Physics 121

Exam #1

January 26, 2018

Name___________________________________

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example $|\vec{p}| \approx m|\vec{v}| = (5kg) \times (2\tfrac{m}{s}) = 10\tfrac{kg \cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- All multiple-choice question are worth 3 points and each free-response part is worth 7 points

Problem #1	/24
Problem #2	/24
Problem #3	/24
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

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1. Consider the arrangement of point charges given by the data on the right.

Charge (μC)	Location (m)
1.2	<0.25,0.50,0>
-1.7	<0,0,0>
2.4	<-0.25,0.50,0>

a. What is the electric potential energy (U) associated with this distribution of charges? Assume that the collection was assembled by bringing each charge in from very far away and placed at their position.

1. $0J$
2. $U=\frac{1}{4\pi\varepsilon_0}\left(\frac{q_1q_2}{r_{12}^2}-\frac{q_2q_3}{r_{23}^2}+\frac{q_1q_3}{r_{13}^2}\right)$
3. $U=\frac{1}{4\pi\varepsilon_0}\left(-\frac{q_1q_2}{r_{12}}+\frac{q_2q_3}{r_{23}}-\frac{q_1q_3}{r_{13}}\right)$
4. $U=\frac{1}{4\pi\varepsilon_0}\left(-\frac{q_1q_2}{r_{12}}-\frac{q_2q_3}{r_{23}}+\frac{q_1q_3}{r_{13}}\right)$
5. $U=\frac{1}{4\pi\varepsilon_0}\left(\frac{q_1q_2}{r_{12}}-\frac{q_2q_3}{r_{23}}+\frac{q_1q_3}{r_{13}}\right)$

b. What is the electric field (magnitude and direction) at a point P in space with coordinates $\langle x,y,z\rangle=\langle -0.25,-0.50,0\rangle m$?

There are two ways to do this problem. One is writing all of the electric fields as vectors and the other is to write the components of the net electric field in each direction and then calculate the magnitude and direction of the electric field. In either case we will need the electric field magnitudes are

$$E_1=\frac{kq_1}{r_1^2}=\frac{9\times10^9\,\frac{Nm^2}{C^2}\times1.2\times10^{-6}C}{\left((1m)^2+(0.5m)^2\right)}=8.6\times10^3\,\tfrac{N}{C};\quad E_2=\frac{kq_2}{r_2^2}=\frac{9\times10^9\,\frac{Nm^2}{C^2}\times1.7\times10^{-6}C}{\left((0.25m)^2+(0.50m)^2\right)}=4.9\times10^4\,\tfrac{N}{C}$$

$$E_3=\frac{kq_3}{r_3^2}=\frac{9\times10^9\,\frac{Nm^2}{C^2}\times2.4\times10^{-6}C}{\left((1m)^2\right)}=2.2\times10^4\,\tfrac{N}{C};\quad \tan\theta=\frac{0.50m}{0.25m}\rightarrow\theta=63.4$$

To calculate the magnitude and direction of the net electric field, the electric field components are given from analysis of a force diagram:

$$E_{net,x}=-E_1\cos\theta+E_2\cos\theta=\left(-8.6\times10^3\,\tfrac{N}{C}+4.9\times10^4\,\tfrac{N}{C}\right)\cos63.4=1.8\times10^4\,\tfrac{N}{C}$$

$$E_{net,y}=-E_1\sin\theta+E_2\sin\theta-E_3=\left(-8.6\times10^3\,\tfrac{N}{C}+4.9\times10^4\,\tfrac{N}{C}\right)\sin63.4-2.2\times10^4\,\tfrac{N}{C}=1.4\times10^4\,\tfrac{N}{C}$$

$$\vec{E}_{net}=\langle 1.8\times10^4,1.4\times10^4,0\rangle\,\tfrac{N}{C}$$

or $E_{net}=\sqrt{E_{net,x}^2+E_{net,y}^2}\,@\phi=\tan^{-1}\left(\frac{E_{net,y}}{E_{net,x}}\right)=2.3\times10^4\,\tfrac{N}{C}\,@\phi=38.6$

To do this problem by adding the vector electric fields, we evaluate

$\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 = \frac{kq_1}{r_1^2}\hat{r}_1 + \frac{kq_2}{r_2^2}\hat{r}_2 + \frac{kq_3}{r_3^2}\hat{r}_3$, where the unit vectors for each of the electric fields at point P need to be determined.

$$\hat{r}_1 = \frac{\langle -0.5,-1,0\rangle m}{\sqrt{(0.5m)^2 + (1m)^2}} = \langle -0.446,-0.893,0\rangle$$

$$\hat{r}_2 = \frac{\langle 0.25,0.5,0\rangle m}{\sqrt{(0.25m)^2 + (0.5m)^2}} = \langle 0.447,0.895,0\rangle$$

$$\hat{r}_3 = \frac{\langle 0,-1,0\rangle m}{\sqrt{(1m)^2}} = \langle 0,-1,0\rangle$$

Then

$$\vec{E}_1 = 8.6\times 10^3\, \tfrac{N}{C}\langle -0.446,-0.893,0\rangle = \langle -3.9,-7.7,0\rangle \times 10^3\, \tfrac{N}{C}$$

$$\vec{E}_2 = 4.9\times 10^4\, \tfrac{N}{C}\langle 0.447,0.895,0\rangle = \langle 2.2,4.4,0\rangle \times 10^4\, \tfrac{N}{C}$$

$$\vec{E}_3 = 2.2\times 10^4\, \tfrac{N}{C}\langle 0,-1,0\rangle = \langle 0,-2.2,0\rangle \times 10^4\, \tfrac{N}{C}$$

And the net electric field is give by

$$\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 = \langle -3.9,-7.7,0\rangle \times 10^3\, \tfrac{N}{C} + \langle 2.2,4.4,0\rangle \times 10^4\, \tfrac{N}{C} + \langle 0,-2.2,0\rangle \times 10^4\, \tfrac{N}{C}$$

$$\vec{E}_{net} = \langle 1.8,1.4,0\rangle \times 10^4\, \tfrac{N}{C}$$

c. Suppose that a ${}^{64}_{29}Cu^{+2}$ ion were placed at point *P* and is released from rest. What initial acceleration would the ${}^{64}_{29}Cu^{+2}$ ion experience? Hint: The mass of a proton and a neutron are $m_p = 1.67\times10^{-27}\,kg$ and $m_n = 1.69\times10^{-27}\,kg$ respectively.

$$\vec{F} = m\vec{a} = q\vec{E} \rightarrow \vec{a} = \frac{q}{m}\vec{E} = \left(\frac{2\times1.6\times10^{-19}\,C}{64\times1.67\times10^{-27}\,kg}\right)\langle 1.8,1.4,0\rangle\times10^{4}\,\tfrac{N}{C}$$

$$\therefore \vec{a} = \langle 5.4,4.3,0\rangle\times10^{10}\,\tfrac{m}{s^2}$$

or if you prefer as a magnitude and direction: $\left|\vec{a}\right| = 6.9\times10^{10}\,\tfrac{m}{s^2}\,@\,\phi = 38.6$

d. How much work would be done to bring the ${}^{64}_{29}Cu^{+2}$ ion in from very far away and put it at point *P*?

$$V_P = V_1 + V_2 + V_3 = \frac{kq_1}{r_1} + \frac{kq_2}{r_2} + \frac{kq_3}{r_3}$$

$$V_P = 9\times10^{9}\,\tfrac{Nm^2}{C^2}\left[\frac{1.2\times10^{-6}\,C}{\sqrt{(0.5m)^2+(1m)^2}} - \frac{1.7\times10^{-6}\,C}{\sqrt{(0.25m)^2+(0.5m)^2}} + \frac{2.4\times10^{-6}\,C}{\sqrt{(1m)^2}}\right]$$

$$V_P = 3.9\times10^{3}\,V$$

$$W = -q\Delta V = -(2e)\Delta V = -2\times1.6\times10^{-19}\,C\times\left[3.9\times10^{3}\,V - 0V\right] = -1.25\times10^{-15}\,J$$

2. We've calculated the electric field due to an extended distribution of charge by using $d\vec{E} = k\dfrac{dq}{r^2}\hat{r}$.

We can extend this idea to calculating electric potentials for distributions of charge by using $V = \int dV = \int k\dfrac{dq}{r}$ where r is the distance between the element of charge dq and the observation point.

a. Consider a thin ring of charge of radius R with constant charge per unit length ($\lambda = \dfrac{Q}{s}$). What is the electric potential at a point $P = \langle 0,0,z \rangle$ along the axis of the ring? Hints: Arc length is given as $s = R\theta$. Write dq as a fraction of the total charge Q for a segment of arc that subtends an angle of $d\theta$. In addition, electric potentials are scalar quantities, so you don't have to write any unit vectors to evaluate the electric potential.

The element of charge is given as $dq = \lambda ds = \left(\dfrac{Q}{s}\right)ds = \left(\dfrac{Q}{2\pi R}\right)Rd\theta = \dfrac{Q}{2\pi}d\theta$.

The distance between a point on the ring and the point P in space is given by $r = \sqrt{R^2 + z^2}$.

The electric potential is therefore $V = \int dV = \frac{kQ}{2\pi\sqrt{R^2+z^2}}\int_0^{2\pi} d\theta = \dfrac{kQ}{\sqrt{R^2+z^2}}$.

b. How much work would be done to bring a point charge $q = -2\mu C$ from very far away and place it at the point $P = \langle 0,0,2 \rangle m$ along the axis of the ring? Assume that $\lambda = 3.2\frac{\mu C}{m}$ and that $R = 0.5m$.

$$W = -q\Delta V = -q\left[\frac{kQ}{\sqrt{R^2+z^2}} - 0\right] = -\frac{kqQ}{\sqrt{R^2+z^2}}$$

$$W = \frac{-\left(9\times10^9 \frac{Nm^2}{C^2}\right)\times\left(-2\times10^{-6}C\right)\times\left(3.2\times10^{-6}\times2\pi\times0.5m\right)}{\sqrt{(0.5m)^2+(2m)^2}} = 0.088J$$

c. What is the electric field at the point $P=\langle 0,0,z\rangle$? Compare your result with that of the electric field of a ring that we derived in class and can be found on the formula sheet. Note: You cannot simply state the electric field from the formula sheet as your answer and say "yours matches." You have to derive the expression for the electric field and there is a harder way (to derive the electric field from first principles) and an easier way (how do electric field and electric potential relate?) to do this problem.

$$\vec{E}=-\left\langle \frac{dV}{dx},\frac{dV}{dy},\frac{dV}{dz}\right\rangle=-\left\langle 0,0,\frac{dV}{dz}\right\rangle=\left\langle 0,0,\frac{kQz}{\left(R^2+z^2\right)^{\frac{3}{2}}}\right\rangle$$

$$E_z=-\frac{dV}{dz}=-kQ\frac{d}{dz}\left[\left(R^2+z^2\right)^{-\frac{1}{2}}\right]=\frac{kQz}{\left(R^2+z^2\right)^{\frac{3}{2}}}$$

Which is the electric field that we determined in class.

d. Suppose that instead of the ring of charge, you had an electric dipole at the origin. The axis of the dipole is pointing along the x-axis with the $+q$ charge located at $\left\langle \frac{s}{2},0,0\right\rangle$ and the $-q$ charge located at $\left\langle -\frac{s}{2},0,0\right\rangle$. At the origin between the two charges the electric field and the electric potential are approximately

1. $\vec{E}=\left\langle \frac{kq}{s^2},0,0\right\rangle$ and $V=\frac{kq}{s}$.

2. $\vec{E}=\left\langle \frac{8kq}{s^2},0,0\right\rangle$ and $V=\frac{2kq}{s}$.

3. $\vec{E}=\left\langle -\frac{8kq}{s^2},0,0\right\rangle$ and $V=0$.

4. $\vec{E}=\left\langle -\frac{8kq}{s^2},0,0\right\rangle$ and $V=\frac{4kq}{s}$.

5. $\vec{E}=\langle 0,0,0\rangle$ and $V=\frac{2kq}{s}$.

3. A parallel plate capacitor is constructed out of two circular metal plates of diameter $D = 0.5m$ separated by $s = 5mm$.

   a. If the capacitor were charged by a battery such that a constant surface charge density of $\sigma = 23.4\frac{\mu C}{m^2}$ was placed on each plate, what are the capacitance of the capacitor and the electric field (in magnitude and direction) that is produced between the plates?

$$C = \frac{\varepsilon_0 A}{s} = \frac{8.85\times10^{-12}\frac{C^2}{Nm^2}\times\pi\left(0.25m\right)^2}{5\times10^{-3}m} = 3.48\times10^{-10}F$$

$$E = \frac{Q}{\varepsilon_0 A} = \frac{\sigma}{\varepsilon_0} = \frac{23.4\times10^{-6}\frac{C}{m^2}}{8.85\times10^{-12}\frac{C^2}{Nm^2}} = 2.64\times10^{6}\frac{N}{C}$$

   The direction of the constant electric field is from the positive plate to the negative plate.

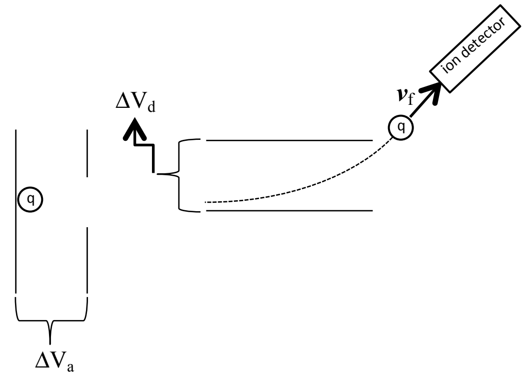
   b. Determine the fringe electric fields (in magnitude and direction) produced outside of the plates and the ratio of a fringe electric field to the electric field between the plates. What can you conclude about the fringe electric fields?

$$E_{fringe} = \frac{Q}{2\varepsilon_0 A}\left(\frac{s}{R}\right) = \frac{E_{inside}}{2}\left(\frac{s}{R}\right) = \frac{2.64\times10^{6}\frac{N}{C}}{2}\left(\frac{5\times10^{-3}m}{0.25m}\right) = 2.64\times10^{4}\frac{N}{C}$$

$$\frac{E_{fringe}}{E_{inside}} = \frac{s}{2R} = \frac{5\times10^{-3}m}{2\times0.25m} = 0.01$$

   The fringe fields point away from the positive plate and toward the negative plate. The conclusion here is that since $E_{fringe} << E_{inside}$ the fringe fields can generally be ignored.

Consider the setup on the right where two sets of capacitors are used. The capacitor on the left is called the accelerator and has a potential difference ΔV_a across the plates while the capacitor on the right is called the deflector and has a potential difference of ΔV_d across its plates. In parts, c and d, the actual values of both ΔV_a and ΔV_d are unknown. A charge $+q$ is accelerated from rest in the accelerator and then enters the deflector. The charge $+q$ is deflected in the y-direction over a distance $\Delta y = y$ by the electric field between the deflector plates and the deflector plates are separated by an amount d, where $d >> y$.



c. To detect the accelerated ions from the left capacitor, the potential difference across the deflector ΔV_d is changed until a signal is seen in the ion detector. When this signal is seen, it is determined that the y-component of the velocity of the ions is v_{fy}. In terms of q, y, d, v_{fy}, and the mass of the charge m, what is the expression for the potential difference across the deflecting capacitor and which deflector plate, the upper or lower, is at the higher electric potential?

The work done on the charge in the deflector region is given by $W = -q\Delta V_y = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}m\left(v_{fy}^2 + v_{fx}^2\right) - \frac{1}{2}mv_i^2$, where ΔV_y is the potential difference over which the charge is accelerated in the electric field.

Since there are no forces in the horizontal direction, $v_{fx} = v_{ix} = v_i$. Thus $W = -q\Delta V_y = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}mv_{fy}^2$.

The electric field between the deflector plates is constant so we can relate ΔV_d to ΔV_y through

$$E = -\frac{\Delta V_y}{y} = -\frac{\Delta V_d}{d} \rightarrow \Delta V_y = \frac{y}{d}\Delta V_d.$$

The difference in potential across the deflector plates is therefore

$$W = -q\Delta V_y = -q\left(\frac{y}{d}\Delta V_d\right) = \frac{1}{2}mv_{fy}^2 \rightarrow \Delta V_d = -\frac{mdv_{fy}^2}{2qy}.$$

d. Which of the following expressions below give the angle, with respect to the horizontal, at which the ion detector was set?

1. $\tan\theta = \frac{y}{d}\left(\frac{\Delta V_d}{\Delta V_a}\right)$. 2. $\tan\theta = \frac{\Delta V_d}{\Delta V_a}$. 3. $\tan\theta = \sqrt{\frac{\Delta V_d}{\Delta V_a}}$. 4. $\tan\theta = \sqrt{\frac{y}{d}\left(\frac{\Delta V_d}{\Delta V_a}\right)}$.

Physics 121 Equation Sheet

Electric Forces, Fields and Potentials

$$\vec{F} = k\frac{Q_1Q_2}{r^2}\hat{r};\ \ \hat{r} = \frac{\vec{r}_o - \vec{r}_s}{\left|\vec{r}_o - \vec{r}_s\right|}$$

$$\vec{E} = \frac{\vec{F}}{q}$$

$$\vec{E}_Q = k\frac{Q}{r^2}\hat{r}$$

$$\vec{p} = q\vec{s} = \alpha\vec{E}$$

$$\left|\vec{E}_{\parallel}\right| = \frac{2kqs}{r^3};\ \text{dipole } r >> s$$

$$\left|\vec{E}_{\perp}\right| = \frac{kqs}{r^3};\ \text{dipole } r >> s$$

$$\left|\vec{E}_{rod}\right| = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{r\sqrt{r^2 + \left(L/2\right)^2}}\right];\ \ \left|\vec{E}_{rod}\right| \sim \frac{1}{4\pi\varepsilon_0}\left(\frac{2Q}{rL}\right)\ \ L >> r$$

$$\left|\vec{E}_{ring}\right| = \frac{1}{4\pi\varepsilon_0}\left[\frac{Qz}{\left(R^2 + z^2\right)^{\frac{3}{2}}}\right]$$

$$\left|\vec{E}_{disk}\right| = \frac{Q}{2\pi\varepsilon_0 R^2}\left[1 - \frac{z}{\sqrt{R^2 + z^2}}\right];\ \ \left|\vec{E}_{disk}\right| \sim \frac{Q}{2\varepsilon_0 A}\left[1 - \frac{z}{R}\right]\ \ z < R;\ \ \left|\vec{E}_{disk}\right| \sim \frac{Q}{2\varepsilon_0 A}\ \ z << R$$

$$\left|\vec{E}_{capacitor}\right| \sim \frac{Q}{\varepsilon_0 A};\ \ \left|\vec{E}_{fringe}\right| \sim \frac{Q}{2\varepsilon_0 A}\left(\frac{s}{R}\right)$$

$$W = -q\Delta V = -\Delta U = \Delta K;\ \ K = \tfrac{1}{2}mv^2$$

$$U = \sum_{i \neq j}\frac{kQ_iQ_j}{r_{ij}};$$

$$V_Q = \frac{kQ}{r};\ \ V_{Q's} = \sum_i \frac{kQ_i}{r_i}$$

$$\Delta V = -\int \vec{E}\cdot d\vec{r}$$

$$E_x = -\frac{\Delta V}{\Delta x};\ \ E_y = -\frac{\Delta V}{\Delta y};\ \ E_z = -\frac{\Delta V}{\Delta z};\ \ \vec{E} = -\left\langle \frac{dV}{dx}, \frac{dV}{dx}, \frac{\Delta V}{\Delta x}\right\rangle$$

Constants

$g = 9.8\,\frac{m}{s^2}$

$1eV = 1.6\times10^{-19}J$

$1e = 1.6\times10^{-19}C$

$\mu_o = 4\pi\times10^{-7}\,\frac{Tm}{A}$

$k = \frac{1}{4\pi\varepsilon_o} = 9\times10^9\,\frac{Nm^2}{C^2}$

$c = 3\times10^8\,\frac{m}{s}$

$\varepsilon_o = 8.85\times10^{-12}\,\frac{C^2}{Nm^2}$

$h = 6.63\times10^{-34}Js$

$m_e = 9.11\times10^{-31}kg = \frac{0.511MeV}{c^2}$

$m_p = 1.67\times10^{-27}kg = \frac{937.1MeV}{c^2}$

$m_n = 1.69\times10^{-27}kg = \frac{948.3MeV}{c^2}$

$1amu = 1.66\times10^{-27}kg = \frac{931.5MeV}{c^2}$

$N_A = 6.02\times10^{23}$

$Ax^2 + Bx + C = 0 \rightarrow x = \frac{-B\pm\sqrt{B^2 - 4AC}}{2A}$