Physics 121

Exam #2

February 26, 2016

Name___________________________________

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example $|\vec{p}| \approx m|\vec{v}| = (5kg)\times\left(2\,\tfrac{m}{s}\right) = 10\,\tfrac{kg\cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- All multiple-choice question are worth 3 points and each free-response part is worth 7 points

Problem #1	/24
Problem #2	/24
Problem #3	/24
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

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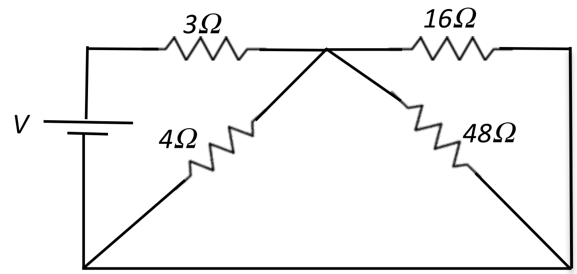
1. Consider the circuit below. A collection of resistors is connected to a battery rated at $\Delta V = 12V$.



a. What are the equivalent resistance of the circuit and the total current produce by the battery?

Let $R_1 = 3\Omega$, $R_2 = 4\Omega$, $R_3 = 48\Omega$, and $R_4 = 16\Omega$ and resistors R_2, R_3, and R_4 are in parallel, and their equivalent resistance is

$$\frac{1}{R_{234}} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} = \frac{1}{4\Omega} + \frac{1}{48\Omega} + \frac{1}{16\Omega}$$

$$R_{234} = 3\Omega$$

R_1 is in series with R_{234} and thus the equivalent resistance of the circuit is

$$R_{eq} = R_1 + R_{234} = 3\Omega + 3\Omega = 6\Omega$$

The current produced by the battery by Ohm's law: $V = IR_{eq} \to I_{total} = I_1 = \dfrac{V}{R_{eq}} = \dfrac{12V}{6\Omega} = 2A$.

b. Apply loop rules to the circuit above and determine the currents in the 4Ω, 48Ω, and 16Ω resistors. Is charge conserved in the circuit? (To earn full credit for charge conservation you must justify your answer. Simply answering yes or no will not do it.)

Loop 1: $V \to R_1 \to R_2 \to V$:

$$\Delta V_B + \Delta V_1 + \Delta V_2 = V - I_1R_1 - I_2R_2 = 0$$
$$I_2 = \frac{V - I_1R_1}{R_2} = \frac{12V - (2A \times 3\Omega)}{4\Omega} = 1.5A$$

Loop 2: $V \to R_1 \to R_3 \to V$:

$$\Delta V_B + \Delta V_1 + \Delta V_3 = V - I_1R_1 - I_3R_3 = 0$$
$$I_3 = \frac{V - I_1R_1}{R_3} = \frac{12V - (2A \times 3\Omega)}{48\Omega} = 0.125A$$

Loop 3: $V \to R_1 \to R_4 \to V$:

$$\Delta V_B + \Delta V_1 + \Delta V_4 = V - I_1R_1 - I_4R_4 = 0$$
$$I_4 = \frac{V - I_1R_1}{R_4} = \frac{12V - (2A \times 3\Omega)}{16\Omega} = 0.375A$$

Conservation of Charge: Yes, since $I_1 = I_2 + I_3 + I_4 \to 2A = 1.5A + 0.125A + 0.375A$.

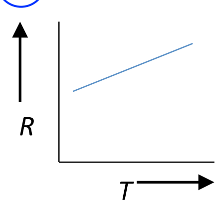
c. Suppose that you have a single resistor with resistance equal to the equivalent resistance of part a. You connect this resistor to a capacitor rated at C. Then you wire the resistor and capacitor to a battery rated at $\Delta V = 10V$ and allow the capacitor to charge fully. Now, after the capacitor is fully charged, suppose that you want to discharge to capacitor through the resistor to 95% of the starting voltage of the capacitor in a time of $t = 3.20$ seconds. What value of C would you need to accomplish this?

$$Q(t) = 0.05Q_{\max} = Q_{\max} e^{-\frac{t}{RC}} \rightarrow C = -\frac{t}{R\ln\left(\frac{0.05Q_{\max}}{Q_{\max}}\right)} = -\frac{3.20s}{6\Omega \ln(0.05)} = 0.17F$$

d. Suppose that in both of your circuits above the wires were made out of copper and further suppose that you want to perform an experiment to investigate the thermal and electrical properties those copper wires. To perform these experiments you take the wire and connect it in series with an ammeter, a power supply V and a switch S. You select a voltage and close the switch so that current flows through the circuit and this raises the wire's temperature. The values in the table below show the results of your experimentation where the trials were conducted at different wire temperatures T. The initial value of the resistance R was measured at a temperature of $T = 293K$ with an ohmmeter. The other values were calculated using Ohm's law. Using the data, which graph below best illustrates the relation between the temperature T and the resistance R?

Trial	T (K)	V (V)	I (A)	R (Ω)
1	293	0	0	4.0
2	373	4.6	0.75	6.1
3	473	10	1.18	8.7
4	573	18	1.60	11.3
5	673	28	2.00	13.9

1.



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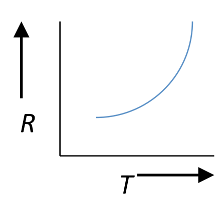
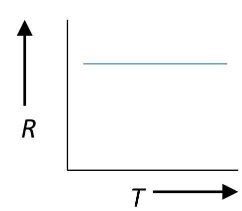


3.



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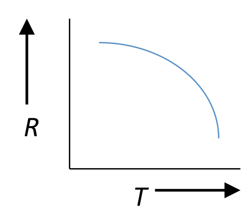
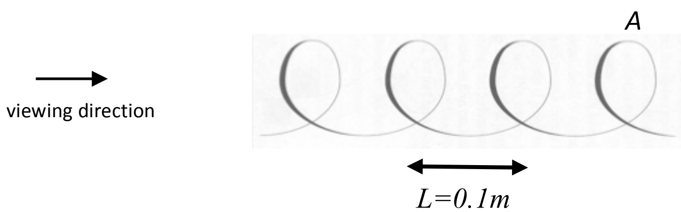


2. Suppose that a proton is moving through a magnetic field and its motion is shown below. If you are on the left side of the page and view the motion of the proton, the proton orbits in a counter-clockwise circle and also moves (across the page from left to right) away from you. The pitch of the proton's orbit is given as $L = 0.1m$, the orbital period of the proton $T = 0.037\mu s$, and the perpendicular component of the proton's velocity is $v_\perp = 2.1\times10^6 \frac{m}{s}$.



a. To produce this motion of the proton, the external magnetic field that the proton is interacting with is

1. directed toward the left side of the page
2. directed toward the right side of the page. By Right Hand Rule
3. directed out of the page.
4. directed into the page.

b. What are the magnitude of the velocity of the proton, the orbital radius of the proton, and the magnitude of the external magnetic field?

The component of the velocity parallel to the field is given by

$$v_{//} = \frac{L}{T} = \frac{0.1m}{0.037\times10^{-6}s} = 2.7\times10^6 \tfrac{m}{s}.$$

Thus the magnitude of the velocity of the proton is

$$v = \sqrt{v_{//}^2 + v_\perp^2} = \sqrt{(2.7)^2 + (2.1)^2}\times10^6 \tfrac{m}{s} = 3.4\times10^6 \tfrac{m}{s}.$$

From the perpendicular component of the velocity, the orbital radius is given as

$$v_\perp = \frac{2\pi r}{T} \rightarrow r = \frac{v_\perp T}{2\pi} = \frac{2.1\times10^6 \frac{m}{s}\times0.037\times10^{-6}s}{2\pi} = 0.0124m = 12.4mm.$$

Thus the magnitude of the magnetic field is given from the magnetic force law.

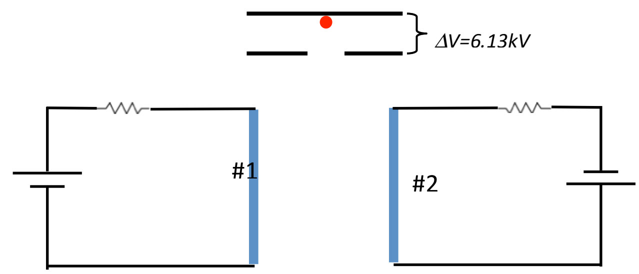
$$\left|\vec{F}_B\right| = qv_\perp B = F_C = m\frac{v_\perp^2}{r} \rightarrow B = \frac{mv_\perp}{qr} = \frac{1.67\times10^{-27}kg\times2.1\times10^6 \frac{m}{s}}{1.6\times10^{-19}C\times0.0124m} = 1.77T.$$

c. Suppose that the magnetic field is turned off at the instant that the proton is at point A and that the proton is coming out of the page at you. At the very instant that the first magnetic field is turned off, a wire oriented vertically, located somewhere to the right of point A suddenly has a current I flowing. This current in the wire is flowing up the plane of the page. What force does the proton feel at this moment? Give both a magnitude and direction. Note this may be a symbolic answer. Take the x-axis to the right, the y-axis up the page and the z-axis out of the page and let r be the distance between the wire and the proton.

The velocity of the proton at this instant is $\vec{v} = \langle 0,0,z \rangle$ and the magnetic field produced by this current flowing up the page in the y-direction is $\vec{B} = \left\langle 0,0,\frac{\mu_0 I}{2\pi r} \right\rangle$. Thus the magnetic force is

$\vec{F}_B = q\vec{v} \times \vec{B} = q\begin{bmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & v \\ 0 & 0 & B \end{bmatrix} = \langle 0,0,0 \rangle$ and the proton feels no magnetic force.

d. Now suppose that you have the setup shown below. The accelerator is used to accelerate an electron and then this electron heads toward the midpoint between the two circuits. What force does the electron feel as it passes between the two circuits along a line through the midpoint between the two circuits? Assume that a long straight wire #1 is connected to a battery rated at $15V$ and a 271Ω resistor, while another long straight wire #2 is connected to a $8V$ battery and a 313Ωresistor and the two wires (#1 & #2) are separated by $0.5m$ and that each blue segment of wire has a length of $0.25m$. (Hint: Take the y-axis vertically up the page, the x-axis to the right and the z-axis coming out of the page at you.)



The currents produced by each circuit are

$I_1 = \frac{V_1}{R_1} = \frac{15V}{271\Omega} = 0.055A$ and

$I_2 = \frac{V_2}{R_2} = \frac{8V}{313\Omega} = 0.026A$. Each wire produces a magnetic field at the midpoint between the two circuits pointing out of the page and the net field is the sum of the two magnetic field

$$\vec{B}_{net} = \vec{B}_1 + \vec{B}_2 = \left\langle 0,0,\frac{\mu_o I_1}{2\pi r_1} + \frac{\mu_o I_2}{2\pi r_2} \right\rangle = \left\langle 0,0,\frac{4\pi \times 10^{-7} \frac{Tm}{A}}{2\pi}\left(\frac{0.055A}{0.25m} + \frac{0.026A}{0.25m}\right)\right\rangle = \left\langle 0,0,6.5\times 10^{-8} \right\rangle T.$$

Thus the magnetic force felt by the electron is given by

$$\vec{F}_B = q\vec{v}\times\vec{B}_{net} = q\begin{bmatrix} x & y & z \\ 0 & -v & 0 \\ 0 & 0 & B_{net} \end{bmatrix} = \left\langle evB_{net},0,0 \right\rangle.$$

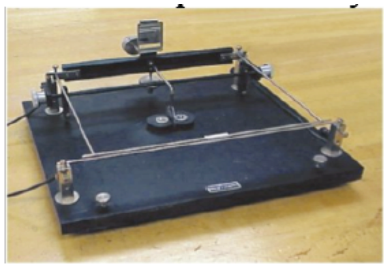
$$\vec{F}_B = \left\langle 1.6\times 10^{-19}C \times 4.61\times 10^{7}\tfrac{m}{s} \times 6.5\times 10^{-8}T,0,0 \right\rangle = \left\langle 4.8\times 10^{-19},0,0 \right\rangle N$$

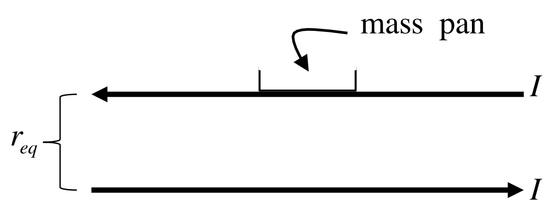
The speed of the electron used in the calculation is given by

$$W = -q\Delta V = \Delta KE \to eV = (\gamma - 1)mc^2 = \left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right)mc^2.$$

$$v = \sqrt{1 - \frac{1}{\left(1+\frac{KE}{mc^2}\right)^2}}\,c = \sqrt{1 - \frac{1}{\left(1+\frac{6.130keV}{511\frac{keV}{c^2}\times c^2}\right)^2}}\,c = 0.154c = 4.61\times 10^{7}\tfrac{m}{s}$$

3. A current balance is a device with two sets of bars through which a current I can pass. The lower bar is fixed, while the upper bar is able to pivot about the back edge (with the mirror) as shown in the figure below. The separation distance r_{eq} between the bars is fixed and this is called the equilibrium separation. When the upper bar is perturbed from equilibrium the experimenter needs to bring the system of bars back into equilibrium by adding masses to the pan attached to the upper bar. This results in the separation between the bars r_{eq} being kept constant.





a. The current in the circuit is produced from a $120V$ battery (not shown, but the wires connecting the battery to the apparatus are on the left side of the picture) connected to a variable resistor (also not shown). A variable resistor is a resistor whose resistance can be changed and suppose that the resistance in this circuit can be changed over a range of resistances from 1Ω to 40Ω. *Derive an expression for the mass that would need to be added to the pan so that the system remains in equilibrium. Evaluate your expression for the largest mass* you would need to add to the pan if the bars have a length of $L = 30cm$ and $r_{eq} = 0.5cm$. Take the x-direction to be to the right, y-direction up the plane of the page and the z-direction out of the page at you.

$$\vec{F}_{net} = \vec{F}_{u,l} + \vec{F}_w = I\int d\vec{l} \times B + \langle 0,-mg,0\rangle = -I\int_0^L \begin{bmatrix} x & y & z \\ dx & 0 & 0 \\ 0 & 0 & B_l \end{bmatrix} + \langle 0,-mg,0\rangle$$

$$\vec{F}_{net} = \langle 0, IB_l L - mg, 0\rangle = \langle 0,0,0\rangle$$

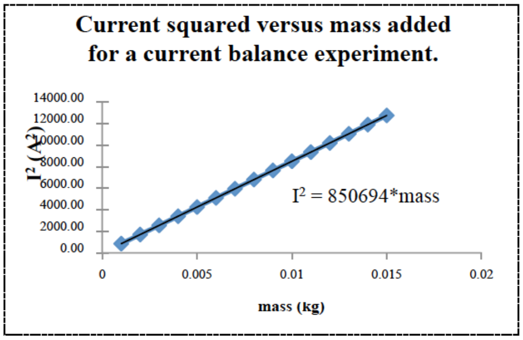
Therefore in the y-direction:

$$IB_l L - mg = 0 \rightarrow m = \frac{IB_l L}{g} = \frac{IL}{g}\left(\frac{\mu_0 I}{2\pi r_{eq}}\right) = \left(\frac{\mu_0 L}{2\pi r_{eq} g}\right) I^2$$

$$m_{lower} = \left(\frac{\mu_0 L}{2\pi r_{eq} g}\right) I^2 = \left(\frac{\mu_0 L}{2\pi r_{eq} g}\right)\left(\frac{V}{R_{\text{large}}}\right)^2 = \left(\frac{4\pi \times 10^{-7}\,\frac{Tm}{A} \times 0.3m}{2\pi \times 0.005m \times 9.8\,\frac{m}{s^2}}\right)\left(\frac{120V}{40\Omega}\right)^2 = 0.000011kg = 0.11g$$

$$m_{upper} = \left(\frac{\mu_0 L}{2\pi r_{eq} g}\right) I^2 = \left(\frac{\mu_0 L}{2\pi r_{eq} g}\right)\left(\frac{V}{R_{\text{small}}}\right)^2 = \left(\frac{4\pi \times 10^{-7}\,\frac{Tm}{A} \times 0.3m}{2\pi \times 0.005m \times 9.8\,\frac{m}{s^2}}\right)\left(\frac{120V}{1\Omega}\right)^2 = 0.0176kg = 1.76g$$

b. Suppose that you go into the lab and use a current balance and take data on the current through the bars versus the mass added to the pan, where the current range is determined from the $120V$ power supply and the variable resistance in part a. You then construct the following graph. What is the experimental value of the permeability of free space, μ_0?



From part a, we have $I^2 = \left(\frac{2\pi r_{eq} g}{\mu_0 L}\right) m$, therefore the slope of the line is related to μ_0. We have

$$\frac{2\pi r_{eq} g}{\mu_0 L} = slope \to \mu_0 = \frac{2\pi r_{eq} g}{(slope)L} = \frac{2\pi \times 0.005m \times 9.8\,\frac{m}{s^2}}{\left(850694\,\frac{A^2}{kg}\right)0.3m} = 1.21\times10^{-6}\,\frac{Tm}{A} = 12.1\times10^{-7}\,\frac{Tm}{A}$$

$$\mu_0 = 3.85\pi\times10^{-7}\,\frac{Tm}{A}$$

c. All of the wires that have been used in this exam (and in class/lab) have been made out of copper. Suppose that the electron mobility of copper is $\mu = 4.38\times10^{-3}\,\frac{m/s}{N/C}$ and that $n = 8.48\times10^{28}\,m^{-3}$.

What is the resistivity of copper?

$$\Delta V = IR \to EL = |e| i\left(\frac{\rho L}{A}\right) \to E = \frac{|e|\rho}{A}(nAv_d) = |e|\rho n v_d = |e|\rho n\mu E$$

$$\rho = \frac{1}{|e|n\mu} = \frac{1}{1.6\times10^{-19}C\times8.48\times10^{28}m^{-3}\times4.38\times10^{-3}\,\frac{m/s}{N/C}}$$

$$\rho = 1.68\times10^{-8}\,\Omega m$$

d. Suppose that we have the current balance apparatus oriented in such a way that the upper and lower wires of the apparatus point along north, with north taken to be to the right in the figure above. A compass is placed at the midpoint between the wires with the face of the compass parallel to the upper wire points north. As you look down on the compass from above, with the current flowing in the apparatus, the compass needle would be deflected

1. North.
2. South.
3. East.
4. West.

Physics 121 Equation Sheet

Electric Forces, Fields and Potentials

$$\vec{F} = k\frac{Q_1Q_2}{r^2}\hat{r}; \quad \hat{r} = \frac{\vec{r}_o - \vec{r}_s}{|\vec{r}_o - \vec{r}_s|}$$

$$\vec{E} = \frac{\vec{F}}{q}$$

$$\vec{E}_Q = k\frac{Q}{r^2}\hat{r}$$

$$\vec{p} = q\vec{s} = \alpha\vec{E}$$

$$|\vec{E}_{\parallel}| = \frac{2kqs}{r^3}; \text{ dipole } r >> s$$

$$|\vec{E}_{\perp}| = \frac{kqs}{r^3}; \text{ dipole } r >> s$$

$$|\vec{E}_{rod}| = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{r\sqrt{r^2 + \left(L/2\right)^2}}\right]; \quad |\vec{E}_{rod}| \sim \frac{1}{4\pi\varepsilon_0}\left(\frac{2Q}{rL}\right) \quad L >> r$$

$$|\vec{E}_{ring}| = \frac{1}{4\pi\varepsilon_0}\left[\frac{Qz}{\left(R^2 + z^2\right)^{\frac{3}{2}}}\right]$$

$$|\vec{E}_{disk}| = \frac{Q}{2\pi\varepsilon_0 R^2}\left[1 - \frac{z}{\sqrt{R^2 + z^2}}\right]; \quad |\vec{E}_{disk}| \sim \frac{Q}{2\varepsilon_0 A}\left[1 - \frac{z}{R}\right] \quad z << R; \quad |\vec{E}_{disk}| \sim \frac{Q}{2\varepsilon_0 A} \quad z << R$$

$$|\vec{E}_{capacitor}| \sim \frac{Q}{\varepsilon_0 A}; \quad |\vec{E}_{fringe}| \sim \frac{Q}{2\varepsilon_0 A}\left(\frac{s}{R}\right)$$

$$W = -q\Delta V = -\Delta U = \Delta K; \quad U = \sum_{i \neq j}\frac{kQ_iQ_j}{r_{ij}};$$

$$V_Q = \frac{kQ}{r}; \quad V_{Q's} = \sum_i \frac{kQ_i}{r_i}$$

$$\Delta V = -\int \vec{E}\cdot d\vec{r}; \quad \oint \vec{E}\cdot d\vec{l} = 0$$

$$E_x = -\frac{\Delta V}{\Delta x}; \quad E_y = -\frac{\Delta V}{\Delta y}; \quad E_z = -\frac{\Delta V}{\Delta z}; \quad \vec{E} = -\left\langle \frac{dV}{dx}, \frac{dV}{dy}, \frac{dV}{dz}\right\rangle$$

$$Q = \left(\frac{\varepsilon_0 A}{s}\right)\Delta V$$

$$I = \frac{\Delta Q}{\Delta t} = i|e| = n|e|Av_d; \quad \vec{v}_d = \mu\vec{E}$$

$$\vec{B} = \frac{\mu_o}{4\pi}\left(\frac{q\vec{v}\times\hat{r}}{r^2}\right)$$

$$d\vec{B} = \frac{\mu_o I}{4\pi}\left(\frac{d\vec{l}\times\hat{r}}{r^2}\right)$$

$$|\vec{B}_{wire}| = \frac{\mu_o LI}{4\pi r\sqrt{\left(\frac{L}{2}\right)^2 + r^2}}; \quad |\vec{B}_{wire}| \approx \frac{\mu_o I}{2\pi r} \quad L >> r$$

$$|\vec{B}| = \frac{\mu_o IR^2}{\ldots}; \quad |\vec{B}| \approx \frac{\mu_o IR^2}{\ldots} \quad z << R$$

Constants

$g = 9.8\,\frac{m}{s^2}$

$1eV = 1.6\times10^{-19}J$

$1e = 1.6\times10^{-19}C$

$\mu_o = 4\pi\times10^{-7}\,\frac{Tm}{A}$

$k = \frac{1}{4\pi\varepsilon_o} = 9\times10^9\,\frac{Nm^2}{C^2}$

$c = 3\times10^8\,\frac{m}{s}$

$\varepsilon_o = 8.85\times10^{-12}\,\frac{C^2}{Nm^2}$

$h = 6.63\times10^{-34}Js$

$m_e = 9.11\times10^{-31}kg = \frac{0.511MeV}{c^2}$

$m_p = 1.67\times10^{-27}kg = \frac{937.1MeV}{c^2}$

$m_n = 1.69\times10^{-27}kg = \frac{948.3MeV}{c^2}$

$1amu = 1.66\times10^{-27}kg = \frac{931.5MeV}{c^2}$

$N_A = 6.02\times10^{23}$

$Ax^2 + Bx + C = 0 \rightarrow x = \frac{-B\pm\sqrt{B^2 - 4AC}}{2A}$

$$V = IR; \quad R = \frac{L}{A} = \frac{\rho L}{\sigma A}$$

$$|\vec{J}| = \frac{I}{A} = \sigma|\vec{E}|$$

$$R_{eq,series} = \sum_{i=1}^{N} R_i$$

$$\frac{1}{R_{eq,parallel}} = \sum_{i=1}^{N}\frac{1}{R_i}$$

$$P = IV = I^2R = \frac{V^2}{R}$$

$$E = \tfrac{1}{2}qV = \tfrac{1}{2}CV^2 = \frac{q^2}{2C}$$

$$Q(t) = Q_{\max}\left(1 - e^{\frac{-t}{RC}}\right); \text{ charge}$$

$$Q(t) = Q_{\max}e^{\frac{-t}{RC}}; \text{ discharge}$$

$$I(t) = I_{\max}e^{\frac{-t}{RC}}$$

$$C_{eq,parallel} = \sum_{i=1}^{N} C_i$$

$$\frac{1}{C_{eq,series}} = \sum_{i=1}^{N}\frac{1}{C_i}$$

$$\vec{F} = q\vec{v} \times \vec{B}$$

$$d\vec{F} = Id\vec{l} \times \vec{B}$$

$$\vec{F}_{net} = \vec{F}_{\perp} + \vec{F}_{\parallel} = p\frac{d\hat{p}}{dt} + \hat{p}\frac{dp}{dt}; \quad \left|\vec{F}_{\perp}\right| = \frac{pv}{r}$$

$$\vec{F}_{Lorentz} = q\vec{E} + q\vec{v} \times \vec{B}$$

$$\omega = \frac{v}{r} = \frac{d\theta}{dt}; \quad \omega = \frac{qB}{\gamma m}$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$K = (\gamma - 1)mc^2$$

$$\vec{p} = \gamma m\vec{v}$$

Loop Rule: $\sum_i \Delta V_i = \sum_i E_i \Delta l_i = 0; \ \oint \vec{E} \cdot d\vec{l} = 0$

Node Rule: $\sum I_{in} = \sum I_{out}$

Cross-Product: $\vec{C} = \vec{A} \times \vec{B} = \left\langle \left(A_y B_z - A_z B_y\right), -\left(A_x B_z - A_z B_x\right), A_x B_y - A_y B_x \right\rangle$