Physics 121

Exam #1

January 29, 2016

Name___________________________________

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example $|\vec{p}| \approx m|\vec{v}| = (5kg)\times(2\,\tfrac{m}{s}) = 10\,\tfrac{kg\cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- All multiple-choice question are worth 3 points and each free-response part is worth 7 points

Problem #1	/24
Problem #2	/24
Problem #3	/24
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

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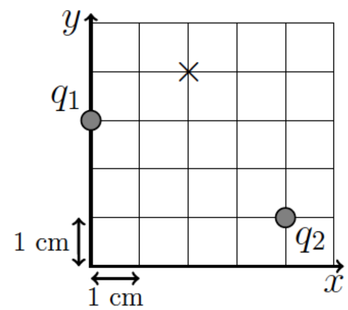
1. Suppose that you have the collection of point charges shown below, where $q_1 = -3\mu C$ and $q_2 = 6\mu C$.

   a. What is the electric field (magnitude and direction) at the point marked by the $\times$?



$$\vec{E}_\times = \vec{E}_1 + \vec{E}_2 = \frac{kQ_1}{r_1^2}\hat{r}_1 + \frac{kQ_2}{r_2^2}\hat{r}_2$$

$$\hat{r}_1 = \frac{\langle 2,4,0\rangle cm - \langle 0,3,0\rangle cm}{\sqrt{(2cm)^2 + (1cm)^2 + (0cm)^2}} = \langle 0.893, 0.447, 0\rangle$$

$$\hat{r}_2 = \frac{\langle 2,4,0\rangle cm - \langle 4,1,0\rangle cm}{\sqrt{(-2cm)^2 + (3cm)^2 + (0cm)^2}} = \langle -0.556, 0.832, 0\rangle$$

$$\vec{E}_\times = 9\times 10^9 \tfrac{Nm^2}{C^2}\left[\frac{-3\times 10^{-6}C}{0.0005m^2}\langle 0.893, 0.447, 0\rangle + \frac{6\times 10^{-6}C}{0.0013m^2}\langle -0.556, 0.832, 0\rangle\right]$$

$$\vec{E}_\times = \langle -7.2, 1.0, 0\rangle \times 10^7 \tfrac{N}{C}$$

   b. Suppose that a point charge $q = -5\mu C$ were placed at the point marked by the $\times$ and released from rest, what would be the initial force on the point charge due to the electric field at the point marked with a $\times$?

$$\vec{F} = q\vec{E} = (-5\times 10^{-6}C)\langle -7.2, 1.0, 0\rangle \times 10^7 \tfrac{N}{C}$$

$$\vec{F} = \langle 360, -50, 0\rangle N$$

c. What is the electric potential at the point marked by the $\times$?

$$V_\times = V_1 + V_2 = \frac{kq_1}{r_1} + \frac{kq_2}{r_2} = 9\times10^9\,\tfrac{Nm^2}{C^2}\left[\frac{-3\times10^{-6}C}{\sqrt{0.0005m^2}} + \frac{6\times10^{-6}C}{\sqrt{0.0013m^2}}\right]$$

$$V_\times = 2.7\times10^5 V$$

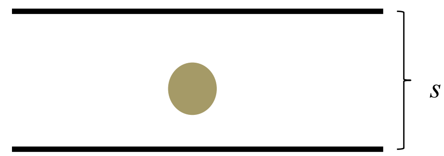
d. Suppose that the point charges q_1 and q_2 were assembled one at a time, each brought in from *very far away*, the work done in assembling the two point charges a distance r_{12} apart is

1. $0J$
2. $W = \dfrac{-q_1 q_2}{4\pi\varepsilon_0 r_{12}^2}$
3. $W = \dfrac{-q_1 q_2}{4\pi\varepsilon_0 r_{12}}$
4. $W = \dfrac{q_1 q_2}{4\pi\varepsilon_0 r_{12}}$

$$W = -\Delta U = -\left[\frac{-q_1 q_2}{4\pi\varepsilon_0 r_{12}} - 0\right] = \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}}$$

2. A small drop of oil with a mass $m = 4.9\times10^{-15}\,kg$ carries an electric charge Q. Suppose that the oil drop is suspended at rest between two parallel circular plates with radii $R = 25cm$ and separation $s = 1mm$ in the vertical direction. The electric potential difference between the top and bottom plates is $V_T - V_B = 100V$.

a. The direction of the electric field between the plates is



1. pointing up the page.
2. pointing down the page.
3. pointing toward the left side of the page.
4. pointing toward the right side of the page.
5. unable to be determined since there is not enough information given in the problem.

Since $V_T - V_B = 100V$, V_T is greater than V_B and the electric field points along decreasing electric potentials so the electric field points down the page.

b. What is the magnitude of the electric field between the plates?

$$E_x = E_z = 0$$

$$E_y = -\frac{\Delta V}{\Delta y} = -\frac{100V}{1\times10^{-3}m} = -1\times10^{5}\,\tfrac{V}{m}$$

$$\left|\vec{E}\right| = \sqrt{E_x^2 + E_y^2 + E_z^2} = 1\times10^{5}\,\tfrac{V}{m}$$

c. What is the total magnitude of the charge on the oil drop, Q?

In order for the net force on the drop to be zero, the electric force on the charge must point up the page in the positive y-direction. Since the electric field points down the page in the negative y-direction, the charge on the drop must be negative to ensure that the force on the drop due to the electric field points in the positive y-direction.

$$\vec{F}_{net} = \vec{0} == \left\langle 0, |Q|E_y - mg, 0 \right\rangle$$

$$\therefore |Q|E_y = mg \rightarrow |Q| = \frac{mg}{E_y} = \frac{4.9\times10^{-15}\,kg\times9.8\,\tfrac{m}{s^2}}{1\times10^{5}\,\tfrac{N}{C}} = 4.8\times10^{-19}\,C$$

d. How many electrons were gained or lost by the drop? (Hint: Determine a number and then state whether they were lost or gained and why this is so.)

$n = \dfrac{q}{e} = \dfrac{4.8\times10^{-19}\,C}{1.6\times10^{-19}\,C} = 3$, so 3 electrons were gained since Q is negative.

3. Suppose that you have a square plate with sides of length $L = 25cm$ where a charge $Q = 5\mu C$ has been spread uniformly over the surface of the plate. Take the origin is taken to be at the center of the plate and that the z-axis is perpendicular to the surface of the plate. A neutral carbon atom is placed at a location along the z-axis of $\langle 0,0,5\rangle cm$.

   a. What are the expressions for the forces (magnitude and directions) on the left and right sides respectively of the carbon atom?

$$\vec{F}_{left} = -q_c\vec{E} = -q_c\left(\frac{Q}{2\varepsilon_0 A}\right)\left[1-\frac{\left(z-\frac{s}{2}\right)}{L}\right]\langle 0,0,1\rangle$$

$$\vec{F}_{right} = q_c\vec{E} = q_c\left(\frac{Q}{2\varepsilon_0 A}\right)\left[1-\frac{\left(z+\frac{s}{2}\right)}{L}\right]\langle 0,0,1\rangle$$

   b. What is the net force (magnitude and direction) on the carbon atom due to the plate of charge? (Hint: The polarizability of carbon, $\alpha = 1.96\times 10^{-40}\,\frac{Cm}{N/C}$.)

$$\vec{F}_{net} = \vec{F}_{left} + \vec{F}_{right} = -q_c\vec{E}$$

$$\vec{F}_{net} = -q_c\left(\frac{Q}{2\varepsilon_0 A}\right)\left[1-\frac{\left(z-\frac{s}{2}\right)}{L}\right]\langle 0,0,1\rangle + q_c\vec{E} + q_c\left(\frac{Q}{2\varepsilon_0 A}\right)\left[1-\frac{\left(z+\frac{s}{2}\right)}{L}\right]\langle 0,0,1\rangle$$

$$F_{net,z} = \frac{q_c Q}{2\varepsilon_0 A}\left[-1+\frac{\left(z-\frac{s}{2}\right)}{L}+1-\frac{\left(z+\frac{s}{2}\right)}{L}\right] = -\frac{q_c Q}{2\varepsilon_0 A}\left[\frac{s}{L}\right] = -\frac{Q}{2\varepsilon_0 AL}\left[q_c s\right] = -\frac{Q}{2\varepsilon_0 AL}\left[\alpha E_{applied}\right]$$

$$\therefore F_{net,z} = -\frac{\alpha Q}{2\varepsilon_0 AL}\left(\frac{Q}{2\varepsilon_0 A}\left[1-\frac{z}{L}\right]\right) = -\frac{\alpha Q^2}{4\varepsilon_0^2 A^4 L}\left[1-\frac{z}{L}\right]$$

$$\vec{F}_{net} \approx -\frac{\alpha Q^2}{4\varepsilon_0^2 L^5}\langle 0,0,1\rangle \quad \text{for } z \ll L$$

c. What is the initial acceleration (magnitude and direction) of the carbon atom due to the plate of charge? (Hint: The molar mass of carbon $m_{^{12}_{6}C} = 12\,\frac{g}{mole}$.)

$$\vec{F}_{net} = m_c\vec{a}_c \rightarrow \vec{a}_c = \frac{\vec{F}_{net}}{m_c} = -\frac{\alpha Q^2}{4m_c\varepsilon_0^2 L^5}\langle 0,0,1\rangle$$

$$\vec{a}_c = -\frac{1.96\times10^{-40}\,\frac{Cm}{N/C}\times\left(5\times10^{-6}C\right)^2}{4\times\left(12\times1.67\times10^{-24}kg\right)\times\left(8.85\times10^{-12}\,\frac{C^2}{Nm^2}\right)^2\times\left(0.25m\right)^5}\langle 0,0,1\rangle$$

$$\vec{a}_c = -0.8\,\tfrac{m}{s^2}\langle 0,0,1\rangle = \langle 0,0,-0.8\rangle\tfrac{m}{s^2}$$

d. Suppose that the neutral carbon atom were placed at a point $\langle 0,0,z'\rangle$ where $|z'| < |z|$. The induced dipole moment at the location $\langle 0,0,z'\rangle$ compared to the induced dipole moment at the location $\langle 0,0,z\rangle$

1. is greater since $|z'| < |z|$. Since $E_{applied}$ gets larger, $\vec{p} = \alpha\vec{E}_{applied}$ gets larger.
2. is less since $|z'| < |z|$.
3. is the same since the dipole moment is an intrinsic property of the neutral carbon atom.
4. is unable to be determined since there is not enough information given about the neutral carbon atom.

Physics 121 Equation Sheet

Electric Forces, Fields and Potentials

$$\vec{F} = k\frac{Q_1Q_2}{r^2}\hat{r}; \quad \hat{r} = \frac{\vec{r}_o - \vec{r}_s}{|\vec{r}_o - \vec{r}_s|}$$

$$\vec{E} = \frac{\vec{F}}{q}$$

$$\vec{E}_Q = k\frac{Q}{r^2}\hat{r}$$

$$\vec{p} = q\vec{s} = \alpha\vec{E}$$

$$\left|\vec{E}_{\parallel}\right| = \frac{2kqs}{r^3}; \text{ dipole } r >> s$$

$$\left|\vec{E}_{\perp}\right| = \frac{kqs}{r^3}; \text{ dipole } r >> s$$

$$\left|\vec{E}_{rod}\right| = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{r\sqrt{r^2 + \left(L/2\right)^2}}\right]; \quad \left|\vec{E}_{rod}\right| \sim \frac{1}{4\pi\varepsilon_0}\left(\frac{2Q}{rL}\right) \quad L >> r$$

$$\left|\vec{E}_{ring}\right| = \frac{1}{4\pi\varepsilon_0}\left[\frac{Qz}{\left(R^2 + z^2\right)^{\frac{3}{2}}}\right]$$

$$\left|\vec{E}_{disk}\right| = \frac{Q}{2\pi\varepsilon_0 R^2}\left[1 - \frac{z}{\sqrt{R^2 + z^2}}\right]; \quad \left|\vec{E}_{disk}\right| \sim \frac{Q}{2\varepsilon_0 A}\left[1 - \frac{z}{R}\right] \quad z < R; \quad \left|\vec{E}_{disk}\right| \sim \frac{Q}{2\varepsilon_0 A} \quad z << R$$

$$\left|\vec{E}_{capacitor}\right| \sim \frac{Q}{\varepsilon_0 A}; \quad \left|\vec{E}_{fringe}\right| \sim \frac{Q}{2\varepsilon_0 A}\left(\frac{s}{R}\right)$$

$$W = -q\Delta V = -\Delta U = \Delta K; \quad U = \sum_{i \neq j}\frac{kQ_iQ_j}{r_{ij}};$$

$$V_Q = \frac{kQ}{r}; \quad V_{Q's} = \sum_i \frac{kQ_i}{r_i}$$

$$\Delta V = -\int \vec{E} \cdot d\vec{r}$$

$$E_x = -\frac{\Delta V}{\Delta x}; \quad E_y = -\frac{\Delta V}{\Delta y}; \quad E_z = -\frac{\Delta V}{\Delta z}; \quad \vec{E} = -\left\langle \frac{dV}{dx}, \frac{dV}{dy}, \frac{dV}{dz} \right\rangle$$

Constants

$g = 9.8\,\frac{m}{s^2}$

$1eV = 1.6 \times 10^{-19} J$

$1e = 1.6 \times 10^{-19} C$

$\mu_o = 4\pi \times 10^{-7}\,\frac{Tm}{A}$

$k = \frac{1}{4\pi\varepsilon_o} = 9 \times 10^9\,\frac{Nm^2}{C^2}$

$c = 3 \times 10^8\,\frac{m}{s}$

$\varepsilon_o = 8.85 \times 10^{-12}\,\frac{C^2}{Nm^2}$

$h = 6.63 \times 10^{-34} Js$

$m_e = 9.11 \times 10^{-31} kg = \frac{0.511 MeV}{c^2}$

$m_p = 1.67 \times 10^{-27} kg = \frac{937.1 MeV}{c^2}$

$m_n = 1.69 \times 10^{-27} kg = \frac{948.3 MeV}{c^2}$

$1amu = 1.66 \times 10^{-27} kg = \frac{931.5 MeV}{c^2}$

$N_A = 6.02 \times 10^{23}$

$Ax^2 + Bx + C = 0 \rightarrow x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$