Name____

Physics 110 Quiz #6, March 4, 2022

Please show all work, thoughts and/or reasoning in order to receive partial credit. The quiz is worth 10 points total.

I affirm that I have carried out my academic endeavors with full academic honesty.

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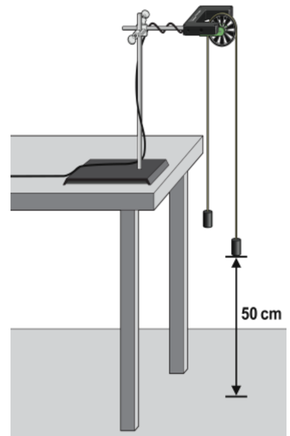
A block of mass $m_1 = 2kg$ on the right is tied by a light rope to a block of mass $m_2 = 1kg$ on the left. The rope is wrapped around a pully of mass $m_p = 0.1kg$ and radius $r_p = 1.25cm$. Suppose the mass on the right is released from rest and allowed to fall through a vertical distance $|\Delta y| = 50cm$.



https://www.vernier.com/experiment/pwv-10_atwoods-machine/

1. From an analysis of the forces and torques in the system, what are the expressions for each of the following:

   Forces on m_1 choosing up as the positive y-direction:

   $F_{TR} - m_1 g = -m_1 a$

   Forces on m_2 choosing up as the positive y-direction:

   $F_{TL} - m_2 g = m_2 a$

   Torque about the pulley assuming that counterclockwise is the positive direction:

   $-r_p F_{TL} + r_p F_{TR} = I\alpha = \frac{1}{2} m_p r_p^2 \alpha = \frac{1}{2} m_p r_p a$

   $\rightarrow -F_{TL} + F_{TR} = \frac{1}{2} m_p a$

2. Using the expressions in question 1, what is the magnitude of the acceleration of m_1 on the right?

$$-(m_2 a + m_2 g) + (m_1 g - m_1 a) = \tfrac{1}{2} m_p a$$
$$(m_1 + m_2) g = \left(m_1 + m_2 + \tfrac{1}{2} m_p\right) a$$
$$a = \left(\frac{m_1 + m_2}{m_1 + m_2 + \frac{1}{2} m_p}\right) g = \left(\frac{2kg + 1kg}{2kg + 1kg + 0.1kg}\right) \times 9.8 \frac{m}{s^2} = 9.5 \frac{m}{s^2}$$

3. What is the translational speed of mass m_1 just before it strikes the ground?

$$v_{fy}^2 = v_{iy}^2 + 2a\Delta x \rightarrow v_{fy} = \sqrt{2a\Delta y} = \sqrt{2 \times 9.5 \frac{m}{s^2} \times 0.5m} = 3.1 \frac{m}{s}$$

4. What is the rotational speed of the pulley just before m_1 hits the ground?

$$v = r_p \omega \rightarrow \omega = \frac{v}{r_p} = \frac{3.1 \frac{m}{s}}{0.0125m} = 246.6 \frac{rad}{s}$$

5. How many revolutions did the pulley make just before the mass m_1 struck the ground?

$$\Delta s = r_p \Delta\theta \rightarrow \Delta\theta = \frac{\Delta s}{r_p} = \frac{0.5m}{0.0125m} = 40rad \times \frac{1rev}{2\pi rad} = 6.4rev$$

Physics 120 Formula Sheet

General Definitions of Motion

$\Delta\vec{r} = \langle \Delta x, \Delta y, \Delta z \rangle = \langle x_f - x_i, y_f - y, z_f - z_i \rangle$

$\vec{v} = \frac{\Delta\vec{r}}{\Delta t} = \langle \frac{\Delta x}{\Delta t}, \frac{\Delta y}{\Delta t}, \frac{\Delta z}{\Delta t} \rangle$

$\vec{a} = \frac{\Delta\vec{v}}{\Delta t} = \langle \frac{\Delta v_x}{\Delta t}, \frac{\Delta v_y}{\Delta t}, \frac{\Delta v_z}{\Delta t} \rangle$

$d\vec{r} = \langle dx, dy, dz \rangle$

$\vec{v} = \langle v_x, v_y, v_z \rangle = \frac{d\vec{r}}{dt} = \langle \frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt} \rangle$

$\vec{a} = \langle a_x, a_y, a_z \rangle = \frac{d\vec{v}}{dt} = \langle \frac{dv_x}{dt}, \frac{dv_y}{dt}, \frac{dv_z}{dt} \rangle$

Geometry

$C = 2\pi r \quad A_{circle} = \pi r^2; \quad A_{rect} = LW$

$A_{triangle} = \frac{1}{2}bh; \quad A_{sphere} = 4\pi r^2$

$V_{sphere} = \frac{4}{3}\pi r^3; \quad V_{cyl} = \pi r^2 h; \quad V_{cone} = \frac{1}{3}\pi r^2 h$

Motion with constant acceleration

$\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2 \rightarrow \langle x_f, y, z_f \rangle = \langle x_i + v_{ix}t + \frac{1}{2}a_x t^2, y_i + v_{iy}t + \frac{1}{2}a_y t^2, z + v_{iz}t + \frac{1}{2}a_z t^2 \rangle$

$\vec{v}_f = \vec{v}_i + \vec{a}t \rightarrow \langle v_{fx}, v_{fy}, v_{fz} \rangle = \langle v_{ix} + a_x t, v_{iy} + a_y t, v_{iz} + a_z t \rangle$

Forces/Momentum

$\vec{p} = m\vec{v}$

$\vec{F}_{net} = \frac{d\vec{p}}{dt} = m\vec{a}$

$\vec{p}_f - \vec{p}_i = \int d\vec{p} = \int \vec{F}_{net} dt$

$\vec{J} = \int \vec{F}_{net} dt$

$\vec{F}_{net} = \vec{F}_{\parallel} + \vec{F}_{\perp} = \frac{d\vec{p}}{dt}\hat{p} + \vec{p}\frac{d\hat{p}}{dt} = m\vec{a}_{\parallel} + m\vec{a}_{\perp}$

$|\vec{F}_{\perp}| = m|\vec{a}_{\perp}| = m\frac{v^2}{r}$

$\vec{F}_G = G\frac{M_1 M_2}{r_{12}^2}\hat{r}_{12} \rightarrow |\vec{F}_G| = G\frac{M_1 M_2}{r_{12}^2}$

$\vec{F}_G = m\vec{g}; \quad \vec{g} = G\frac{M_{cb}}{(R_{cb} + h)}\hat{r}$

$|\vec{F}_{fr}| = \mu|\vec{F}_N|$

$\vec{F}_s = -k\Delta\vec{r}$

Constants

$g = 9.8\frac{m}{s^2}; \quad G = 6.67 \times 10^{-11}\frac{Nm^2}{kg^2}$

$v_{sound} = 343\frac{m}{s}; \quad v_{light} = c = 3 \times 10^8\frac{m}{s}$

$N_A = 6.02 \times 10^{23}$

Vectors

$\vec{C} = \vec{A} + \vec{B} \rightarrow \langle C_x, C_y, C_z \rangle = \langle A_x + B_x, A_y + B_y, A_z + B_z \rangle + \langle C_x, C_y, C_z \rangle; \quad \overrightarrow{|C|} = \sqrt{C_x^2 + C_y^2 + C_z^2}$

$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}|\cos\theta = A_x B_x + A_y B_y + A_z B_z$

$\vec{A} \times \vec{B} = |\vec{A}||\vec{B}|\sin\theta = \langle a_y b_z - b_y a_z, a_z b_x - b_z a_x, a_x b_y - b_x a_y \rangle$

Work and Energy

$$W_T = \int dW_T = \int \vec{F} \cdot d\vec{r} = \Delta K_T = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 = \frac{p_f^2}{2m} - \frac{p_i^2}{2m}$$

$$W_R = \int dW_R = \int \vec{\tau} \cdot d\vec{\theta} = \Delta K_R = \tfrac{1}{2}I\omega_f^2 - \tfrac{1}{2}I\omega_i^2 = \frac{L_f^2}{2I} - \frac{L_i^2}{2I}$$

$$W_{net} = W_T + W_R = \Delta E_{sys} = \begin{cases} 0 \\ W_{fr} \end{cases}$$

$$W_{net} = -\sum \Delta U = \Delta K_T + \Delta K_R$$

$$\Delta E_{sys} = \Delta K + \Delta U_g + \Delta U_s = \begin{cases} 0 \\ W_{fr} \end{cases}$$

$$U_g = mgy$$

$$U_s = \tfrac{1}{2}kx^2$$

Rotational Motion

$$s = r\theta \rightarrow ds = rd\theta$$

$$\frac{ds}{dt} = r\frac{d\theta}{dt} \rightarrow v = r\omega; \quad \omega = \frac{d\theta}{dt}$$

$$a = \frac{dv}{dt} = r\frac{d\omega}{dt} = r\alpha; \quad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$\theta_f = \theta_i + \omega_i t + \tfrac{1}{2}\alpha t^2$$

$$\omega_f = \omega_i + \alpha t$$

$$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$$

Rotational Forces/Momentum

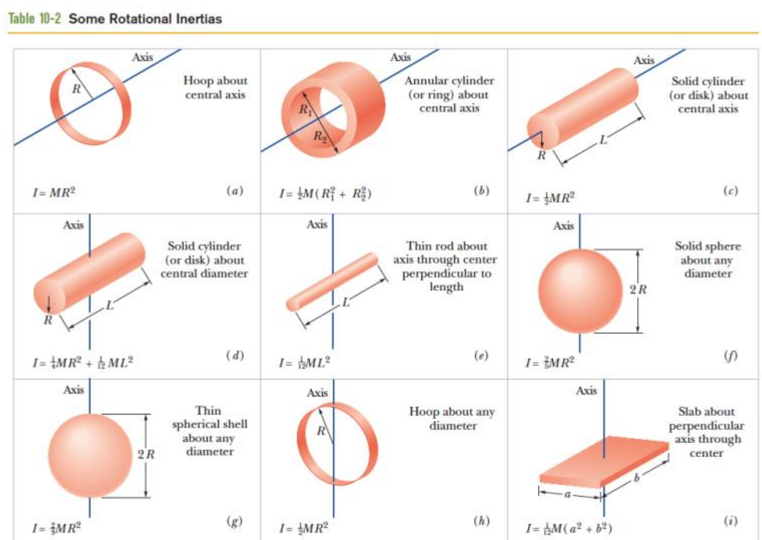
$$\vec{\tau} = \vec{r} \times \vec{F} = \frac{d\vec{L}}{dt} = I\vec{\alpha}$$

$$|\vec{\tau}| = rF\sin\theta = r_\perp F = rF_\perp$$

$$\vec{L} = I\vec{\omega}$$

$$I = \int r^2 dm$$

$$\vec{L}_f = \vec{L}_i + \int \vec{\tau}_{net} dt$$



Some moments of inertia from Halliday, Resnick, & Walker, 10th edition.