Name____

Physics 110 Quiz #5, February 16, 2022

Please show all work, thoughts and/or reasoning in order to receive partial credit. The quiz is worth 10 points total.

I affirm that I have carried out my academic endeavors with full academic honesty.

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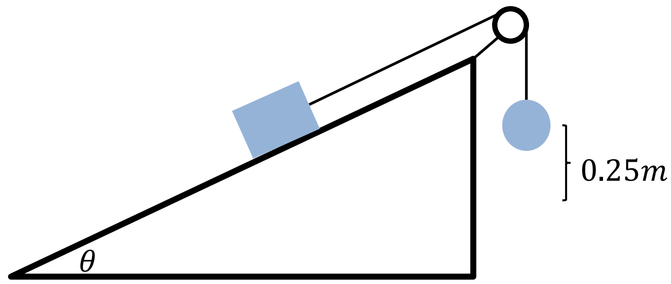
A block of mass $m_1 = 2kg$ is tied by a light rope to a sphere of mass $m_2 = 4kg$. The ramp is inclined at angle $\theta = 42^0$measured with respect to the horizontal.

1. If the $4kg$ sphere is released from rest and is allowed to fall through a distance $d = 0.25m$, what is the speed of the sphere if the ramp is considered frictionless?



$$\Delta E = \Delta K_1 + \Delta U_{g1} + \Delta K_2 + \Delta U_{g2} = 0$$
$$\frac{1}{2}m_1 v_{1f}^2 + m_1 g y_{1f} + \frac{1}{2}m_2 v_{2f}^2 - m_2 g y_{2f} = 0$$
$$\frac{1}{2}(m_1 + m_2)v_f^2 = m_2 g d - m_1 g d \sin\theta$$
$$v_f = \sqrt{\frac{2(m_2 - m_1 \sin\theta)gd}{m_1 + m_2}}$$
$$v_f = \sqrt{\frac{2(4kg - 2kg\sin 42) \times 9.8\frac{m}{s^2} \times 0.25m}{2kg + 4kg}} = 1.47\frac{m}{s}$$

2. Suppose instead that the $4kg$ sphere were again released from rest and is allowed to fall through a distance $d = 0.25m$ but this time the ramp is not frictionless. If the speed of the sphere is $v = 1.2\frac{m}{s}$, what is the coefficient of friction between the block and the ramp?

$$\Delta E = \Delta K_1 + \Delta U_{g1} + \Delta K_2 + \Delta U_{g2} = W_{fr}$$
$$\frac{1}{2}m_1 v_{1f}^2 + m_1 g y_{1f} + \frac{1}{2}m_2 v_{2f}^2 - m_2 g y_{2f} = -\mu(m_1 g\cos\theta)d$$
$$\mu = -\frac{\frac{1}{2}(m_1 + m_2)v_f^2 - m_2 g d + m_1 g d\sin\theta}{m_1 g d\cos\theta}$$
$$\mu = -\frac{\frac{1}{2}(2kg + 4kg)\left(1.2\frac{m}{s}\right)^2 + (2kg\sin 42 - 4kg) \times 9.8\frac{m}{s^2} \times 0.25m}{2kg \times 9.8\frac{m}{s^2} \times 0.25m\cos 42}$$
$$\mu = 0.6$$

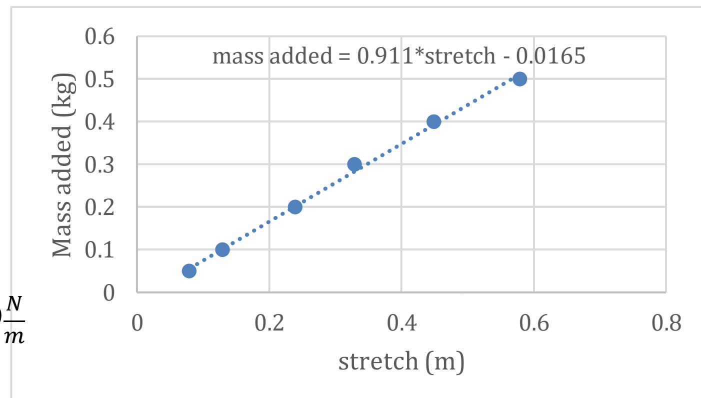
3. What is the ratio of the change in gravitational potential energy of the sphere to the change in gravitational potential energy of the block?

$$\Delta U_{g1} = m_1 g(y_{1f} - y_{1i}) = m_1 g d \sin\theta = 2kg \times 9.8\tfrac{m}{s^2} \times 0.25m \times \sin 42 = 3.28J$$

$$\Delta U_{g2} = m_2 g(y_{2f} - y_{2i}) = -m_2 g d = -4kg \times 9.8\tfrac{m}{s^2} \times 0.25m = -9.8J$$

$$\frac{\Delta U_{g2}}{\Delta U_{g1}} = \frac{-9.8J}{3.28J} = -3$$

A spring of stiffness k is used in a separate experiment. To determine the stiffness of the spring, the spring is initially suspended vertically, and various masses are hung and the corresponding stretch of the spring from equilibrium is measured. A plot of the mass added to the spring versus the stretch of the spring is shown below.



4. What is the stiffness k of the spring?

$$F_s - F_w = ma_y = 0$$

$$ky = mg \rightarrow m = \frac{k}{g}y$$

$$\frac{k}{g} = 0.91\frac{kg}{m} \rightarrow k = 0.91\frac{kg}{m} \times 9.8\frac{m}{s^2} = 8.9\frac{N}{m}$$

5. Suppose that the spring is laid horizontal and the $2kg$ mass is attached to the spring at its equilibrium length. The surface is frictionless and the $2kg$ mass is pulled out to a distance of $0.75m$ from equilibrium and released from rest. What is the speed of the mass when the spring returns to its equilibrium length?

$$\Delta E = \Delta K + \Delta U_s \rightarrow 0 = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}kx_i^2 \rightarrow v_f = \sqrt{\frac{k}{m}}x_i = \sqrt{\frac{8.9\frac{N}{m}}{2kg}} \times 0.75m = 1.6\frac{m}{s}$$

Physics 120 Formula Sheet

General Definitions of Motion

$\Delta\vec{r} = \langle \Delta x, \Delta y, \Delta z\rangle = \langle x_f - x_i, y_f - y, z_f - z_i\rangle$

$\vec{v} = \frac{\Delta\vec{r}}{\Delta t} = \langle \frac{\Delta x}{\Delta t}, \frac{\Delta y}{\Delta t}, \frac{\Delta z}{\Delta t}\rangle$

$\vec{a} = \frac{\Delta\vec{v}}{\Delta t} = \langle \frac{\Delta v_x}{\Delta t}, \frac{\Delta v_y}{\Delta t}, \frac{\Delta v_z}{\Delta t}\rangle$

$d\vec{r} = \langle dx, dy, dz\rangle$

$$\vec{v} = \langle v_x, v_y, v_z\rangle = \frac{d\vec{r}}{dt} = \langle \frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\rangle$$

$$\vec{a} = \langle a_x, a_y, a_z\rangle = \frac{d\vec{v}}{dt} = \langle \frac{dv_x}{dt}, \frac{dv_y}{dt}, \frac{dv_z}{dt}\rangle$$

Geometry

$C = 2\pi r \;\; A_{circle} = \pi r^2;\;\; A_{rect} = LW$

$A_{triangle} = \frac{1}{2}bh;\;\; A_{sphere} = 4\pi r^2$

$V_{sphere} = \frac{4}{3}\pi r^3;\;\; V_{cyl} = \pi r^2 h;\;\; V_{cone} = \frac{1}{3}\pi r^2 h$

Motion with constant acceleration

$\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2 \rightarrow \langle x_f, y, z_f\rangle = \langle x_i + v_{ix}t + \frac{1}{2}a_x t^2, y_i + v_{iy}t + \frac{1}{2}a_y t^2, z + v_{iz}t + \frac{1}{2}a_z t^2\rangle$

$\vec{v}_f = \vec{v}_i + \vec{a}t \rightarrow \langle v_{fx}, v_{fy}, v_{fz}\rangle = \langle v_{ix} + a_x t, v_{iy} + a_y t, v_{iz} + a_z t\rangle$

Forces/Momentum

$\vec{p} = m\vec{v}$

$\vec{F}_{net} = \frac{d\vec{p}}{dt} = m\vec{a}$

$\vec{p}_f - \vec{p}_i = \int d\vec{p} = \int \vec{F}_{net}dt$

$$\vec{J} = \int \vec{F}_{net}dt$$

$$\vec{F}_{net} = \vec{F}_{\parallel} + \vec{F}_{\perp} = \frac{d\vec{p}}{dt}\hat{p} + \vec{p}\frac{d\hat{p}}{dt} = m\vec{a}_{\parallel} + m\vec{a}_{\perp}$$

$$|\vec{F}_{\perp}| = m|\vec{a}_{\perp}| = m\frac{v^2}{r}$$

$$\vec{F}_G = G\frac{M_1 M_2}{r_{12}^2}\hat{r}_{12} \rightarrow |\vec{F}_G| = G\frac{M_1 M_2}{r_{12}^2}$$

$$\vec{F}_G = m\vec{g};\quad \vec{g} = G\frac{M_{cb}}{(R_{cb} + h)}\hat{r}$$

$|\vec{F}_{fr}| = \mu|\vec{F}_N|$

$\vec{F}_s = -k\Delta\vec{r}$

Constants

$g = 9.8\frac{m}{s^2};\;\; G = 6.67 \times 10^{-11}\frac{Nm^2}{kg^2}$

$v_{sound} = 343\frac{m}{s};\;\; v_{light} = c = 3 \times 10^8\,\frac{m}{s}$

$N_A = 6.02 \times 10^{23}$

Vectors

$\vec{C} = \vec{A} + \vec{B} \rightarrow \langle C_x, C_y, C_z\rangle = \langle A_x + B_x, A_y + B_y, A_z + B_z\rangle + \langle C_x, C_y, C_z\rangle;\quad |\vec{C}| = \sqrt{C_x^2 + C_y^2 + C_z^2}$

$\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta = A_x B_x + A_y B_y + A_z B_z$

$\vec{A}\times\vec{B} = |\vec{A}||\vec{B}|\sin\theta = \langle a_y b_z - b_y a_z, a_z b_x - b_z a_x, a_x b_y - b_x a_y\rangle$

Work and Energy

$$W_T = \int dW_T = \int \vec{F} \cdot d\vec{r} = \Delta K_T = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 = \frac{p_f^2}{2m} - \frac{p_i^2}{2m}$$

$$W_R = \int dW_R = \int \vec{\tau} \cdot d\vec{\theta} = \Delta K_R = \tfrac{1}{2}I\omega_f^2 - \tfrac{1}{2}I\omega_i^2 = \frac{L_f^2}{2I} - \frac{L_i^2}{2I}$$

$$W_{net} = W_T + W_R = \Delta E_{sys} = \begin{cases} 0 \\ W_{fr} \end{cases}$$

$$W_{net} = -\sum \Delta U = \Delta K_T + \Delta K_R$$

$$\Delta E_{sys} = \Delta K + \Delta U_g + \Delta U_s = \begin{cases} 0 \\ W_{fr} \end{cases}$$

$$U_g = mgy$$

$$U_s = \tfrac{1}{2}kx^2$$

Rotational Motion

$$s = r\theta \rightarrow ds = rd\theta$$

$$\frac{ds}{dt} = r\frac{d\theta}{dt} \rightarrow v = r\omega; \quad \omega = \frac{d\theta}{dt}$$

$$a = \frac{dv}{dt} = r\frac{d\omega}{dt} = r\alpha; \quad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$\theta_f = \theta_i + \omega_i t + \tfrac{1}{2}\alpha t^2$$

$$\omega_f = \omega_i + \alpha t$$

$$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$$

Rotational Forces/Momentum

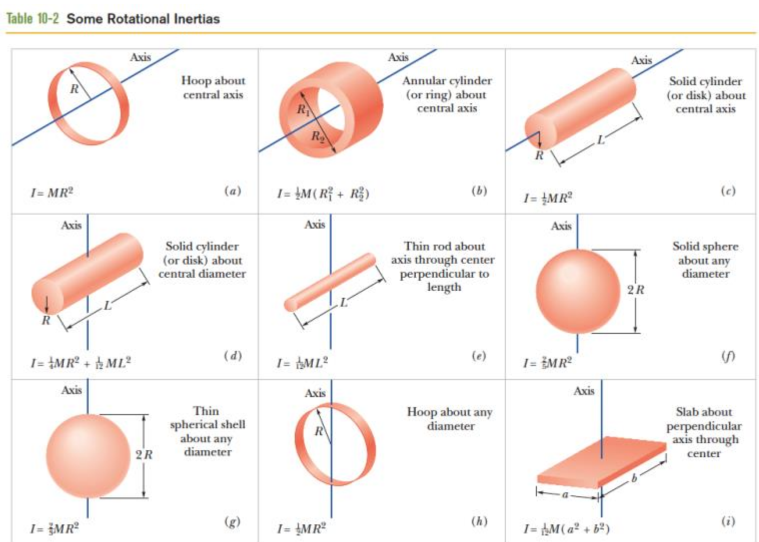
$$\vec{\tau} = \vec{r} \times \vec{F} = \frac{d\vec{L}}{dt} = I\vec{\alpha}$$

$$|\vec{\tau}| = rF\sin\theta = r_\perp F = rF_\perp$$

$$\vec{L} = I\vec{\omega}$$

$$I = \int r^2 dm$$

$$\vec{L}_f = \vec{L}_i + \int \vec{\tau}_{net} dt$$



Some moments of inertia from Halliday, Resnick, & Walker, 10[th] edition.