Physics 121

Exam #2

October 28, 2022

Name____________________________________

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example, $|\vec{p}| \approx m|\vec{v}| = (5kg) \times (2\,\frac{m}{s}) = 10\,\frac{kg \cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- All free-response part is worth 6 points.

Problem #1	/24
Problem #2	/24
Problem #3	/24
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

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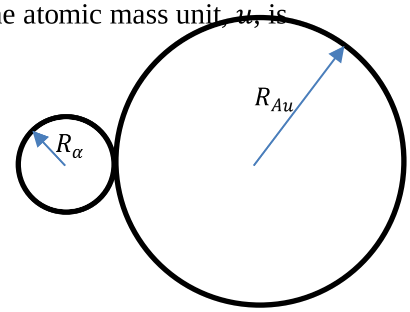
1. Small particles, such as protons or alphas (helium nuclei) are often used to probe the elemental composition of a target.

   a. Suppose that an alpha particle is initially very far from a target made of gold. As the alpha particle approaches a gold nucleus in the target, the alpha particle slows down and comes to rest. Suppose that the alpha particle was to come to rest just as it touches the "surface" of a gold nucleus, as shown below. If the radii of the alpha particle and a gold nucleus are $1.90 \times 10^{-15} m$ and $6.98 \times 10^{-15} m$ respectively, how much energy (in MeV) is stored in the system as an electric potential energy? Hint: The alpha particle has a mass of $4u$ and a charge $+2e$, while gold has a mass $197u$ and has 79 protons in its nucleus. The atomic mass unit, u, is approximately equal to the mass of a proton.



$$U_e = qV = Q_\alpha \times \frac{kQ_{Au}}{r} = \frac{k(2e)(79e)}{R_\alpha + R_{Au}}$$

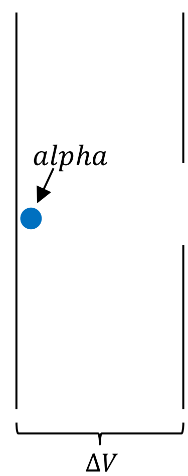
$$U_e = \frac{9 \times 10^9 \frac{Nm^2}{C^2} \times 158 \times \left(1e \times \frac{1.6\times10^{-19} C}{1e}\right)^2}{1.90 \times 10^{-15} m + 6.98 \times 10^{-15} m}$$

$$U_e = 4.1 \times 10^{-12} J \times \frac{1eV}{1.6 \times 10^{-19} J} \times \frac{1MeV}{1 \times 10^6 eV} = 25.6MeV$$

   b. Very far away from the gold target (and the gold nuclei) we need to accelerate the alpha particle so that it has enough energy to come to rest when it just touches the surface of a gold nucleus. We generally use a capacitor system to accomplish this. Through what potential difference, ΔV, would the alpha particle need to be accelerated so that it could come to rest just as it touches the surface of a gold nucleus?

$$W = -q\Delta V_{acc} = 25.6MeV$$

$$\Delta V_{acc} = -\frac{W}{Q_\alpha} = -\frac{25.6MeV}{2e} = -12.8MV = -1.28 \times 10^7 V$$

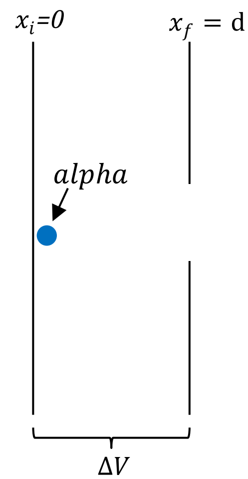


c. What was the speed of the alpha particle when it leaves the accelerator but is still very far away from the gold target?

$$W = -q\Delta V_{acc} = 25.6MeV = \Delta K = \frac{1}{2}m_\alpha v_{f\alpha}^2 - \frac{1}{2}m_\alpha v_{i\alpha}^2 = \frac{1}{2}m_\alpha v_{f\alpha}^2$$

$$v_{f\alpha} = \sqrt{\frac{2W}{m_\alpha}} = \sqrt{\frac{2 \times 25.6 \times 10^6 eV \times \frac{1.6\times10^{-19}J}{1eV}eV}{4 \times 1.67 \times 10^{-27}kg}} = 2.48 \times 10^7 \frac{m}{s}$$

d. Through what electric field between the plates was the alpha particle accelerated if the alpha particle starts out at $x_i = 0m$ and ends at $x_f = d = 0.25m$?



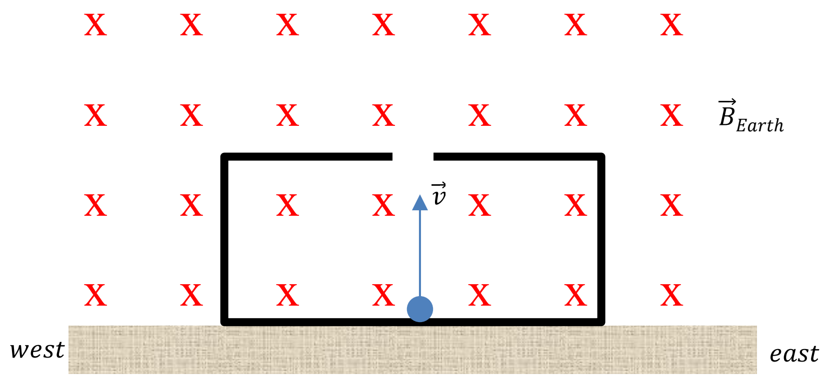
$$\Delta V_{acc} = -\int_{x_i}^{x_f} \vec{E} \cdot d\vec{x}$$

$$\Delta V_{acc} = -E_x \int_{x_i}^{x_f} dx = -E_x \Delta x|_0^d = -E_x d$$

$$E_x = -\frac{\Delta V_{acc}}{d} = -\frac{(-1.28 \times 10^7 V)}{0.25m}$$

$E_x = 1.28 \times 10^7 \frac{V}{m}$ to the right or $\vec{E} = \langle 1.28,0,0 \rangle \times 10^7 \frac{V}{m}$

2. A proton is in a box that contains an electric field $\vec{E}$. The box is in the Earth's magnetic field ($B_{Earth} = 5.2 \times 10^{-5}T$) which points north, and the proton moves up away from the Earth's surface with a speed $v = 7.2 \times 10^{6}\frac{m}{s}$.



a. In the box above, draw the direction of the electric field inside of the box so that there is no change in the trajectory of the proton as it moves upward in the box. To earn full credit, explain why you drew the electric field as you did.

By the RHR the magnetic force is to the west. Therefore, to keep the proton moving up away from the earth, the electric force must point east. Since we have a proton and it feels a force in the direction of the electric field, the electric field must also point east.

b. What is the magnitude of the electric field inside of the box while the proton in moving upwards away from the Earth's surface?

$$F_x: \ -F_B + F_E = 0 \rightarrow -evB + eE = 0$$

$$\rightarrow E = vB = 7.2 \times 10^{6}\frac{m}{s} \times 5.2 \times 10^{-5}T = 374.4\frac{N}{C}$$

c. When the proton leaves the box through the hole in the upper surface, the proton is subject only to the Earth's magnetic field. Which way does the proton move in the magnetic field and what is the radius of its orbit?

By the RHR, the magnetic force is to the west, so the proton moves in a circle of radius R initially toward the west.

$$F_B = qvB\sin\theta = evB\sin\theta = ma_c = m\frac{(v\sin\theta)^2}{R} \rightarrow R = \frac{mv\sin\theta}{eB}$$

$$R = \frac{1.67\times10^{-27}kg\times7.2\times10^6\frac{m}{s}\times\sin 90}{1.6\times10^{-19}C\times5.2\times10^{-5}T} = 1445.2m$$

d. When the proton is in the magnetic field only region it takes the proton a time T to complete one full orbit through the magnetic field at speed v. Suppose that a second proton were given an initial speed of $2v$ and this second proton moves upward away from the Earth's surface through the box and out of the hole at the upper surface. To get this second proton out of the box, what would have to happen to the magnitude and direction of the electric field inside of the box and when the proton leaves the box and enters the magnetic field only region, what is its new orbital period in terms of T?

Since $E = vB$ (by equating the electric and magnetic forces as in part a), and since $B$is constant in magnitude and direction, if v increases by a factor of 2, so too does the electric field E in magnitude. The direction of the electric field E remains unchanged.

The original orbital period T is $v = \frac{2\pi R}{T} \rightarrow T = \frac{2\pi R}{v} = \frac{2\pi R}{v} = \frac{2\pi R}{\frac{eBR}{m}} = \frac{2\pi m}{eB}$

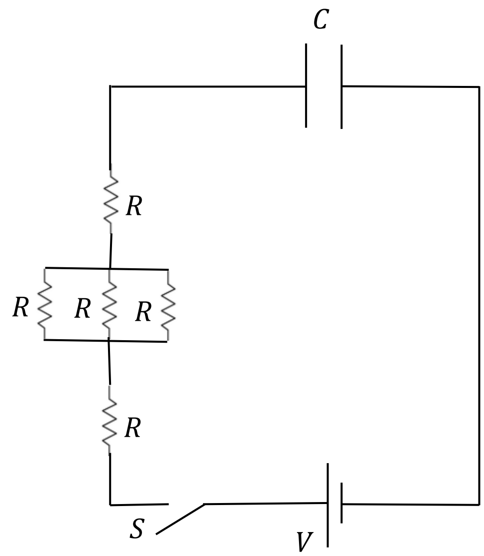
The speed is determined from the magnetic force: $F_B = evB = m\frac{v^2}{R} \rightarrow v = \frac{eRB}{m}$.

The new orbital period T' is $v' = \frac{2\pi R'}{T'} \rightarrow T' = \frac{2\pi R'}{v'} = \frac{4\pi R}{2v} = \frac{2\pi}{\frac{eBR}{m}} = \frac{2\pi m}{eB}$

The new orbital radius is determined from the magnetic force:

$$F_B = ev'B = m\frac{v'^2}{R'} \rightarrow R' = \frac{mv'}{eB} = \frac{2mv}{eB} = 2R.$$

3. A resistor R, uncharged parallel-plate capacitor C, and a battery V are connected to an open switch S as shown below.



a. The resistance of each resistor in the circuit is $R = 100k\Omega$ and the resistors and capacitor are connected to a $V = 6700V$ battery. When the switch S is closed the capacitor begins to charge. Between the plates of the capacitor there is a $0.2\mu m$ piece of material with a dielectric constant $\kappa = 210$. If the time constant of the circuit is needed to be $\tau = 0.5hr$, what is the area A of a plate in the capacitor?

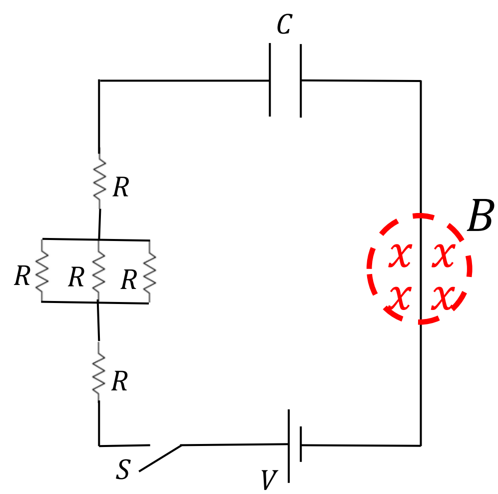
$$R_{eq} = R + R + \left(\frac{1}{R} + \frac{1}{R} + \frac{1}{R}\right)^{-1} = 2R + \frac{R}{3} = \frac{7}{3}R$$

$$= \frac{7}{3} \times 100k\Omega = 233.3k\Omega$$

$$\tau = R_{eq}C \rightarrow C = \frac{\tau}{R_{eq}} = \frac{0.5hr \times \frac{3600s}{1hr}}{233.3 \times 10^3\Omega} = 0.0077F = 7.7 \times 10^{-3}F$$

$$C = \frac{\kappa\varepsilon_0 A}{d} \rightarrow A = \frac{Cd}{\kappa\varepsilon_0} = \frac{7.7 \times 10^{-3}F \times 0.2 \times 10^{-6}m}{210 \times 8.85 \times 10^{-12}\frac{C^2}{Nm^2}} = 0.83m^2$$

b. Measurements of the current in the circuit show that the current varies in time as the capacitor charges according to $I = I_{max}e^{-\frac{t}{R_{eq}C}}$ and part of the right-hand side of the circuit passes through a pair of circular poles of a magnet 5cm in diameter with a strength $B = 1.5T$. At a time $t = 3\tau$, where τ is the time constant of the circuit, what is the magnetic force on the right-hand side of the wire?



$$I = I_{max}e^{-\frac{t}{R_{eq}C}} = \frac{V_{max}}{R_{eq}}e^{-\frac{3\tau}{\tau}} = \frac{6700V}{233.3\times10^3\Omega}e^{-3}$$

$I = 0.00143A = 1.43mA$

$F_B = ILB = 0.00143A \times 0.05m \times 1.5T$

$F_B = 1.1 \times 10^{-4}N$ to the right

c. At a time $t = 3\tau$, what is the power dissipated across all the resistors in the circuit if the current is again given by $I = I_{max}e^{-\frac{t}{R_{eq}C}}$?

$$I_{max} = \frac{V_{max}}{R_{eq}} = \frac{6700V}{233.3 \times 10^3\Omega} = 0.0287A$$

$$P = \left(I_{max}e^{-\frac{t}{R_{eq}C}}\right)^2 R_{eq} = (0.0287e^{-6}A)^2 \times 233.3 \times 10^3\Omega = 0.476\text{W}$$

Or, $V = V_{max}\left(1 - e^{-\frac{t}{R_{eq}C}}\right) = V_{max}\left(1 - e^{-\frac{3\tau}{\tau}}\right) = 0.95V_{max}$

$V_{max} = V_C + V_R \rightarrow V_R = V_{max} - 0.95V_{max} = 0.05V_{max}$

$P = \frac{V^2}{R_{eq}} = \frac{(0.05V_{max})^2}{R_{eq}} = \frac{(0.05\times6700V)^2}{233.3\times10^3\Omega} = 0.480W$

Or, $P = IV = 0.00143A \times 0.05 \times 6700V = 0.479W$

The differences between each method are due to rounding.

d. Suppose that the wire that is between the poles of the magnet in part b had a radius of $r_{wire} = 0.25mm$. What Hall voltage would be measured across the width of the wire at a time $t = 3\tau$? Hints: The wire is made of nickel with $\rho_{Ni} = 8908\frac{kg}{m^3}, M_{Ni} = 59\frac{g}{mol}, \ \rho = 6.99 \times 10^{-8}\Omega m$ and nickel donates 2 charge carriers per nickel atom.

$$V_{Hall} = wv_dB = \frac{IwB}{neA} = \frac{I(2r_{wire})B}{ne\left(\pi r_{wire}^2\right)} = \frac{2IB}{\pi ner_{wire}}$$

$$V_{Hall} = \frac{2 \times 0.00143A \times 1.5T}{\pi \times 1.8 \times 10^{29}m^{-3} \times 1.6 \times 10^{-19}C \times 0.25 \times 10^{-3}m} = 1.9 \times 10^{-10}V = 0.19nV$$

Where, $n = \left[\frac{\rho_{Ni}}{M_{Ni}}N_A\right] \times 2 = \frac{2\frac{charge\ carriers}{atom\ of\ Ni} \times 8908\frac{kg}{m^3} \times 6.02 \times 10^{23}\frac{atoms\ of\ Ni}{mol\ of\ Ni}}{0.059\frac{kg}{mol\ of\ Ni}}$

$$n = 1.8 \times 10^{29}\frac{charge\ carriers}{m^3}$$