Physics 121

Exam #2

February 23, 2018

Name___________________________________

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example $|\vec{p}| \approx m|\vec{v}| = (5kg)\times(2\,\frac{m}{s}) = 10\,\frac{kg\cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- All multiple-choice question are worth 3 points and each free-response part is worth 7 points

Problem #1	/24
Problem #2	/24
Problem #3	/24
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

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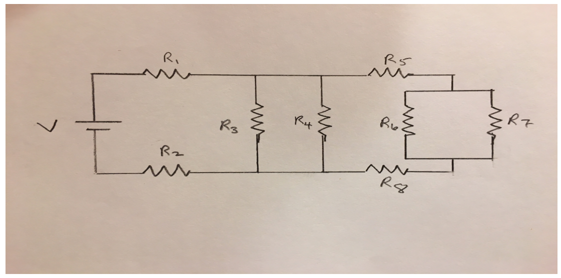
1. Consider the following circuit in which an ideal battery $V = 10V$ is connected to a network of resistors, each of which has a resistance of $R = 100\Omega$.



a. What is the total current produced by the battery?

R_6 and R_7 are in parallel, $\frac{1}{R_{67}} = \frac{1}{R_6} + \frac{1}{R_7} = \frac{2}{R} \to R_{67} = \frac{R}{2}$.

R_5, R_{67} and R_8 are in series, $R_{5678} = R_5 + R_{67} + R_8 = R + \frac{R}{2} + R = \frac{5R}{2}$.

R_3, R_4 and R_{5678} are in parallel, $\frac{1}{R_{345678}} = \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_{5678}} = \frac{1}{R} + \frac{1}{R} + \frac{2}{5R} \to R_{345678} = \frac{5R}{12}$.

R_1, R_2 and R_{345678} are in series, $R_{12345678} = R_{eq} = R_1 + R_2 + R_{345678} = R + R + \frac{5R}{12} = \frac{29R}{12}$.

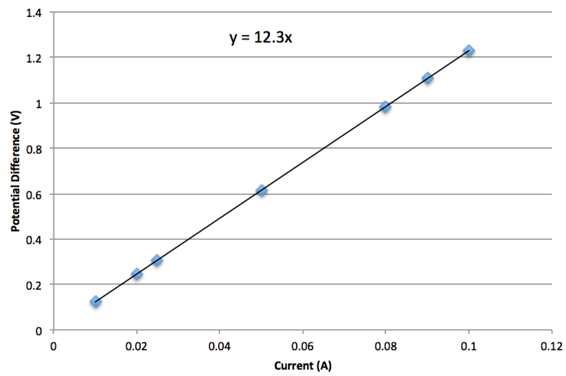
The total current: $I_{total} = \frac{V}{R_{eq}} = \frac{V}{\frac{29R}{12}} = \frac{10V}{\frac{29}{12}(100\Omega)} = 0.0414A = 41.4mA$

b. What is the power dissipated across the resistor R_4?

$$V - V_{R_1} - V_{R_2} - V_{R_{345678}} = 0$$
$$\to V_{R_{345678}} = V - V_{R_1} - V_{R_2} = V - I_{total}(R_1 + R_2) = 10V - 0.0414A(200\Omega) = 1.72V$$

$$P_4 = \frac{V_4^2}{R_4} = \frac{V_{345678}^2}{R_4} = \frac{(1.72V)^2}{100\Omega} = 0.0297W = 29.7mW$$

c. Consider the graph below that shows the potential difference across a resistor versus the current through the resistor, which is constructed out of an unknown material. The resistor is made from a $100m$ long wire with $1mm$ diameter and if a current $39mA$ of flows through the wire, what is the drift velocity of the charge carriers in the wire? The data in the table on the right may be useful.



Element	ρ (x10^{-8}Ωm)	n (x10^{28}m^{-3})
Ag	1.62	5.86
Cu	1.69	8.47
Au	2.35	5.90
Al	2.75	6.02
W	5.25	6.32
Fe	9.68	8.41

$$R=\frac{\rho l}{A}\rightarrow \rho=\frac{RA}{L}=\frac{12.3\Omega\times\pi\times\left(0.5\times10^{-3}m\right)^2}{100m}=9.7\times10^{-8}\Omega m\rightarrow Fe$$

$$I=neAv_d\rightarrow v_d=\frac{I}{neA}=\frac{39\times10^{-3}A}{8.41\times10^{28}m^{-3}\times1.6\times10^{-19}C\times\pi\times\left(0.5\times10^{-3}m\right)^2}=3.7\times10^{-6}\tfrac{m}{s}=3.7\tfrac{\mu m}{s}$$

d. Which of the following would give the magnitude of the electric field in the wire when connected to a battery of potential V ?

1. $E=\rho J$
2. $E=\frac{V}{L}$.
3. $E=\rho nev_d$.
4. All of the above.
5. None of the above.

2. The Earth's atmosphere is able to act as a capacitor, with one plate the ground and the other plate the clouds and in between the plates an air gap. Air, however, is not a perfect insulator and can be made to conduct, so that the separation of charges from the cloud to ground can be bridged. Such an event is called a lightning strike. Suppose that a uniform layer of clouds exits around the surface of the Earth at a distance of $s = 5000m$ and that a potential difference of $\Delta V = 5.5 \times 10^5 V$ exists between the clouds and the ground everywhere.

   a. Suppose that the resistance of the air between the clouds and the ground is $R = 300\Omega$, what are the time constant for the Earth-cloud system ***and*** how much charge is on each one of the "plates" in the system? (Hint: The surface area of a sphere is $4\pi r^2$ and $R_{Earth} = 6.4 \times 10^6 m$.)

$$C = \frac{\kappa\varepsilon_0 A}{s} = \frac{8.85 \times 10^{-12} \frac{C^2}{Nm^2}\left(4\pi\left(6.4 \times 10^6 m\right)^2\right)}{5000m} = 0.91F$$

$$\tau = RC = 300\Omega \times 0.91F = 273.3s$$

$$Q = CV = 0.91F \times 5.5 \times 10^5 V = 4.55 \times 10^5 C$$

   b. How long would it take to discharge the Earth-cloud system to approximately 0.1% of its initial charge?

$$Q(t) = 0.001Q_{\max} = Q_{\max} e^{-\frac{t}{RC}} \rightarrow \ln(0.001) = -\frac{t}{273.3s} \rightarrow t = 1888s$$

c. If each lightning strike transfers approximately $25C$ worth of charge, about how many lightning strikes occur around the earth in one day? (Hint: Assume that as soon as the Earth-cloud system discharges to 0.1% of its initial charge, the Earth-cloud system instantaneously recharges to the charge in part a.)

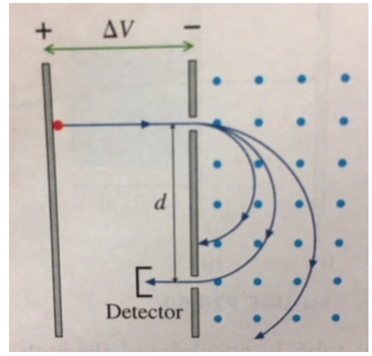
$$\#\tfrac{\text{discharges}}{\text{day}} = 86400\frac{\text{s}}{\text{day}} \times \frac{\#\text{discharges}}{1888s} = 45.8\tfrac{\text{discharges}}{\text{day}}$$

and every discharge corresponds to $\#\tfrac{\text{stikes}}{\text{discharge}} = 4.55\times10^5\tfrac{C}{strike}\times\tfrac{1strike}{25C} = 18200\tfrac{\text{stikes}}{\text{discharge}}$.

Therefore the number in one day is approximately:
$\#\tfrac{strikes}{day} = 45.8\tfrac{\text{discharges}}{\text{day}}\times18200\tfrac{\text{stikes}}{\text{discharge}} = 8.3\times10^5\tfrac{strikes}{day}$

d. The 300Ω resistance of the air corresponds to a very humid day. Suppose that on a less humid day the resistance of the air is higher than that on a humid day, which of the following would occur assuming everything else remains the same?
1. The number of lightning strikes would increase.
2. The number of lightning strikes would remain the same.
3. The number of lightning strikes would decrease.
4. How the number changes would not be able to be determined with the information given.

3. A mass spectrometer is an analytical instrument used to identify the various molecules in a sample by measuring their charge-to-mass ratio q/m. The sample is ionized and the positive ions are accelerated through a potential difference ΔV, and then enter a region of uniform magnetic field. The magnetic filed bends the ions in to circular trajectories, but after just half a circle they either strike the wall or pass through a small opening to a detector. As the accelerating voltage is slowly increased, different ions reach the detector and are measured. Consider the mass spectrometer shown below with a magnetic field $B = 200mT$ and a $d = 8.00cm$ spacing between the entrance and exit holes.



a. What accelerating potential difference is required to detect CO^+ ions? Some atomic masses are shown in the table on the right.

Element	Atomic mass (amu)
C	12.000
N	14.003
O	15.995

$$F_b = qv_\perp B = \frac{mv_\perp^2}{R} \rightarrow v_\perp = \frac{qRB}{m} = \frac{edB}{2m}$$

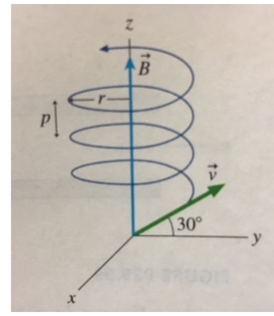
$$W = -q\Delta V = -e\Delta V = \tfrac{1}{2}mv_f^2 \rightarrow \Delta V = -\frac{m}{2e}v_f^2 = -\frac{m}{2e}\left(\frac{edB}{2m}\right)^2 = -\frac{ed^2B^2}{8m}$$

$$\therefore \Delta V = -\frac{ed^2B^2}{8m} = -\frac{1.6\times10^{-19}C\times(0.08m)^2\times(200\times10^{-3}T)^2}{8\left(27.995u\times\frac{1.66\times10^{-27}kg}{1u}\right)} = -110.2V$$

b. Suppose that you wanted to see N_2^+ ions rather than the CO^+ ions. N_2^+ has nominally the same mass as a CO^+ ion but because of small but measurable differences in the accelerating voltages the two ion species are easily separable. To see the N_2^+ ions the acceleration voltage needed would

1. need to decrease from the value used for the CO^+ ions.
2. remain unchanged from the value used for the CO^+ ions.
3. need to increase from the value used for the CO^+ ions.
4. be unable to be determined from the information given in the problem.

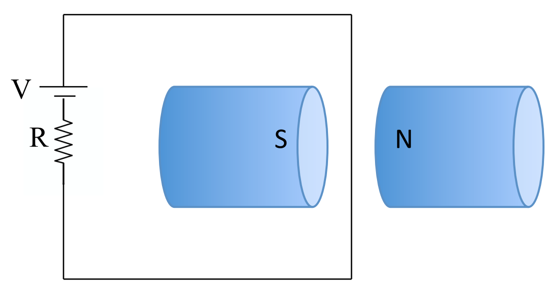
c. Suppose that instead of the mass spectrometer setup you had a uniform magnetic field that points along the z-axis with a value of $|\vec{B}| = 30mT$. An electron enters the magnetic filed with a speed $|\vec{v}| = 5\times10^6 \frac{m}{s}$ at an angle 30^0 above the x-y plane as shown in the figure on the right. What is the radius of the circular orbit about the magnetic field line and the distance between the windings of the helix, called the pitch, p?



$$F_b = qv_\perp B = \frac{mv_\perp^2}{R} \rightarrow R = \frac{mv_\perp}{eB} = \frac{9.11\times10^{-31}kg\times5\times10^6\frac{m}{s}\sin60}{1.6\times10^{-19}C\times30\times10^{-3}T} = 8.22\times10^{-4}m$$

$$v_\| = \frac{p}{T} \rightarrow p = v_\| T = v_\|\left(\frac{2\pi R}{v_\perp}\right) = \left(\frac{v\cos\phi}{v\sin\phi}\right)2\pi R = \frac{2\pi R}{\tan\phi} = \frac{2\pi\times8.22\times10^{-4}m}{\tan60} = 2.98\times10^{-3}m$$

d. Suppose instead of electrons in an external field you had the following setup in which a wire of constant linear mass density $\lambda = \frac{mass}{length}$ is held between the poles of a magnet in the plane of the page. The magnets are both circular with radius r. If an ideal battery provided the current in the circuit, what is the initial acceleration of the wire?



$$|\vec{F}_B| = ILB = \frac{V}{R}rB = ma \rightarrow |\vec{a}| = \frac{VrB}{mR} = \frac{VB}{\lambda R}$$

direction by the RHR is into the page.

Physics 121 Equation Sheet

Electric Forces, Fields and Potentials

$\vec{F} = k\frac{Q_1Q_2}{r^2}\hat{r};\ \hat{r} = \frac{\vec{r}_o - \vec{r}_s}{|\vec{r}_o - \vec{r}_s|}$

$\vec{E} = \frac{\vec{F}}{q} = \int d\vec{E} = \int dE\hat{r} = \int \frac{kdq}{r^2}\hat{r}$

$\vec{E}_Q = k\frac{Q}{r^2}\hat{r}$

$|\vec{E}_{\parallel}| = \frac{2kqs}{r^3}$; dipole $r >> s$

$|\vec{E}_{\perp}| = \frac{kqs}{r^3}$; dipole $r >> s$

$|\vec{E}_{rod}|_{\perp} = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{r\sqrt{r^2 + \left(\frac{L}{2}\right)^2}}\right]$

$|\vec{E}_{rod}|_{\parallel} = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{r(L+r)}\right];\ |\vec{E}_{rod}|_{\parallel} \sim \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{rL}\right)\quad L >> r$

$|\vec{E}_{ring}| = \frac{1}{4\pi\varepsilon_0}\left[\frac{Qz}{\left(R^2+z^2\right)^{\frac{3}{2}}}\right]$

$|\vec{E}_{disk}| = \frac{Q}{2\pi\varepsilon_0 R^2}\left[1 - \frac{z}{\sqrt{R^2+z^2}}\right];\quad |\vec{E}_{disk}| \sim \frac{Q}{2\varepsilon_0 A}\left[1 - \frac{z}{R}\right]\quad z << R;\quad |\vec{E}_{disk}| \sim \frac{Q}{2\varepsilon_0 A}\quad z << R$

$|\vec{E}_{capacitor}| \sim \frac{Q}{\varepsilon_0 A};\quad |\vec{E}_{fringe}| \sim \frac{Q}{2\varepsilon_0 A}\left(\frac{s}{R}\right)$

$W = -q\Delta V = -\Delta U = \Delta K;\ U = \sum_{i\neq j}\frac{kQ_iQ_j}{r_{ij}};$

$V_Q = \frac{kQ}{r};\ V_{Q's} = \sum_i \frac{kQ_i}{r_i}$

$\Delta V = -\int \vec{E}\cdot d\vec{r}$

$E_x = -\frac{\Delta V}{\Delta x};\ E_y = -\frac{\Delta V}{\Delta y};\ E_z = -\frac{\Delta V}{\Delta z};\ \vec{E} = -\left\langle \frac{dV}{dx}, \frac{dV}{dy}, \frac{dV}{dz}\right\rangle$

$Q = \left(\frac{\kappa\varepsilon_0 A}{s}\right)\Delta V$

$U = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$

$Q = Q_{\max}\left(1 - e^{-\frac{t}{RC}}\right);\ Q = Q_{\max}e^{-\frac{t}{RC}}$

$I = \frac{dQ}{dt} = n|e|Av_d = \int \vec{J}\cdot d\vec{A}$

$n = \frac{\rho}{M}N_A$

$\vec{J} = n|e|\vec{v}_d = \sigma\vec{E} = \frac{1}{\rho}\vec{E} \rightarrow V = IR;\ R = \frac{\rho L}{A}$

$P = IV = I^2R = \frac{V^2}{R}$

$\rho = \rho_0\left(1 + \alpha\Delta T\right)$

$\vec{F} = q\vec{v}\times\vec{B};\ |\vec{F}| = qvB\sin\theta$

$\vec{B} = \frac{\mu_o}{4\pi}\left(\frac{q\vec{v}\times\hat{r}}{r^2}\right)$

$d\vec{B} = \frac{\mu_o I}{4\pi}\left(\frac{d\vec{l}\times\hat{r}}{r^2}\right)$

$|\vec{B}_{wire}| = \frac{\mu_o LI}{4\pi r\sqrt{\left(\frac{L}{2}\right)^2 + r^2}};\quad |\vec{B}_{wire}| \approx \frac{\mu_o I}{2\pi r}\quad L >> r$

$|\vec{B}_{ring}| = \frac{\mu_o IR^2}{2\left(z^2+R^2\right)^{\frac{3}{2}}};\ |\vec{B}_{ring}| \approx \frac{\mu_o IR^2}{2z^3}\quad z << R$

Constants

$g = 9.8\,\frac{m}{s^2}$

$1eV = 1.6\times10^{-19}J$

$1e = 1.6\times10^{-19}C$

$\mu_o = 4\pi\times10^{-7}\,\frac{Tm}{A}$

$k = \frac{1}{4\pi\varepsilon_o} = 9\times10^9\,\frac{Nm^2}{C^2}$

$c = 3\times10^8\,\frac{m}{s}$

$\varepsilon_o = 8.85\times10^{-12}\,\frac{C^2}{Nm^2}$

$h = 6.63\times10^{-34}Js$

$m_e = 9.11\times10^{-31}kg = \frac{0.511MeV}{c^2}$

$m_p = 1.67\times10^{-27}kg = \frac{937.1MeV}{c^2}$

$m_n = 1.69\times10^{-27}kg = \frac{948.3MeV}{c^2}$

$1amu = 1.66\times10^{-27}kg = \frac{931.5MeV}{c^2}$

$N_A = 6.02\times10^{23}$

$Ax^2 + Bx + C = 0 \rightarrow x = \frac{-B\pm\sqrt{B^2-4AC}}{2A}$

$C_{eq} = \sum C_i;\ \frac{1}{C_{eq}} = \sum\frac{1}{C_i}$

$R_{eq} = \sum R_i;\ \frac{1}{R_{eq}} = \sum\frac{1}{R_i}$