Name________________________________________

Physics 120 Quiz #6, February 21, 2020

*Please show all work, thoughts and/or reasoning in order to receive partial credit. The quiz is worth 10 points total.*

*I affirm that I have carried out my academic endeavors with full academic honesty.*

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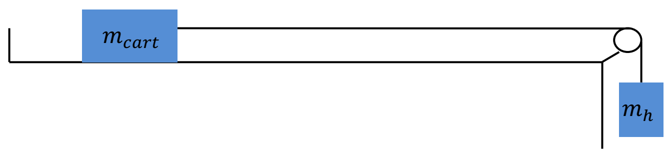
A cart of mass $m_{cart} = 0.60kg$ is attached to a hanging mass $m_h = 0.25kg$ by a light rope passing over a massless pulley. The hanging mass is released from rest, as shown below. The horizontal surface is considered frictionless unless otherwise specified.




a. What is the change in gravitational potential energy of the system of the two masses $m_{cart}$ and $m_h$ after the hanging mass $m_h$ falls a distance of $\Delta y = 1.2m$?

$$\Delta U_g = \Delta U_{g1} + \Delta U_{g2} = (m_{cart} g y_{f,cart} - m_{cart} g y_{i,cart}) + (m_h g y_{f,h} - m_h g y_{i,h})$$

$$\Delta U_g = (m_h g y_{f,h} - m_h g y_{i,h}) = -m_h g y_{i,h} = -0.25kg \times 9.8\frac{m}{s^2} \times 1.2m = -2.94J$$

b. If the horizontal surface is frictionless, what is the change in kinetic energy of the cart and hanging mass system?

$$\Delta E_{system} = \Delta K_{cart} + \Delta K_h + \Delta U_{g,cart} + \Delta U_{g,h} = \Delta K_{cart} + \Delta K_h + \Delta U_{g,h} = 0$$

$$\rightarrow \Delta K_{cart} + \Delta K_h = -\Delta U_{g,h} = 2.94J$$

c. If the horizontal surface is frictionless, what is the speed of the hanging mass $m_h$ after it falls a distance of $\Delta y = 1.2m$?

$$\Delta K_{cart} + \Delta K_h = \left(\tfrac{1}{2}m_{cart}v_{f,cart}^2 - \tfrac{1}{2}m_{cart}v_{i,cart}^2\right) + \left(\tfrac{1}{2}m_h v_{f,h}^2 - \tfrac{1}{2}m_h v_{i,h}^2\right) = -\Delta U_{g,h}$$

$$\tfrac{1}{2}m_{cart}v_{f,cart}^2 + \tfrac{1}{2}m_h v_{f,h}^2 = \tfrac{1}{2}(m_{cart} + m_h)v_f^2 = -\Delta U_{g,h}$$

$$\rightarrow v_f = \sqrt{\frac{-2 \times \Delta U_{g,h}}{m_{cart} + m_h}} = \sqrt{\frac{-2 \times (-2.94J)}{0.60kg + 0.25kg}} = 2.6\frac{m}{s}$$

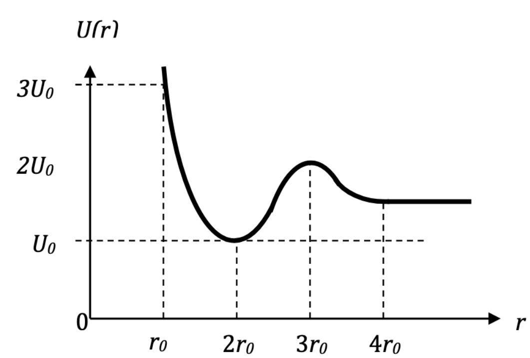
d. Suppose that there is friction between the cart and the horizontal surface, with coefficient of friction $\mu_k = 0.3$. What is the speed of the hanging mass $m_h$ in this case, after it falls a distance of $\Delta y = 1.2m$ after being released from rest?

$$\Delta E_{system} = \Delta K_{cart} + \Delta K_h + \Delta U_{g,h} = W_{fr} = -\mu_k F_N x$$
$$-\mu_k m_{cart} g x = -\mu_k m_{cart} g y = \tfrac{1}{2}m_{cart}v_f^2 + \tfrac{1}{2}m_h v_f^2 + -m_h g y$$

$$\tfrac{1}{2}(m_{cart} + m_h)v_f^2 = m_h g y - \mu_k m_{cart} g y = (m_h - \mu_k m_{cart})gy$$

$$v_f = \sqrt{\frac{2(m_h - \mu_k m_{cart})gy}{m_{cart} + m_h}} = \sqrt{\frac{2(0.25kg - 0.3 \times 0.6kg) \times 9.8\frac{m}{s^2} \times 1.2m}{0.25kg + 0.6kg}} = 1.4\frac{m}{s}$$

e. Consider the graph shown on right of the potential energy of a particle of mass $m$ moving in one dimension as a function of distance. If the particle is released from rest at a location $r_o$ and moves between $r_o$ and $3r_o$, the speed of the particle is most likely given by which of the following?




1. $v = \sqrt{\frac{3U_0}{2m}}$.
2. $v = \sqrt{\frac{4U_0}{m}}$.
3. $v = \sqrt{\frac{6U_0}{m}}$.
4. $v = \sqrt{\frac{U_0}{m}}$.
5. $v = \sqrt{\frac{2U_0}{m}}$.

# Physics 120 Formula Sheet

## General Definitions of Motion

$\Delta\vec{r} = \langle \Delta x, \Delta y, \Delta z\rangle = \langle x_f - x_i, y_f - y, z_f - z_i\rangle$

$\vec{v} = \frac{\Delta\vec{r}}{\Delta t} = \langle \frac{\Delta x}{\Delta t}, \frac{\Delta y}{\Delta t}, \frac{\Delta z}{\Delta t}\rangle$

$\vec{a} = \frac{\Delta\vec{v}}{\Delta t} = \langle \frac{\Delta v_x}{\Delta t}, \frac{\Delta v_y}{\Delta t}, \frac{\Delta v_z}{\Delta t}\rangle$

$d\vec{r} = \langle dx, dy, dz\rangle$

$$\vec{v} = \langle v_x, v_y, v_z\rangle = \frac{d\vec{r}}{dt} = \langle \frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\rangle$$

$$\vec{a} = \langle a_x, a_y, a_z\rangle = \frac{d\vec{v}}{dt} = \langle \frac{dv_x}{dt}, \frac{dv_y}{dt}, \frac{dv_z}{dt}\rangle$$

## Geometry

$C = 2\pi r \quad A_{circle} = \pi r^2; \quad A_{rect} = LW$

$A_{triangle} = \frac{1}{2}bh; \quad A_{sphere} = 4\pi r^2$

$V_{sphere} = \frac{4}{3}\pi r^3; \quad V_{cyl} = \pi r^2 h; \quad V_{cone} = \frac{1}{3}\pi r^2 h$

## Motion with constant acceleration

$\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2 \rightarrow \langle x_f, y, z_f\rangle = \langle x_i + v_{ix}t + \frac{1}{2}a_x t^2, y_i + v_{iy}t + \frac{1}{2}a_y t^2, z + v_{iz}t + \frac{1}{2}a_z t^2\rangle$

$\vec{v}_f = \vec{v}_i + \vec{a}t \rightarrow \langle v_{fx}, v_{fy}, v_{fz}\rangle = \langle v_{ix} + a_x t, v_{iy} + a_y t, v_{iz} + a_z t\rangle$

## Forces/Momentum

$\vec{p} = m\vec{v}$

$\vec{F}_{net} = \frac{d\vec{p}}{dt} = m\vec{a}$

$\vec{p}_f - \vec{p}_i = \int d\vec{p} = \int \vec{F}_{net} dt$

$$\vec{J} = \int \vec{F}_{net} dt$$

$$\vec{F}_{net} = \vec{F}_\parallel + \vec{F}_\perp = \frac{dp}{dt}\hat{p} + p\frac{d\hat{p}}{dt} = m\vec{a}_\parallel + m\vec{a}_\perp$$

$$|\vec{F}_\perp| = m|\vec{a}_\perp| = m\frac{v^2}{r}$$

$$\vec{F}_G = G\frac{M_1 M_2}{r_{12}^2}\hat{r}_{12} \rightarrow |\vec{F}_G| = G\frac{M_1 M_2}{r_{12}^2}$$

$$\vec{F}_G = m\vec{g}; \quad \vec{g} = G\frac{M_{cb}}{(R_{cb} + h)}\hat{r}$$

$|\vec{F}_{fr}| = \mu|\vec{F}_N|$

$\vec{F}_s = -k\Delta\vec{r}$

## Constants

$g = 9.8\frac{m}{s^2}; \quad G = 6.67 \times 10^{-11}\frac{Nm^2}{kg^2}$

$v_{sound} = 343\frac{m}{s}; \quad v_{light} = c = 3 \times 10^8\,\frac{m}{s}$

$N_A = 6.02 \times 10^{23}$

## Vectors

$\vec{C} = \vec{A} + \vec{B} \rightarrow \langle C_x, C_y, C_z\rangle = \langle A_x + B_x, A_y + B_y, A_z + B_z\rangle + \langle C_x, C_y, C_z\rangle; \quad \overline{|C|} = \sqrt{C_x^2 + C_y^2 + C_z^2}$

$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}|\cos\theta = A_x B_x + A_y B_y + A_z B_z$

$\vec{A} \times \vec{B} = |\vec{A}||\vec{B}|\sin\theta = \langle a_y b_z - b_y a_z, a_z b_x - b_z a_x, a_x b_y - b_x a_y\rangle$

## Work and Energy

$$W_T = \int dW_T = \int \vec{F} \cdot d\vec{r} = \Delta K_T = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 = \frac{p_f^2}{2m} - \frac{p_i^2}{2m}$$

$$W_R = \int dW_R = \int \vec{\tau} \cdot d\vec{\theta} = \Delta K_R = \tfrac{1}{2}I\omega_f^2 - \tfrac{1}{2}I\omega_i^2$$

$$W_{net} = W_T + W_R = \Delta E_{sys} = \begin{cases} 0 \\ W_{fr} \end{cases}$$

$$W_{net} = -\sum \Delta U = \Delta K_T + \Delta K_R$$

$$U_g = mgy$$

$$U_s = \tfrac{1}{2}kx^2$$

## Rotational Motion

$$s = r\theta \rightarrow ds = rd\theta$$

$$\frac{ds}{dt} = r\frac{d\theta}{dt} \rightarrow v = r\omega; \;\; \omega = \frac{d\theta}{dt}$$

$$a = \frac{dv}{dt} = r\frac{d\omega}{dt} = r\alpha; \;\; \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$\theta_f = \theta_i + \omega_i t + \tfrac{1}{2}\alpha t^2$$

$$\omega_f = \omega_i + \alpha t$$

$$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$$

## Rotational Forces/Momentum

$$\vec{\tau} = \vec{r} \times \vec{F} = \frac{d\vec{L}}{dt} = I\vec{\alpha}$$

$$\tau = rF\sin\theta = r_\perp F = rF_\perp$$

$$\vec{L} = I\vec{\omega}$$

$$I = \int r^2 dm$$

$$\vec{L}_f = \vec{L}_i + \int \vec{\tau}_{net} dt$$