Physics 121

Exam #1

April 26, 2019

Name____________________________________

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example $|\vec{p}| \approx m|\vec{v}| = (5kg)\times\left(2\frac{m}{s}\right) = 10\frac{kg\cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- All multiple-choice question are worth 3 points and each free-response part is worth 7 points

Problem #1	/24
Problem #2	/24
Problem #3	/24
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

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1. Suppose that you have a collection of point charges where point charge $q_1 = -3\mu C$ is located at the point $P_1 = \langle 0,7,0 \rangle cm$ and point charge $q_2 = 4\mu C$ is located at point $P_2 = \langle 12,0,0 \rangle cm$.

a. What is the electric potential at a point $P = \langle 0,0,0 \rangle cm$ due to point charges q_1 and q_2?

$$V_P = V_{q_1} + V_{q_1} = \frac{kq_1}{r_{1P}} + \frac{kq_2}{r_{2P}} = 9\times10^9 \tfrac{Nm^2}{C^2}\left[\frac{-3\times10^{-6}C}{0.07m} + \frac{4\times10^{-6}C}{0.12m}\right] = -9\times10^4 V$$

b. What is the electric field at a point $P = \langle 0,0,0 \rangle cm$ due to point charges q_1 and q_2? Be sure to express your answer either as a vector or as a magnitude and a direction, where the direction is measured with respect to the positive x-axis.

c.

$$\vec{E}_P = \vec{E}_{q_1} + \vec{E}_{q_2} = \frac{kq_1}{r_{1P}^2}\hat{r}_{1P} + \frac{kq_2}{r_{2P}^2}\hat{r}_{2P} = \frac{kq_1}{r_{1P}^2}\left[\frac{\langle 0,0,0\rangle - \langle 0,y,0\rangle}{y}\right] + \frac{kq_1}{r_{1P}^2}\left[\frac{\langle 0,0,0\rangle - \langle x,0,0\rangle}{x}\right]$$

$$\vec{E}_P = \left(\frac{9\times10^9 \tfrac{Nm^2}{C^2}\times\left(-3\times10^{-6}C\right)}{(0.07m)^2}\langle 0,-1,0\rangle\right) + \left(\frac{9\times10^9 \tfrac{Nm^2}{C^2}\times\left(4\times10^{-6}C\right)}{(0.12m)^2}\langle -1,0,0\rangle\right)$$

$$\vec{E}_P = \langle 0,5.5,0\rangle\times10^6 \tfrac{N}{C} + \langle -2.5,0,0\rangle\times10^6 \tfrac{N}{C}$$

$$\therefore \vec{E}_P = \langle -2.5,5.5,0\rangle\times10^6 \tfrac{N}{C};$$

$$\therefore \left|\vec{E}_P\right| = 6.0\times10^6 \tfrac{N}{C} @ \theta = 114.4^0$$

d. Suppose that a third point charge q_3 were brought in from very far away and placed at the point $P = \langle 0,0,0 \rangle cm$. Which of the following expressions would give the change in electric potential energy of the system?

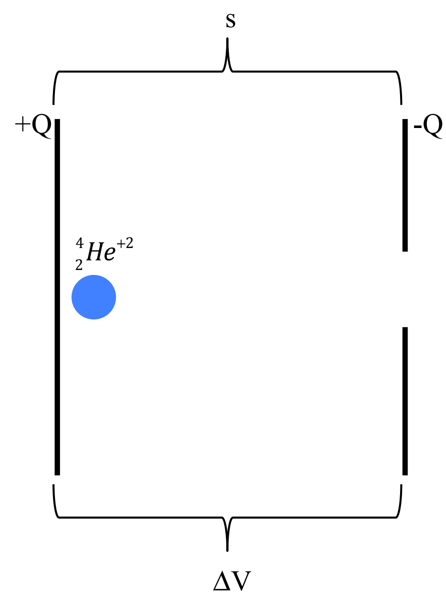
1. $\Delta U_E = \frac{1}{4\pi\varepsilon_0}\left[-\frac{q_1q_2}{r_{12}} - \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right]$.
2. $\Delta U_E = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} - \frac{q_2q_3}{r_{23}}\right]$.
3. $\Delta U_E = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1q_2}{r_{12}^2} + \frac{q_1q_3}{r_{13}^2} - \frac{q_2q_3}{r_{23}^2}\right]$.
4. $\Delta U_E = \frac{1}{4\pi\varepsilon_0}\left[-\frac{q_1q_2}{r_{12}^2} - \frac{q_1q_3}{r_{13}^2} + \frac{q_2q_3}{r_{23}^2}\right]$.
5. None of the above gives the correct expression.

e. Suppose that the third point charge had a value $q_3 = +2\mu C$. If q_3 had a mass $m_{q_3} = 100g$ and was released from rest at point $P = \langle 0,0,0 \rangle cm$, what initial acceleration would q_3 experience? Be sure to express your answer either s a vector or as a magnitude and a direction where the direction is measured with respect to the positive x-axis.

$$\vec{F} = q\vec{E} = m\vec{a} \rightarrow \vec{a} = \frac{q}{m}\vec{E}_P = \left(\frac{2\times10^{-6}C}{0.1kg}\right)\times\langle -2.5,5.5,0\rangle\times10^6 \tfrac{N}{C} = \langle -50,110,0\rangle N\tfrac{m}{s^2}$$

$$|\vec{F}| = m|\vec{a}| = |q\vec{E}_P| \rightarrow |\vec{a}| = \frac{q}{m}|\vec{E}_P| = \left(\frac{2\times10^{-6}C}{0.1kg}\right)\times 6.0\times10^6 \tfrac{N}{C} = 120\tfrac{m}{s^2} @ \theta = 114.4^0$$

2. Suppose that you have the particle accelerator shown on the right. The system is constructed from a set of parallel metal plates and designed to accelerate positive charges from rest to a high rate of speed. Suppose that helium ions ($^{4}_{2}He^{+2}$) are initially at rest at the left plate and are accelerated toward the right plate, which has a small hole, drilled in it. The plates are square with sides $L = 10cm$ and are separated by $s = 1mm$. A battery (not shown) charges the plates of the capacitor and magnitude of the charge on a plate is $Q = 292\mu C$. $^{4}_{2}He^{+2}$ are helium atoms with their two electrons removed and the helium ions have a mass of approximately $4m_p$, where m_p is the mass of a proton.



a. Through what potential difference (ΔV) are the helium ions accelerated?

$$Q = CV = \left(\frac{\varepsilon_0 A}{s}\right)V$$

$$\rightarrow V = \frac{s}{\varepsilon_0 A}Q = \frac{1\times10^{-3}m}{8.85\times10^{-12}\frac{C^2}{Nm^2}\times(0.1m)^2}\times292\times10^{-6}C$$

$$V = 3.3\times10^{6}V = 3.3MV$$

$$\therefore \Delta V = -3.3\times10^{6}V = -3.3MV$$

b. When the helium ions leave the hole in the right plate, what is their speed?

$$W = -q\Delta V = \Delta K = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 = \tfrac{1}{2}mv_f^2$$

$$\rightarrow v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{-2q\Delta V}{m}} = \sqrt{\frac{-2\times(2\times1.6\times10^{-19}C)\times(-3.3\times10^{6}V)}{4\times1.67\times10^{-27}kg}} = 1.8\times10^{7}\,\tfrac{m}{s}$$

c. Suppose that the helium ions that leave the hole in the right plate are directed at a target composed of lead. ($^{208}_{82}Pb$: $Z_{Pb} = 82$ and $m_{Pb} = 208u \sim 208m_p$). Treating the helium ions and the lead nuclei as point charges, how close does a helium ion get to the lead nucleus? Assume that the helium nuclei start very far away from the lead target.

$$W = -q\Delta V = \Delta K$$

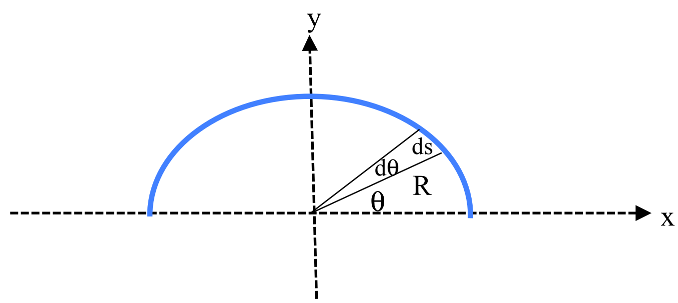
$$-q\left[\frac{kQ}{r_f} - \frac{kQ}{r_i}\right] = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 = -\tfrac{1}{2}mv_i^2$$

$$\rightarrow r_f = \frac{2kqQ}{mv_i^2} = \frac{2\times 9\times 10^9 \frac{Nm^2}{C^2}\times\left(2\times 1.6\times 10^{-19}C\right)\times\left(82\times 1.6\times 10^{-19}C\right)}{4\times 1.67\times 10^{-27}kg\left(1.8\times 10^7 \frac{m}{s}\right)^2} = 3.4\times 10^{-14}m$$

d. Suppose that you wanted to perform the same experiment but wanted to use electrons instead. Which change to the experimental setup below would allow you to accelerate electrons instead of positive charges?

1. Changing the magnitude of the charge on the plates.
2. Changing the sign of the charges on the plates.
3. Change the separation between the plates.
4. Change the area of the plates.
5. There is no change that could be made so that electrons can be accelerated.

3. Consider the half-ring of charge centered on the origin as shown below. The half-ring has a radius R and carries a uniform positive charge ($+Q$) over its length. Suppose that we eventually want to calculate the electric field at say the origin ($P = \langle 0,0,0 \rangle$). To do this we can treat this system as if it were composed of a large collection of smaller charges (dq) and determine the contribution of each of these to the total electric field at the point in question.



a. Express the small piece of charge in terms of the total charge $+Q$, the radius of the half-ring R and the differential angle $d\theta$ and any other constants you need. Hint: ds is the piece of the arc of the ring that bounds dq.

$$dq = (\text{fraction of})Q = \left(\frac{ds}{\pi R}\right)Q = \left(\frac{Rd\theta}{\pi R}\right)Q$$

$$\therefore dq = \frac{Q}{\pi}d\theta$$

b. What is the unit vector from dq to the point $P = \langle 0,0,0 \rangle$?

$$\hat{r} = \frac{\langle 0,0,0\rangle - \langle x,y,0\rangle}{|\vec{r}|} = \frac{\langle -x,-y,0\rangle}{\sqrt{x^2+y^2}} = \frac{\langle -R\cos\theta, -R\sin\theta, 0\rangle}{R}$$

$$\hat{r} = \langle -\cos\theta, -\sin\theta, 0\rangle$$

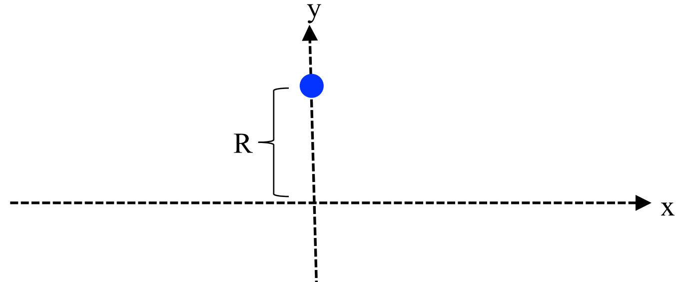
c. What is the expression for the differential element $d\vec{E}$ of the electric field at point $P = \langle 0,0,0 \rangle$? Just write the complete simplified expression. Do not integrate the expression that you get.

$$d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\hat{r} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\pi R^2}d\theta\langle -\cos\theta, -\sin\theta, 0\rangle$$

$$d\vec{E} = \left\langle -\frac{1}{4\pi\varepsilon_0}\frac{Q}{\pi R^2}\cos\theta d\theta, -\frac{1}{4\pi\varepsilon_0}\frac{Q}{\pi R^2}\sin\theta d\theta, 0 \right\rangle$$

d. The electric field at point due to the half-ring of charge (if you integrated the express for $d\vec{E}$ from part c) is $\vec{E} = \left\langle 0, -\frac{Q}{2\pi^2\varepsilon_0 R^2}, 0 \right\rangle$. Suppose that the half-ring of charge were collapsed to a point charge Q located a distance R from the origin as shown below. In this case, which of the following would give the magnitude of the electric field at the origin due to the half-ring of charge to that of the point charge?

1. The magnitude of the electric field due to the half-ring would be a factor of 2 larger than the electric field of the point charge.
2. The magnitude of the electric field due to the half-ring would be a factor of $\frac{2}{\pi}$ larger than the electric field of the point charge.
3. The magnitudes of the electric field of the half-ring and the point charge would be the same.
4. The magnitude of the electric field due to the half-ring would be a factor of 2 smaller than the electric field of the point charge.
5. The magnitude of the electric field due to the half-ring would be a factor of $\frac{2}{\pi}$ smaller than the electric field of the point charge.



Physics 121 Equation Sheet

Electric Forces, Fields and Potentials

$\vec{F} = k\frac{Q_1Q_2}{r^2}\hat{r};\ \hat{r} = \frac{\vec{r}_o - \vec{r}_s}{|\vec{r}_o - \vec{r}_s|}$

$\vec{E} = \frac{\vec{F}}{q} = \int d\vec{E} = \int dE\hat{r} = \int \frac{kdq}{r^2}\hat{r}$

$\vec{E}_Q = k\frac{Q}{r^2}\hat{r}$

$|\vec{E}_{\parallel}| = \frac{2kqs}{r^3}$; dipole $r >> s$

$|\vec{E}_{\perp}| = \frac{kqs}{r^3}$; dipole $r >> s$

$|\vec{E}_{rod}|_{\perp} = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{r\sqrt{r^2 + \left(\frac{L}{2}\right)^2}}\right]$

$|\vec{E}_{rod}|_{\parallel} = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{r(L+r)}\right];\ |\vec{E}_{rod}|_{\parallel} \sim \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{rL}\right)\ \ L >> r$

$|\vec{E}_{ring}| = \frac{1}{4\pi\varepsilon_0}\left[\frac{Qz}{(R^2+z^2)^{\frac{3}{2}}}\right]$

$|\vec{E}_{disk}| = \frac{Q}{2\pi\varepsilon_0 R^2}\left[1 - \frac{z}{\sqrt{R^2+z^2}}\right];\ \ |\vec{E}_{disk}| \sim \frac{Q}{2\varepsilon_0 A}\left[1 - \frac{z}{R}\right]\ \ z << R;\ \ |\vec{E}_{disk}| \sim \frac{Q}{2\varepsilon_0 A}\ \ z << R$

$|\vec{E}_{capacitor}| \sim \frac{Q}{\varepsilon_0 A};\ \ |\vec{E}_{fringe}| \sim \frac{Q}{2\varepsilon_0 A}\left(\frac{s}{R}\right)$

$W = -q\Delta V = -\Delta U = \Delta K;\ U = \sum_{i \neq j}\frac{kQ_iQ_j}{r_{ij}};$

$V_Q = \frac{kQ}{r};\ V_{Q's} = \sum_i \frac{kQ_i}{r_i}$

$\Delta V = -\int \vec{E}\cdot d\vec{r}$

$E_x = -\frac{\Delta V}{\Delta x};\ E_y = -\frac{\Delta V}{\Delta y};\ E_z = -\frac{\Delta V}{\Delta z};\ \vec{E} = -\left\langle \frac{dV}{dx}, \frac{dV}{dy}, \frac{dV}{dz}\right\rangle$

$Q = \left(\frac{\kappa\varepsilon_0 A}{s}\right)\Delta V$

$U = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$

$Q = Q_{\max}\left(1 - e^{-\frac{t}{RC}}\right);\ Q = Q_{\max}e^{-\frac{t}{RC}}$

$I = \frac{dQ}{dt} = n|e|Av_d = \int \vec{J}\cdot d\vec{A}$

$n = \frac{\rho}{M}N_A$

$\vec{J} = n|e|\vec{v}_d = \sigma\vec{E} = \frac{1}{\rho}\vec{E} \rightarrow V = IR;\ R = \frac{\rho L}{A}$

$P = IV = I^2R = \frac{V^2}{R}$

$\rho = \rho_0(1 + \alpha\Delta T)$

$\vec{F} = q\vec{v}\times\vec{B};\ |\vec{F}| = qvB\sin\theta$

$\vec{B} = \frac{\mu_o}{4\pi}\left(\frac{q\vec{v}\times\hat{r}}{r^2}\right)$

$d\vec{B} = \frac{\mu_o I}{4\pi}\left(\frac{d\vec{l}\times\hat{r}}{r^2}\right)$

$|\vec{B}_{wire}| = \frac{\mu_o LI}{4\pi r\sqrt{\left(\frac{L}{2}\right)^2 + r^2}};\ \ |\vec{B}_{wire}| \approx \frac{\mu_o I}{2\pi r}\ \ L >> r$

$|\vec{B}_{ring}| = \frac{\mu_o IR^2}{2(z^2+R^2)^{\frac{3}{2}}};\ |\vec{B}_{ring}| \approx \frac{\mu_o IR^2}{2z^3}\ \ z << R$

Constants

$g = 9.8\frac{m}{s^2}$

$1e = 1.6\times10^{-19}C$

$k = \frac{1}{4\pi\varepsilon_o} = 9\times10^9\frac{Nm^2}{C^2}$

$\varepsilon_o = 8.85\times10^{-12}\frac{C^2}{Nm^2}$

$1eV = 1.6\times10^{-19}J$

$\mu_o = 4\pi\times10^{-7}\frac{Tm}{A}$

$c = 3\times10^8\frac{m}{s}$

$h = 6.63\times10^{-34}Js$

$m_e = 9.11\times10^{-31}kg = \frac{0.511MeV}{c^2}$

$m_p = 1.67\times10^{-27}kg = \frac{937.1MeV}{c^2}$

$m_n = 1.69\times10^{-27}kg = \frac{948.3MeV}{c^2}$

$1amu = 1.66\times10^{-27}kg = \frac{931.5MeV}{c^2}$

$N_A = 6.02\times10^{23}$

$Ax^2 + Bx + C = 0 \rightarrow x = \frac{-B\pm\sqrt{B^2-4AC}}{2A}$

$\vec{v}_f = \langle v_{fx}, v_{fy}, v_{fz}\rangle = \langle v_{ix} + a_xt, v_{iy} + a_yt, v_{iz} + a_zt\rangle$

$\Delta\vec{r}_f = \langle x_f - x_i, y_f - y_i, z_f - z_i\rangle = \langle v_{ix}t + \frac{1}{2}a_xt^2, v_{iy}t + \frac{1}{2}a_yt^2, v_{iz}t + \frac{1}{2}a_zt^2\rangle$