 120 Formula Sheet

General Definitions of Motion

$\Delta\vec{r} = \langle \Delta x, \Delta y, \Delta z\rangle = \langle x_f - x_i, y_f - y, z_f - z_i\rangle$

$\vec{v} = \frac{\Delta\vec{r}}{\Delta t} = \langle \frac{\Delta x}{\Delta t}, \frac{\Delta y}{\Delta t}, \frac{\Delta z}{\Delta t}\rangle$

$\vec{a} = \frac{\Delta\vec{v}}{\Delta t} = \langle \frac{\Delta v_x}{\Delta t}, \frac{\Delta v_y}{\Delta t}, \frac{\Delta v_z}{\Delta t}\rangle$

$d\vec{r} = \langle dx, dy, dz\rangle$

$$\vec{v} = \langle v_x, v_y, v_z\rangle = \frac{d\vec{r}}{dt} = \langle \frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\rangle$$

$$\vec{a} = \langle a_x, a_y, a_z\rangle = \frac{d\vec{v}}{dt} = \langle \frac{dv_x}{dt}, \frac{dv_y}{dt}, \frac{dv_z}{dt}\rangle$$

Geometry

$C = 2\pi r \;\; A_{circle} = \pi r^2; \;\; A_{rect} = LW$

$A_{triangle} = \frac{1}{2}bh; \;\; A_{sphere} = 4\pi r^2$

$V_{sphere} = \frac{4}{3}\pi r^3; \;\; V_{cyl} = \pi r^2 h; \;\; V_{cone} = \frac{1}{3}\pi r^2 h$

Motion with constant acceleration

$\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2 \rightarrow \langle x_f, y, z_f\rangle = \langle x_i + v_{ix}t + \frac{1}{2}a_x t^2, y_i + v_{iy}t + \frac{1}{2}a_y t^2, z + v_{iz}t + \frac{1}{2}a_z t^2\rangle$

$\vec{v}_f = \vec{v}_i + \vec{a}t \rightarrow \langle v_{fx}, v_{fy}, v_{fz}\rangle = \langle v_{ix} + a_x t, v_{iy} + a_y t, v_{iz} + a_z t\rangle$

Forces/Momentum

$\vec{p} = m\vec{v}$

$\vec{F}_{net} = \frac{d\vec{p}}{dt} = m\vec{a}$

$\vec{p}_f - \vec{p}_i = \int d\vec{p} = \int \vec{F}_{net}dt$

$$\vec{F}_{net} = \vec{F}_{\parallel} + \vec{F}_{\perp} = \frac{dp}{dt}\hat{p} + p\frac{d\hat{p}}{dt} = m\vec{a}_{\parallel} + m\vec{a}_{\perp}$$

$$|\vec{F}_{\perp}| = m|\vec{a}_{\perp}| = m\frac{v^2}{r}$$

$$\vec{F}_G = G\frac{M_1 M_2}{r_{12}^2}\hat{r}_{12} \rightarrow |\vec{F}_G| = G\frac{M_1 M_2}{r_{12}^2}$$

$$\vec{F}_G = m\vec{g}; \quad \vec{g} = G\frac{M_{cb}}{(R_{cb} + h)}\hat{r}$$

$|\vec{F}_{fr}| = \mu|\vec{F}_N|$

$\vec{F}_s = -k\Delta\vec{r}$

Constants

$g = 9.8\frac{m}{s^2}; \;\; G = 6.67 \times 10^{-11}\frac{Nm^2}{kg^2}$

$v_{sound} = 343\frac{m}{s}; \;\; v_{light} = c = 3 \times 10^8\,\frac{m}{s}$

$N_A = 6.02 \times 10^{23}$

Vectors

$\vec{C} = \vec{A} + \vec{B} \rightarrow \langle C_x, C_y, C_z\rangle = \langle A_x + B_x, A_y + B_y, A_z + B_z\rangle + \langle C_x, C_y, C_z\rangle; \;\; |\vec{C}| = \sqrt{C_x^2 + C_y^2 + C_z^2}$

$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}|\cos\theta = A_x B_x + A_y B_y + A_z B_z$

$\vec{A} \times \vec{B} = |\vec{A}||\vec{B}|\sin\theta = \langle a_y b_z - b_y a_z, a_z b_x - b_z a_x, a_x b_y - b_x a_y\rangle$

Work and Energy

$$W_T = \int dW_T = \int \vec{F} \cdot d\vec{r} = \Delta K_T = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 = \frac{p_f^2}{2m} - \frac{p_i^2}{2m}$$

$$W_R = \int dW_R = \int \vec{\tau} \cdot d\vec{\theta} = \Delta K_R = \tfrac{1}{2}I\omega_f^2 - \tfrac{1}{2}I\omega_i^2$$

$$W_{net} = W_T + W_R = \Delta E_{sys} = \begin{cases} 0 \\ -W_{fr} \end{cases}$$

$$W_{net} = -\sum \Delta U = \Delta K_T + \Delta K_R$$

$$U_g = mgy$$

$$U_s = \tfrac{1}{2}kx^2$$

Center-of-Mass

Collisions

Rotational Motion