Physics 121

Exam #1

September 30, 2022

Name____________________________________

Please read and follow these instructions carefully:

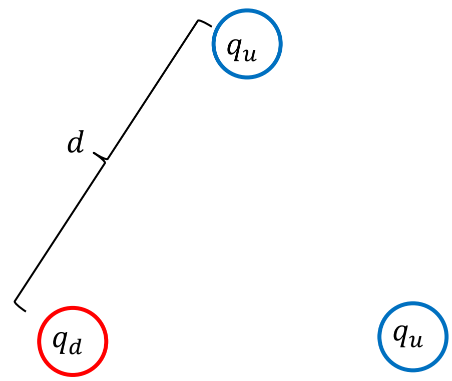
- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example $|\vec{p}| \approx m|\vec{v}| = (5kg) \times (2\,\frac{m}{s}) = 10\,\frac{kg \cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- All free-response part is worth 6 points.

Problem #1	/24
Problem #2	/24
Problem #3	/24
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

____

1. When we've done problem involving protons, we've assumed them to be point charges, meaning that they are a point in space with no extent. In actuality, protons are not points, but a bound state consisting of three quarks. Suppose that you have two "up" quarks with a charge of $q_u = +\frac{2}{3}e$ and one "down" quark with charge $q_d = -\frac{1}{3}e$ located on the vertices of an equilateral triangle. The distance between each charge is $d = 2.6 \times 10^{-15} m$ as shown on the right.



a. What is the electrostatic force on the down quark due to the two up quarks? Assume that the quarks are point charges. Assume that the two up quarks are already at their locations when you bring the down quark in.

$$\vec{F}_{net,d} = \vec{F}_{d,u} + \vec{F}_{d,u} = \frac{kq_u q_d}{d^2}\left[\frac{\langle 0,0,0\rangle - \langle \frac{d}{2}, \frac{\sqrt{3}d}{2}, 0\rangle}{d}\right] + \frac{kq_u q_d}{d^2}\left[\frac{\langle 0,0,0\rangle - \langle d,0,0\rangle}{d}\right]$$

$$\vec{F}_{net,d} = \frac{k\left(\frac{2}{3}e\right)\left(-\frac{1}{3}e\right)}{d^2}\langle -\frac{1}{2}, -\frac{\sqrt{3}}{2}, 0\rangle + \frac{k\left(\frac{2}{3}e\right)\left(-\frac{1}{3}e\right)}{d^2}\langle -1,0,0\rangle$$

$$\vec{F}_{net,d} = \frac{2ke^2}{9d^2}\langle \frac{3}{2}, \frac{\sqrt{3}}{2}, 0\rangle = \frac{2 \times 9 \times 10^9 \frac{Nm^2}{C^2} \times (1.6 \times 10^{-19} C)^2}{9 \times (2.6 \times 10^{-15} m)^2}\langle \frac{3}{2}, \frac{\sqrt{3}}{2}, 0\rangle = \langle 11.4, 6.6, 0\rangle N$$

As a magnitude and direction: $13.2N$ @ $\phi = 30.1^0$

b. What is the net electric field at the down quark's location due to the two up quarks?

$$\vec{F}_{net,d} = q\vec{E}_{net,d} \rightarrow \vec{E}_{net,d} = \frac{\vec{F}_{net,d}}{q_d} = \frac{\langle 11.4, 6.6, 0\rangle N}{-\frac{1}{3} \times 1.6 \times 10^{-19} C} = \langle -2.1, -1.24, 0\rangle \times 10^{20} \frac{N}{C}$$

As a magnitude and direction: $2.4 \times 10^{20} \frac{N}{C}$ @ -30.6^0 or 30.6^0 below the negative x-axis.

c. What is the value of the electric potential (voltage) at the down quark's location due the two up quarks?

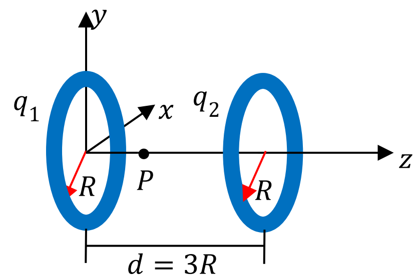
$$V_d = V_{u,d} + V_{u,d} = \frac{kq_u}{d} + \frac{kq_u}{d} = \frac{2kq_u}{d} = \frac{4ke}{3d} = \frac{4 \times 9 \times 10^9 \frac{Nm^2}{C^2} \times 1.6 \times 10^{-19} C}{3 \times 2.6 \times 10^{-15} m}$$

$$V_d = 7.4 \times 10^5 V$$

d. How much work would it take to brin the down quark in from very far away and place it at its current location?

$$W = -q\Delta V = -\left(-\frac{1}{3}e\right)\Delta V = \frac{1}{3} \times 1.6 \times 10^{-19} C \times 7.4 \times 10^5 V = 3.94 \times 10^{-14} J$$

2. Suppose that you have two parallel nonconducting rings with their central axes along a common line. Ring 1 has uniform charge q_1 and radius R, while ring 2 has uniform charge q_2 and radius R. The rings are separated by a distance $d = 3R$.



a. What is the electric field at point $P = \langle 0,0,R\rangle$ from the ring of charge q_1? Express your answer in terms of k, q_1, and R.

$$\vec{E}_1 = \frac{kq_1 z}{(R^2+z^2)^{\frac{3}{2}}}\left[\frac{\langle 0,0,R\rangle - \langle 0,0,0\rangle}{R}\right] = \frac{kq_1 R}{(R^2+R^2)^{\frac{3}{2}}}\langle 0,0,1\rangle = \frac{kq_1}{\left(\sqrt{2}\right)^3 R^2}\langle 0,0,1\rangle$$

b. What is the electric field at point $P = \langle 0,0,R\rangle$ from the ring of charge q_2? Express your answer in terms of k, q_2, and R.

$$\vec{E}_2 = \frac{kq_2 z}{(R^2+z^2)^{\frac{3}{2}}}\left[\frac{\langle 0,0,R\rangle - \langle 0,0,3R\rangle}{2R}\right] = \frac{kq_2(2R)}{(R^2+(2R)^2)^{\frac{3}{2}}}\langle 0,0,-1\rangle = \frac{2kq_2}{\left(\sqrt{5}\right)^3 R^2}\langle 0,0,-1\rangle$$

c. Suppose that at point $P = \langle 0,0,R\rangle$ the net eledctric field were known to be zero. What is the ratio of q_1 to q_2? That is, what is ${}^{q_1}/_{q_2}$?

$$\vec{E}_{net} = \langle 0,0,0\rangle = \vec{E}_1 + \vec{E}_2 = \frac{kq_1}{\left(\sqrt{2}\right)^3 R^2}\langle 0,0,1\rangle + \frac{2kq_2}{\left(\sqrt{5}\right)^3 R^2}\langle 0,0,-1\rangle$$

$$0 = \frac{kq_1}{\left(\sqrt{2}\right)^3 R^2} - \frac{2kq_2}{\left(\sqrt{5}\right)^3 R^2} \rightarrow \frac{q_1}{\left(\sqrt{2}\right)^3} = \frac{2q_2}{\left(\sqrt{5}\right)^3} \rightarrow \frac{q_1}{q_2} = 2\left(\sqrt{\frac{2}{5}}\right)^3 = 0.506$$

d. The numbers in this part are NOT based on the values found in part c. Suppose that $q_1 = +3nC$ and $q_2 = -4nC$. What is the net electric field at the midpoint between the two rings? Let $R = 10cm$.

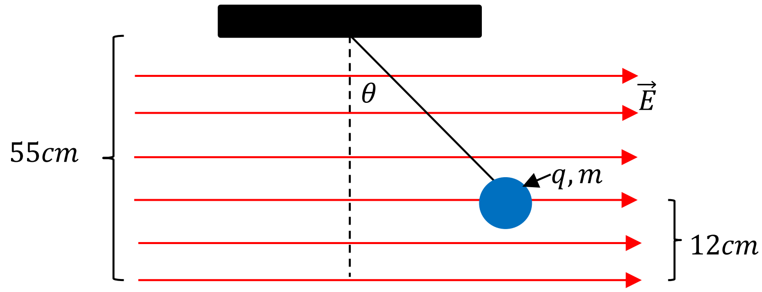
$$\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 = \frac{kq_1 z}{(R^2+z^2)^{\frac{3}{2}}}\left[\frac{\langle 0,0,\frac{3}{2}R\rangle - \langle 0,0,0\rangle}{\frac{3}{2}R}\right] + \frac{kq_2 z}{(R^2+z^2)^{\frac{3}{2}}}\left[\frac{\langle 0,0,\frac{3}{2}R\rangle - \langle 0,0,3R\rangle}{\frac{3}{2}R}\right]$$

$$\vec{E}_{net} = \frac{kq_1\left(\frac{3}{2}R\right)}{\left(R^2+\left(\frac{3}{2}R\right)^2\right)^{\frac{3}{2}}}\left[\frac{\langle 0,0,\frac{3}{2}R\rangle - \langle 0,0,0\rangle}{\frac{3}{2}R}\right] + \frac{kq_2\left(\frac{3}{2}R\right)}{\left(R^2+\left(\frac{3}{2}R\right)^2\right)^{\frac{3}{2}}}\left[\frac{\langle 0,0,\frac{3}{2}R\rangle - \langle 0,0,3R\rangle}{\frac{3}{2}R}\right]$$

$$\vec{E}_{net} = \frac{3kq_1R}{2\left(\sqrt{\frac{13}{4}}\right)^3 R^3}\langle 0,0,1\rangle + \frac{3kq_2R}{2\left(\sqrt{\frac{13}{4}}\right)^3 R^3}\langle 0,0,-1\rangle = \frac{3k}{2\left(\sqrt{\frac{13}{4}}\right)^3 R^2}\langle 0,0,q_1-q_2\rangle$$

$$\vec{E}_{net} = \frac{3\times 9\times 10^9\frac{Nm^2}{C^2}}{2\times\left(\sqrt{\frac{13}{4}}\right)^3\times(0.1m)^2}\langle 0,0,3-(-4)\rangle\times 10^{-9}C = \langle 0,0,1613\rangle\frac{N}{C}$$

3. A $m = 1g$ point charge is suspended at the end of an insulating cord of length $L = 55cm$ is observed to be in equilibrium in a uniform horizontal electric field of magnitude $|\vec{E}| = 15000\frac{N}{C}$ when the pendulum's position is $12cm$ above its lowest vertical position.



a. What is the magnitude of the tension force in the cord in this configuration?

$$\vec{F}_{net} = \langle 0,0,0\rangle = \vec{F}_T + \vec{F}_W + \vec{F}_E = \langle -F_T \sin\theta, F_T \cos\theta, 0\rangle + \langle 0, -mg, 0\rangle + \langle qE, 0, 0\rangle$$

In the y-direction: $0 = F_T \cos\theta - mg \rightarrow F_T = \frac{mg}{\cos\theta} = \frac{0.001kg \times 9.8\frac{m}{s^2}}{\cos 38.6} = 0.0125N$

b. What is the sign of the point charge on the end of the cord? Be sure to explain fully your choice for the sign of the point charge and why you chose this.

Since the point charge moved in the direction of the electric field from hanging vertical, the point charge must be positive since it felt an electric force in the direction of the electric field.

c. What is the magnitude of the charge on the point charge on the end of the cord?

In the x-direction: $0 = -F_T \sin\theta + qE \rightarrow q = \frac{F_T \sin\theta}{E} = \frac{0.0125N \sin 38.6}{15000\frac{N}{C}} = 5.2 \times 10^{-7}C = 0.52\mu C$

d. Suppose that the electric field were generated by a set of parallel square metal plates with sides of length $L = 1m$ that have equal and opposite charges on them. How much charge in magnitude would have been placed on these plates to generate this uniform electric field?

$$E = \frac{Q}{\varepsilon_0 A} \rightarrow Q = \varepsilon_0 EA = 8.85 \times 10^{-12}\frac{C^2}{Nm^2} \times 15000\frac{N}{C} \times (1m)^2 = 1.33 \times 10^{-7}C = 13.3\mu C$$

Physics 121 Equation Sheet

$\vec{F} = k\frac{Q_1Q_2}{r^2}\hat{r};\ \hat{r} = \frac{\vec{r}_o - \vec{r}_s}{|\vec{r}_o - \vec{r}_s|}$

$\vec{E} = \frac{\vec{F}}{q} = \int d\vec{E} = \int dE\hat{r} = \int \frac{kdq}{r^2}\hat{r}$

$\vec{E}_Q = k\frac{Q}{r^2}\hat{r}$

$|\vec{E}_{\|}| = \frac{2kqs}{r^3}$; dipole $r >> s$

$|\vec{E}_{\perp}| = \frac{kqs}{r^3}$; dipole $r >> s$

$|\vec{E}_{rod}|_{\perp} = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{r\sqrt{r^2 + \left(L/2\right)^2}}\right]$

$|\vec{E}_{rod}|_{\|} = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{r(L+r)}\right];\ |\vec{E}_{rod}|_{\|} \sim \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{rL}\right)\quad L >> r$

$|\vec{E}_{ring}| = \frac{1}{4\pi\varepsilon_0}\left[\frac{Qz}{(R^2+z^2)^{\frac{3}{2}}}\right]$

$|\vec{E}_{disk}| = \frac{Q}{2\pi\varepsilon_0 R^2}\left[1 - \frac{z}{\sqrt{R^2+z^2}}\right];\quad |\vec{E}_{disk}| \sim \frac{Q}{2\varepsilon_0 A}\left[1 - \frac{z}{R}\right]\quad z << R;\quad |\vec{E}_{disk}| \sim \frac{Q}{2\varepsilon_0 A}\quad z << R$

$|\vec{E}_{capacitor}| \sim \frac{Q}{\varepsilon_0 A};\quad |\vec{E}_{fringe}| \sim \frac{Q}{2\varepsilon_0 A}\left(\frac{s}{R}\right)$

$W = -q\Delta V = -\Delta U = \Delta K;\ U = \sum_{i \neq j}\frac{kQ_iQ_j}{r_{ij}};$

$V_Q = \frac{kQ}{r};\ V_{Q's} = \sum_i \frac{kQ_i}{r_i}$

$\Delta V = -\int \vec{E}\cdot d\vec{r}$

$E_x = -\frac{\Delta V}{\Delta x};\ E_y = -\frac{\Delta V}{\Delta y};\ E_z = -\frac{\Delta V}{\Delta z};\ \vec{E} = -\left\langle \frac{dV}{dx}, \frac{dV}{dy}, \frac{dV}{dz}\right\rangle$

$Q = \left(\frac{\kappa\varepsilon_0 A}{s}\right)\Delta V$

$U = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$

$Q = Q_{\max}\left(1 - e^{-\frac{t}{RC}}\right);\ Q = Q_{\max}e^{-\frac{t}{RC}}$

$I = \frac{dQ}{dt} = n|e|Av_d = \int \vec{J}\cdot d\vec{A}$

$n = \frac{\rho}{M}N_A$

$\vec{J} = n|e|\vec{v}_d = \sigma\vec{E} = \frac{1}{\rho}\vec{E} \rightarrow V = IR;\ R = \frac{\rho L}{A}$

$P = IV = I^2R = \frac{V^2}{R}$

$\rho = \rho_0(1 + \alpha\Delta T)$

$\vec{F} = q\vec{v}\times\vec{B};\ |\vec{F}| = qvB\sin\theta$

$\vec{B} = \frac{\mu_o}{4\pi}\left(\frac{q\vec{v}\times\hat{r}}{r^2}\right)$

$d\vec{B} = \frac{\mu_o I}{4\pi}\left(\frac{d\vec{l}\times\hat{r}}{r^2}\right)$

$|\vec{B}_{wire}| = \frac{\mu_o LI}{4\pi r\sqrt{\left(\frac{L}{2}\right)^2 + r^2}};\quad |\vec{B}_{wire}| \approx \frac{\mu_o I}{2\pi r}\quad L >> r$

$|\vec{B}_{ring}| = \frac{\mu_o IR^2}{2(z^2+R^2)^{\frac{3}{2}}};\ |\vec{B}_{ring}| \approx \frac{\mu_o IR^2}{2z^3}\quad z << R$

Constants

$g = 9.8\frac{m}{s^2}$

$1e = 1.6\times10^{-19}C$

$k = \frac{1}{4\pi\varepsilon_o} = 9\times10^9\frac{Nm^2}{C^2}$

$\varepsilon_o = 8.85\times10^{-12}\frac{C^2}{Nm^2}$

$1eV = 1.6\times10^{-19}J$

$\mu_o = 4\pi\times10^{-7}\frac{Tm}{A}$

$c = 3\times10^8\frac{m}{s}$

$h = 6.63\times10^{-34}Js$

$m_e = 9.11\times10^{-31}kg = \frac{0.511MeV}{c^2}$

$m_p = 1.67\times10^{-27}kg = \frac{937.1MeV}{c^2}$

$m_n = 1.69\times10^{-27}kg = \frac{948.3MeV}{c^2}$

$1amu = 1.66\times10^{-27}kg = \frac{931.5MeV}{c^2}$

$N_A = 6.02\times10^{23}$

$Ax^2 + Bx + C = 0 \rightarrow x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

$\vec{v}_f = \langle v_{fx}, v_{fy}, v_{fz}\rangle = \langle v_{ix} + a_x t, v_{iy} + a_y t, v_{iz} + a_z t\rangle$

$\Delta\vec{r}_f = \langle x_f - x_i, y_f - y_i, z_f - z_i\rangle = \langle v_{ix}t + \frac{1}{2}a_x t^2, v_{iy}t + \frac{1}{2}a_y t^2, v_{iz}t + \frac{1}{2}a_z t^2\rangle$