Name____

Physics 120 Quiz #7, March 6, 2020

Please show all work, thoughts and/or reasoning in order to receive partial credit. The quiz is worth 10 points total.

I affirm that I have carried out my academic endeavors with full academic honesty.

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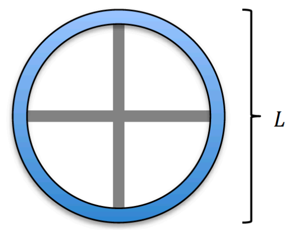
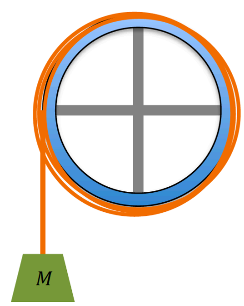
A pair of long, thin, rods, each of length L and mass M are connected to a hoop of mass M and radius $\frac{L}{2}$ to form a 4-spoked wheel as shown on the right.



a. Assuming moments of inertia are additive, what is the total moment of inertia of the 4-spoked wheel assembly for an axis of rotation through the center of the assembly perpendicular to the plane of the wheel?

$$I_{total} = I_{hoop} + I_{rod} + I_{rod} = M\left(\frac{L}{2}\right)^2 + \frac{1}{12}ML^2 + \frac{1}{12}ML^2 = \frac{5}{12}ML^2$$

b. Suppose that the 4-spoked wheel is mounted to a low-friction axle through its center and several turns of light rope are wrapped onto the wheel as shown on the right. A mass M is attached to the assembly and the system is released from rest. What is the tension in the rope attached to the mass as the mass falls?



$$|\vec{\tau}_{net}| = \vec{r} \times \vec{F} = \frac{L}{2}F_T \sin 90 = \frac{L}{2}F_T$$

$$\vec{\tau}_{net} = I\vec{\alpha} = \langle 0,0,+\frac{L}{2}F_T\rangle + I\langle 0,0,+\alpha\rangle \rightarrow \frac{L}{2}F_T = I\alpha = I\left(\frac{2a}{L}\right) \rightarrow a = \frac{L^2F_T}{4I}$$

$$\vec{F}_{net} = \vec{F}_T + \vec{F}_W = M\vec{a} \rightarrow \langle 0, F_T - Mg, 0\rangle = M\langle 0, -a, 0\rangle \rightarrow F_T - Mg = -Ma \rightarrow a = \frac{Mg - F_T}{M}$$

$$a = \frac{L^2F_T}{4I} = \frac{Mg - F_T}{M} \rightarrow F_T = \frac{Mg}{\left(\frac{ML^2}{4I} + 1\right)} = \frac{Mg}{\left(\frac{ML^2}{4\left(\frac{5}{12}ML^2\right)} + 1\right)} = \frac{5}{8}Mg$$

$$F_T = \frac{5}{8}Mg$$

c. What is the magnitude of the angular acceleration of the assembly after the wheel has turned one full revolution?

$$\tau = I\alpha \rightarrow \alpha = \frac{\tau}{I} = \frac{\frac{L}{2}F_T}{I} = \frac{\frac{L}{2}\left(\frac{5}{8}Mg\right)}{\frac{5}{12}ML^2} = \frac{3g}{4L}$$

Or $a = r\alpha \rightarrow \frac{L^2 F_T}{4I} = \frac{L^2\left(\frac{5}{8}Mg\right)}{4\left(\frac{5}{12}ML^2\right)} = \frac{12}{32}g = \frac{L}{2}\alpha \rightarrow \alpha = \frac{3g}{4L}$

d. What is the magnitude of the angular velocity of the assembly after the wheel has made one full revolution?

$$v_{fy}^2 = v_{iy}^2 + 2a_y\Delta y = 2a_y\Delta y = 2\left(\frac{12}{32}g\right)\left(2\pi\frac{L}{2}\right) = \frac{3\pi gL}{4} = (r\omega)^2 = \left(\frac{L}{2}\omega\right)^2$$

$$\frac{L^2\omega}{4} = \frac{3\pi gL}{4} \rightarrow \omega = \sqrt{\frac{3\pi g}{L}}$$

e. What is the angular momentum of the wheel assembly only, after the wheel has made one full revolution?

$$L_{wheel} = I\omega = \frac{5}{12}ML^2\left(\sqrt{\frac{3\pi g}{L}}\right) = \sqrt{\frac{75\pi g M^2 L^4}{144L}} = \frac{1}{12}ML\sqrt{75\pi gL}$$

Physics 120 Formula Sheet

General Definitions of Motion

$\Delta\vec{r} = \langle \Delta x, \Delta y, \Delta z \rangle = \langle x_f - x_i, y_f - y, z_f - z_i \rangle$

$\vec{v} = \frac{\Delta \vec{r}}{\Delta t} = \langle \frac{\Delta x}{\Delta t}, \frac{\Delta y}{\Delta t}, \frac{\Delta z}{\Delta t} \rangle$

$\vec{a} = \frac{\Delta \vec{v}}{\Delta t} = \langle \frac{\Delta v_x}{\Delta t}, \frac{\Delta v_y}{\Delta t}, \frac{\Delta v_z}{\Delta t} \rangle$

$d\vec{r} = \langle dx, dy, dz \rangle$

$$\vec{v} = \langle v_x, v_y, v_z \rangle = \frac{d\vec{r}}{dt} = \langle \frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt} \rangle$$

$$\vec{a} = \langle a_x, a_y, a_z \rangle = \frac{d\vec{v}}{dt} = \langle \frac{dv_x}{dt}, \frac{dv_y}{dt}, \frac{dv_z}{dt} \rangle$$

Geometry

$C = 2\pi r \;\; A_{circle} = \pi r^2; \;\; A_{rect} = LW$

$A_{triangle} = \frac{1}{2}bh; \;\; A_{sphere} = 4\pi r^2$

$V_{sphere} = \frac{4}{3}\pi r^3; \;\; V_{cyl} = \pi r^2 h; \;\; V_{cone} = \frac{1}{3}\pi r^2 h$

Motion with constant acceleration

$\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2 \rightarrow \langle x_f, y, z_f \rangle = \langle x_i + v_{ix}t + \frac{1}{2}a_x t^2, y_i + v_{iy}t + \frac{1}{2}a_y t^2, z + v_{iz}t + \frac{1}{2}a_z t^2 \rangle$

$\vec{v}_f = \vec{v}_i + \vec{a}t \rightarrow \langle v_{fx}, v_{fy}, v_{fz} \rangle = \langle v_{ix} + a_x t, v_{iy} + a_y t, v_{iz} + a_z t \rangle$

Forces/Momentum

$\vec{p} = m\vec{v}$

$\vec{F}_{net} = \frac{d\vec{p}}{dt} = m\vec{a}$

$\vec{p}_f - \vec{p}_i = \int d\vec{p} = \int \vec{F}_{net} dt$

$$\vec{J} = \int \vec{F}_{net} dt$$

$$\vec{F}_{net} = \vec{F}_{\parallel} + \vec{F}_{\perp} = \frac{d\vec{p}}{dt}\hat{p} + \vec{p}\frac{d\hat{p}}{dt} = m\vec{a}_{\parallel} + m\vec{a}_{\perp}$$

$$|\vec{F}_{\perp}| = m|\vec{a}_{\perp}| = m\frac{v^2}{r}$$

$$\vec{F}_G = G\frac{M_1 M_2}{r_{12}^2}\hat{r}_{12} \rightarrow |\vec{F}_G| = G\frac{M_1 M_2}{r_{12}^2}$$

$$\vec{F}_G = m\vec{g}; \;\; \vec{g} = G\frac{M_{cb}}{(R_{cb} + h)}\hat{r}$$

$|\vec{F}_{fr}| = \mu|\vec{F}_N|$

$\vec{F}_s = -k\Delta\vec{r}$

Constants

$g = 9.8\frac{m}{s^2}; \;\; G = 6.67 \times 10^{-11}\frac{Nm^2}{kg^2}$

$v_{sound} = 343\frac{m}{s}; \;\; v_{light} = c = 3 \times 10^8 \frac{m}{s}$

$N_A = 6.02 \times 10^{23}$

Vectors

$\vec{C} = \vec{A} + \vec{B} \rightarrow \langle C_x, C_y, C_z \rangle = \langle A_x + B_x, A_y + B_y, A_z + B_z \rangle + \langle C_x, C_y, C_z \rangle; \;\; |\vec{C}| = \sqrt{C_x^2 + C_y^2 + C_z^2}$

$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}|\cos\theta = A_x B_x + A_y B_y + A_z B_z$

$\vec{A} \times \vec{B} = |\vec{A}||\vec{B}|\sin\theta = \langle a_y b_z - b_y a_z, a_z b_x - b_z a_x, a_x b_y - b_x a_y \rangle$

Work and Energy

$$W_T = \int dW_T = \int \vec{F} \cdot d\vec{r} = \Delta K_T = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 = \frac{p_f^2}{2m} - \frac{p_i^2}{2m}$$

$$W_R = \int dW_R = \int \vec{\tau} \cdot d\vec{\theta} = \Delta K_R = \tfrac{1}{2}I\omega_f^2 - \tfrac{1}{2}I\omega_i^2 = \frac{L_f^2}{2I} - \frac{L_i^2}{2I}$$

$$W_{net} = W_T + W_R = \Delta E_{sys} = \begin{cases} 0 \\ W_{fr} \end{cases}$$

$$W_{net} = -\sum \Delta U = \Delta K_T + \Delta K_R$$

$$U_g = mgy$$

$$U_s = \tfrac{1}{2}kx^2$$

Rotational Motion

$$s = r\theta \rightarrow ds = rd\theta$$

$$\frac{ds}{dt} = r\frac{d\theta}{dt} \rightarrow v = r\omega; \quad \omega = \frac{d\theta}{dt}$$

$$a = \frac{dv}{dt} = r\frac{d\omega}{dt} = r\alpha; \quad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$\theta_f = \theta_i + \omega_i t + \tfrac{1}{2}\alpha t^2$$

$$\omega_f = \omega_i + \alpha t$$

$$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$$

Rotational Forces/Momentum

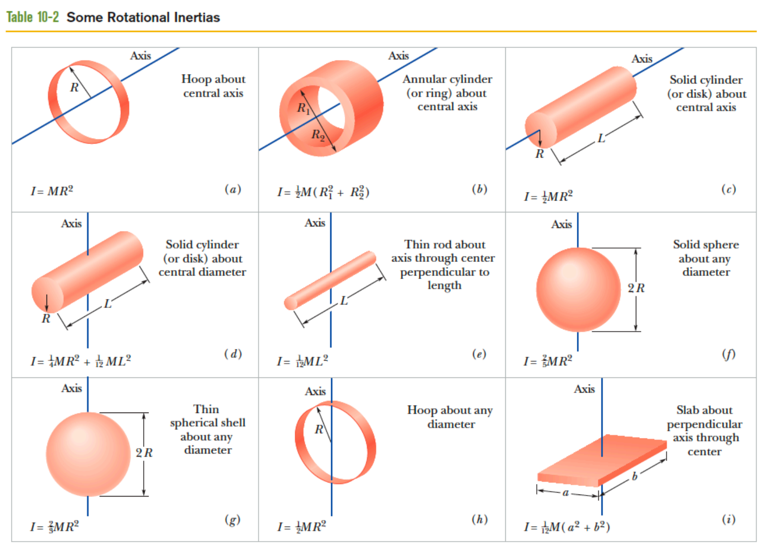
$$\vec{\tau} = \vec{r} \times \vec{F} = \frac{d\vec{L}}{dt} = I\vec{\alpha}$$

$$|\vec{\tau}| = rF\sin\theta = r_\perp F = rF_\perp$$

$$\vec{L} = I\vec{\omega}$$

$$I = \int r^2 dm$$

$$\vec{L}_f = \vec{L}_i + \int \vec{\tau}_{net} dt$$



Some moments of inertia from Halliday, Resnick, & Walker, 10th edition.