Physics 121

Exam #2

May 24, 2019

Name___________________________________

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example $|\vec{p}| \approx m|\vec{v}| = (5kg)\times(2\,\tfrac{m}{s}) = 10\,\tfrac{kg\cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- All multiple-choice question are worth 3 points and each free-response part is worth 7 points

Problem #1	/24
Problem #2	/24
Problem #3	/24
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

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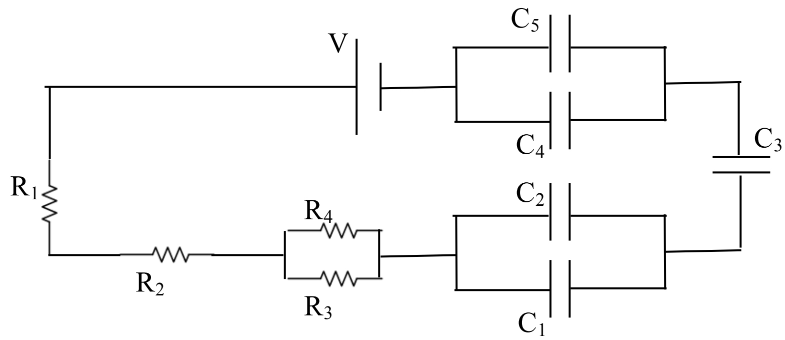
1. Consider the following circuit in which some resistors and capacitors are wired to a $20V$ battery. Each capacitor is rated at $10000\mu F$ while each of the resistors is rated at $2k\Omega$.



a. What are the equivalent resistance and equivalent capacitance of the circuit?

$$\frac{1}{R_{34}} = \frac{1}{R_3} + \frac{1}{R_4} = \frac{1}{2k\Omega} + \frac{1}{2k\Omega} = \frac{2}{2k\Omega} \rightarrow R_{34} = 1k\Omega$$

$$\mathrm{R}_{eq} = R_1 + R_2 + R_{34} = 2k\Omega + 2k\Omega + 1k\Omega = 5k\Omega$$

$$C_{12} + C_1 + C_2 = 10000\mu F + 10000\mu F = 20000\mu F$$

$$C_{45} + C_4 + C_5 = 10000\mu F + 10000\mu F = 20000\mu F$$

$$\frac{1}{C_{eq}} = \frac{1}{C_{12}} + \frac{1}{C_3} + \frac{1}{C_{45}} = \frac{1}{20000\mu F} + \frac{1}{10000\mu F} + \frac{1}{20000\mu F}$$

$$C_{eq} = 5000\mu F$$

b. What is the time constant that characterizes the circuit and how much charge is stored in the system when the capacitors are fully charged?

$$\tau = R_{eq} C_{eq} = 5000\Omega \times 5000 \times 10^{-6} F = 25s$$

$$Q_{total} = C_{eq} V = 5000 \times 10^{-6} F \times 20V = 0.1C$$

c. Recalling that the electric power is the rate at which work is done, or the rate at which energy is transferred, derive an expression for how much the power developed by the battery changes with time. Then evaluate your expression using values from the problem, in the limit that the capacitors have had a sufficiently long time to charge.

$$P=\frac{dW}{dt}=\frac{d}{dt}\left(\tfrac{1}{2}C_{eq}V(t)^{2}\right)=\frac{d}{dt}\left(\tfrac{1}{2}C_{eq}V_{\max}^{2}\left(1-e^{-\frac{t}{R_{eq}C_{eq}}}\right)^{2}\right)=\tfrac{1}{2}C_{eq}V_{\max}^{2}\frac{d}{dt}\left(\left(1-e^{-\frac{t}{R_{eq}C_{eq}}}\right)^{2}\right)$$

$$P=2\left(\tfrac{1}{2}C_{eq}V_{\max}^{2}\right)\left(\frac{1}{R_{eq}C_{eq}}\right)\left(1-e^{-\frac{t}{R_{eq}C_{eq}}}\right)=\frac{V_{\max}^{2}}{R_{eq}}\left(1-e^{-\frac{t}{R_{eq}C_{eq}}}\right)$$

$$\text{as}\quad t\rightarrow\infty,\quad e^{-\frac{t}{R_{eq}C_{eq}}}\rightarrow 0$$

$$\therefore P=\frac{V_{\max}^{2}}{R_{eq}}=\frac{(20V)^{2}}{5000\Omega}=0.08W=80mW$$

d. Suppose that you removed the battery from the circuit and connected the resistor combination directly to the capacitor combination. Compared to the time constant for the charging circuit, which of the following would give the time constant that characterizes discharge of the capacitor circuit?

1. The time constant of the discharging circuit would be smaller that the time constant for the charging circuit since the battery has been removed.
2. The time constant of the discharging circuit would be larger that the time constant for the charging circuit since the battery has been removed.
3. The time constant of the discharging circuit would be smaller that the time constant for the charging circuit since capacitance of the circuit decreases when the battery is removed.
4. The time constant of the discharging circuit would be larger than the time constant for the charging circuit since the capacitance of the circuit increases when the battery is removed.
5. None of the above would give the correct relation between the time constants for the charging and discharging circuits.

2. Suppose that an electron were accelerated from rest through a potential difference $\Delta V = 620V$. The electron acquires a velocity, after being accelerated, of $\vec{v} = \langle 0, 0.96, 1.12 \rangle \times 10^7 \frac{m}{s}$, at which point a uniform magnetic field $\vec{B} = \langle 0, 5, 0 \rangle \times 10^{-5} T$ is turned on everywhere in space.

a. What is the magnetic force the electron experiences due to the electron's interaction with the external magnetic field? You may ignore any effects due to the earth's magnetic field.

$$\vec{F}_B = q\vec{v} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -ev_y & -ev_z \\ 0 & B & 0 \end{vmatrix} = \langle ev_z B, 0, 0 \rangle$$

$$|\vec{F}_B| = ev_z B = 1.6 \times 10^{-19} C \times 1.12 \times 10^7 \tfrac{m}{s} \times 5 \times 10^{-5} T = 8.96 \times 10^{-17} N$$

$$\therefore \vec{F}_B = \langle ev_z B, 0, 0 \rangle = \langle 8.96, 0, 0 \rangle \times 10^{-17} N$$

b. At what angle was the velocity vector of the electron oriented with respect to the external magnetic field?

$$\vec{F}_B = q\vec{v} \times \vec{B} \rightarrow |\vec{F}_B| = |q\vec{v} \times \vec{B}| = e|\vec{v}||\vec{B}|\sin\theta$$

$$\sin\theta = \frac{|\vec{F}_B|}{e|\vec{v}||\vec{B}|} = \frac{8.96 \times 10^{-17} N}{1.6 \times 10^{-19} C \times 1.476 \times 10^7 \tfrac{m}{s} \times 5 \times 10^{-5} T} = 0.7588$$

$$\therefore \theta = \sin^{-1}(0.7588) = 49.4^0$$

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2} = \sqrt{(0)^2 + (0.96)^2 + (1.12)^2} \times 10^7 \tfrac{m}{s} = 1.476 \times 10^7 \tfrac{m}{s}$$

c. What are the orbital radius and orbital period of the electron about the external magnetic field?

$$\vec{F}_B = q\vec{v}\times\vec{B} = \langle ev_z B,0,0\rangle = \left\langle \frac{mv_z^2}{R} \right\rangle$$

$$R = \frac{mv_z}{eB} = \frac{9.11\times10^{-31}kg\times1.12\times10^{7}\,\frac{m}{s}}{1.6\times10^{-19}C\times5\times10^{-5}T} = 1.28m$$

$$v_\perp = v\sin\theta = \frac{2\pi R}{T} \rightarrow T = \frac{2\pi R}{v\sin\theta} = \frac{2\pi\times1.28m}{1.12\times10^{7}\,\frac{m}{s}} = 7.2\times10^{-7}s = 0.72\mu s$$

d. Which of the following most likely describes the motion of the electron through this external magnetic field?

1. A straight line.
2. A circle of radius R about the external magnetic field.
3. A helix wound counterclockwise about the external magnetic field.
4. A helix wound clockwise about the external magnetic field.
5. The exact motion of the electron in the external magnetic field cannot be determined.

3. A $2mm$ diameter, $0.5m$ long piece of gold is used as a conductive device, where the resistivity, density and molar mass of gold are $2.44\times10^{-8}\Omega m$, $19{,}300\frac{kg}{m^3}$, and $0.197kg$ respectively. The conductive device is connected to a $3V$ battery. Assume that gold provides one free charge carrier per atom.

a. What is the drift speed for the charge carriers in gold?

$$I = neAv_d \rightarrow v_d = \frac{I}{neA} = \frac{772.5A}{5.9\times10^{28}m^{-3}\times1.6\times10^{-19}C\times\pi\left(1\times10^{-3}m\right)^2} = 0.026\tfrac{m}{s} = 2.6\tfrac{cm}{s}$$

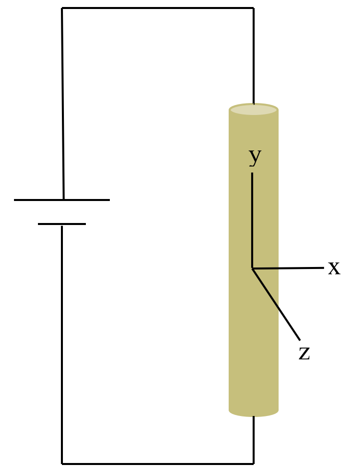
$$n = \left(\frac{\rho}{M}N_A\right)\times1\frac{\text{charge carrier}}{\text{atom}} = \left(\frac{19300\frac{kg}{m^3}}{0.197\frac{kg}{mol}}\times6.02\times10^{23}\frac{atoms}{mol}\right)\times1\frac{\text{charge carrier}}{\text{atom}} = 5.9\times10^{28}m^{-3}$$

$$I = \frac{V}{R} = \frac{VA}{\rho L} = \frac{3V\times\pi\left(1\times10^{-3}m\right)^2}{2.44\times10^{-8}\Omega m\times0.5m} = 772.5A$$

b. Suppose that the gold conductive device were placed in an external magnetic field of strength $\left|\vec{B}\right| = 0.25T$ oriented perpendicular to the direction of the current flow. What Hall potential difference would you record across the diameter of the gold conductor?

$$v_d = \frac{V_{Hall}}{wB} \rightarrow V_{Hall} = v_d wB = 0.026\tfrac{m}{s}\times2\times10^{-3}m\times0.25T = 1.33\times10^{-5}V = 13.3\mu V$$

c. Consider the gold conductive device oriented as shown below. Taking the origin of the coordinate system to be at the center of the device, what are the magnitude and direction of the magnetic field created a point $P = \langle 10,0,0 \rangle cm$? Assume that the gold conductor can be considered a long straight wire.



$$\left|\vec{B}\right| = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7} \frac{Tm}{A} \times 772.5A}{2\pi \times 0.1m} = 1.55 \times 10^{-3} T = 1.55mT$$

$$\vec{B} = \langle 0,0,1.6 \rangle \times 10^{-3} T$$

d. Suppose instead of the circuit above, you had an electron moving with velocity $\vec{v} = \langle 0,v,0 \rangle$. Which of the following would most likely give the magnetic field due to the motion of the electron at a point $P = \langle x,0,0 \rangle$ when the electron is located at $\langle 0,0,0 \rangle$?

1. $\vec{B} = \left\langle \frac{\mu_0 ev}{4\pi x^2},0,0 \right\rangle$.
2. $\vec{B} = \left\langle -\frac{\mu_0 ev}{4\pi x^2},0,0 \right\rangle$.
3. $\vec{B} = \left\langle 0,\frac{\mu_0 ev}{4\pi x^2},0 \right\rangle$.
4. $\vec{B} = \left\langle 0,-\frac{\mu_0 ev}{4\pi x^2},0 \right\rangle$.
5. $\vec{B} = \left\langle 0,0,\frac{\mu_0 ev}{4\pi x^2} \right\rangle$
6. $\vec{B} = \left\langle 0,0,-\frac{\mu_0 ev}{4\pi x^2} \right\rangle$

Physics 121 Equation Sheet

Electric Forces, Fields and Potentials

$\vec{F} = k\frac{Q_1Q_2}{r^2}\hat{r};\ \hat{r} = \frac{\vec{r}_o - \vec{r}_s}{|\vec{r}_o - \vec{r}_s|}$

$\vec{E} = \frac{\vec{F}}{q} = \int d\vec{E} = \int dE\hat{r} = \int \frac{kdq}{r^2}\hat{r}$

$\vec{E}_Q = k\frac{Q}{r^2}\hat{r}$

$|\vec{E}_{\parallel}| = \frac{2kqs}{r^3}$; dipole $r >> s$

$|\vec{E}_{\perp}| = \frac{kqs}{r^3}$; dipole $r >> s$

$|\vec{E}_{rod}|_{\perp} = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{r\sqrt{r^2 + \left(L/2\right)^2}}\right]$

$|\vec{E}_{rod}|_{\parallel} = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{r(L+r)}\right]; |\vec{E}_{rod}|_{\parallel} \sim \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{rL}\right)\ L >> r$

$|\vec{E}_{ring}| = \frac{1}{4\pi\varepsilon_0}\left[\frac{Qz}{(R^2+z^2)^{\frac{3}{2}}}\right]$

$|\vec{E}_{disk}| = \frac{Q}{2\pi\varepsilon_0 R^2}\left[1 - \frac{z}{\sqrt{R^2+z^2}}\right];\quad |\vec{E}_{disk}| \sim \frac{Q}{2\varepsilon_0 A}\left[1 - \frac{z}{R}\right]\ z << R;\quad |\vec{E}_{disk}| \sim \frac{Q}{2\varepsilon_0 A}\ z << R$

$|\vec{E}_{capacitor}| \sim \frac{Q}{\varepsilon_0 A};\quad |\vec{E}_{fringe}| \sim \frac{Q}{2\varepsilon_0 A}\left(\frac{s}{R}\right)$

$W = -q\Delta V = -\Delta U = \Delta K;\ U = \sum_{i \neq j}\frac{kQ_iQ_j}{r_{ij}};$

$V_Q = \frac{kQ}{r};\ V_{Q's} = \sum_i \frac{kQ_i}{r_i}$

$\Delta V = -\int \vec{E}\cdot d\vec{r}$

$E_x = -\frac{\Delta V}{\Delta x};\ E_y = -\frac{\Delta V}{\Delta y};\ E_z = -\frac{\Delta V}{\Delta z};\ \vec{E} = -\left\langle \frac{dV}{dx}, \frac{dV}{dy}, \frac{dV}{dz}\right\rangle$

$Q = \left(\frac{\kappa\varepsilon_0 A}{s}\right)\Delta V$

$U = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$

$Q = Q_{\max}\left(1 - e^{-\frac{t}{RC}}\right);\ Q = Q_{\max}e^{-\frac{t}{RC}}$

$I = \frac{dQ}{dt} = n|e|Av_d = \int \vec{J}\cdot d\vec{A}$

$n = \frac{\rho}{M}N_A$

$\vec{J} = n|e|\vec{v}_d = \sigma\vec{E} = \frac{1}{\rho}\vec{E} \rightarrow V = IR; R = \frac{\rho L}{A}$

$P = IV = I^2R = \frac{V^2}{R} = \frac{dW}{dt}$

$\rho = \rho_0(1 + \alpha\Delta T)$

$\vec{F} = q\vec{v}\times\vec{B};\ |\vec{F}| = qvB\sin\theta$

$\vec{B} = \frac{\mu_o}{4\pi}\left(\frac{q\vec{v}\times\hat{r}}{r^2}\right)$

$d\vec{B} = \frac{\mu_o I}{4\pi}\left(\frac{d\vec{l}\times\hat{r}}{r^2}\right)$

$|\vec{B}_{wire}| = \frac{\mu_o LI}{4\pi r\sqrt{\left(\frac{L}{2}\right)^2 + r^2}};\ |\vec{B}_{wire}| \approx \frac{\mu_o I}{2\pi r}\ L >> r$

$|\vec{B}_{ring}| = \frac{\mu_o IR^2}{2(z^2+R^2)^{\frac{3}{2}}};\ |\vec{B}_{ring}| \approx \frac{\mu_o IR^2}{2z^3}\ z << R$

Constants

$g = 9.8\frac{m}{s^2}$

$1e = 1.6\times10^{-19}C$

$k = \frac{1}{4\pi\varepsilon_o} = 9\times10^9\frac{Nm^2}{C^2}$

$\varepsilon_o = 8.85\times10^{-12}\frac{C^2}{Nm^2}$

$1eV = 1.6\times10^{-19}J$

$\mu_o = 4\pi\times10^{-7}\frac{Tm}{A}$

$c = 3\times10^8\frac{m}{s}$

$h = 6.63\times10^{-34}Js$

$m_e = 9.11\times10^{-31}kg = \frac{0.511MeV}{c^2}$

$m_p = 1.67\times10^{-27}kg = \frac{937.1MeV}{c^2}$

$m_n = 1.69\times10^{-27}kg = \frac{948.3MeV}{c^2}$

$1amu = 1.66\times10^{-27}kg = \frac{931.5MeV}{c^2}$

$N_A = 6.02\times10^{23}$

$Ax^2 + Bx + C = 0 \rightarrow x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

$\vec{v}_f = \langle v_{fx}, v_{fy}, v_{fz}\rangle = \langle v_{ix} + a_xt, v_{iy} + a_yt, v_{iz} + a_zt\rangle$

$\Delta\vec{r}_f = \langle x_f - x_i, y_f - y_i, z_f - z_i\rangle = \langle v_{ix}t + \frac{1}{2}a_xt^2, v_{iy}t + \frac{1}{2}a_yt^2, v_{iz}t + \frac{1}{2}a_zt^2\rangle$

$\vec{F} = I\vec{L}\times\vec{B}; |\vec{F}| = ILB\sin\theta$

$V_{Hall} = wv_dB = \frac{IwB}{eAn}$

$\frac{1}{R_{eq}} = \sum_i \frac{1}{R_i}$

$R_{eq} = \sum_i R_i$

$C_{eq} = \sum_i C_i$

$\frac{1}{C_{eq}} = \sum_i \frac{1}{C_i}$

$\phi_B = \int \vec{B}\cdot d\vec{A}$

$\varepsilon = -N\frac{d\phi_B}{dt}$

$\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta$

$|\vec{A}\times\vec{B}| = |\vec{A}||\vec{B}|\sin\theta$

$\vec{A}\times\vec{B} = \langle A_yB_z - A_zB_y, A_zB_x - A_xB_z, A_xB_y - A_yB_x\rangle$