Name____

Physics 121 Quiz #3, January 22, 2016

Please show all work, thoughts and/or reasoning in order to receive partial credit. The quiz is worth 10 points total.

I affirm that I have carried out my academic endeavors with full academic honesty.

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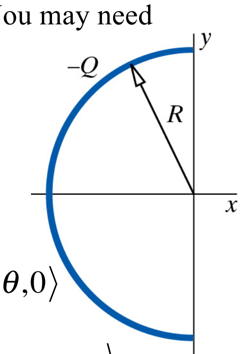
Consider a semi-circular arc of radius $R = 10cm$ with its center at the origin as shown below. The rod carries a uniformly distributed charge $Q = -0.3nC$.

a. What is the expression for the vector electric field at the origin? Hints: 1. Measure angles in radians with respect to the positive x-axis. That is, $+x \rightarrow \theta = 0;\ +y \rightarrow \theta = \frac{\pi}{2}$, etc. 2. You may need $\int \cos\theta\, d\theta = \sin\theta$ and $\int \sin\theta\, d\theta = -\cos\theta$.



$$\vec{E}_0 = \int dE = \int \frac{dq}{4\pi\varepsilon_0 r^2}\hat{r}$$

$$dq = -\left(\frac{d\theta}{\pi}\right)Q; \qquad \hat{r} = \frac{\vec{r}_o - \vec{r}_s}{|\vec{r}_o - \vec{r}_s|} = \frac{\langle 0,0,0\rangle - \langle R\cos\theta, R\sin\theta, 0\rangle}{R} = \langle -\cos\theta, -\sin\theta, 0\rangle$$

$$\vec{E}_0 = \int dE = -\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \frac{1}{4\pi\varepsilon_0 R^2}\left(\frac{d\theta}{\pi}\right)Q\langle -\cos\theta, -\sin\theta, 0\rangle = \frac{Q}{4\pi^2\varepsilon_0 R^2}\left\langle \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \cos\theta\, d\theta, \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \sin\theta\, d\theta, 0\right\rangle$$

$$\vec{E}_0 = \frac{Q}{4\pi^2\varepsilon_0 R^2}\left\langle \sin\theta\Big|_{\frac{\pi}{2}}^{\frac{3\pi}{2}}, -\cos\theta\Big|_{\frac{\pi}{2}}^{\frac{3\pi}{2}}, 0\right\rangle$$

$$\vec{E}_0 = \left\langle \frac{-2Q}{4\pi^2\varepsilon_0 R^2}, 0, 0\right\rangle$$

b. Suppose that a Cu^{+2} ion with mass $m_{Cu^{+2}} = 1.055\times 10^{-25}\,kg$ and charge $q_{Cu^{+2}} = 2e$ were placed at the origin. If the Cu^{+2} ion were released from rest, what initial acceleration would the Cu^{+2} experience?

The force is given by Newton's second law.

$$\vec{F}_{Cu^{+2}} = m_{Cu^{+2}}\vec{a}_{Cu^{+2}} \rightarrow \vec{a}_{Cu^{+2}} = \frac{\vec{F}_{Cu^{+2}}}{m_{Cu^{+2}}} = \left(\frac{2e}{m_{Cu^{+2}}}\right)\vec{E}_0 = \left\langle \frac{-2e2Q}{m4\pi^2\varepsilon_0 R^2}, 0, 0\right\rangle$$

$$\vec{a}_{Cu^{+2}} = \left\langle \frac{-eQ}{m\pi^2\varepsilon_0 R^2}, 0, 0\right\rangle = \left\langle \frac{1.6\times 10^{-19}C \times \left(-0.3\times 10^{-9}C\right)}{\pi^2 \times 1.055\times 10^{-25}kg \times 8.85\times 10^{-12}\frac{C^2}{Nm^2} \times (0.1m)^2}, 0, 0\right\rangle$$

$$\vec{a}_{Cu^{+2}} = \langle -5.21, 0, 0\rangle \times 10^8 \tfrac{m}{s^2}$$

Physics 121 Equation Sheet

Electric Forces, Fields and Potentials

$$\vec{F} = k\frac{Q_1Q_2}{r^2}\hat{r}; \quad \hat{r} = \frac{\vec{r}_o - \vec{r}_s}{\left|\vec{r}_o - \vec{r}_s\right|}$$

$$\vec{E} = \frac{\vec{F}}{q}$$

$$\vec{E}_Q = k\frac{Q}{r^2}\hat{r}$$

$$\vec{p} = q\vec{s} = \alpha\vec{E}$$

$$\left|\vec{E}_{\parallel}\right| = \frac{2kqs}{r^3}; \text{ dipole } r >> s$$

$$\left|\vec{E}_{\perp}\right| = \frac{kqs}{r^3}; \text{ dipole } r >> s$$

$$\left|\vec{E}_{rod}\right| = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{r\sqrt{r^2 + \left(L/2\right)^2}}\right]; \quad \left|\vec{E}_{rod}\right| \sim \frac{1}{4\pi\varepsilon_0}\left(\frac{2Q}{rL}\right) \quad L >> r$$

$$\left|\vec{E}_{ring}\right| = \frac{1}{4\pi\varepsilon_0}\left[\frac{Qz}{\left(R^2 + z^2\right)^{\frac{3}{2}}}\right]$$

$$\left|\vec{E}_{disk}\right| = \frac{Q}{2\pi\varepsilon_0 R^2}\left[1 - \frac{z}{\sqrt{R^2 + z^2}}\right]; \quad \left|\vec{E}_{disk}\right| \sim \frac{Q}{2\varepsilon_0 A}\left[1 - \frac{z}{R}\right] \quad z << R; \quad \left|\vec{E}_{disk}\right| \sim \frac{Q}{2\varepsilon_0 A} \quad z << R$$

$$\left|\vec{E}_{capacitor}\right| \sim \frac{Q}{\varepsilon_0 A}; \quad \left|\vec{E}_{fringe}\right| \sim \frac{Q}{2\varepsilon_0 A}\left(\frac{s}{R}\right)$$

Constants

$g = 9.8\,\frac{m}{s^2}$

$1e = 1.6\times10^{-19}C$

$k = \frac{1}{4\pi\varepsilon_o} = 9\times10^{9}\,\frac{Nm^2}{C^2}$

$\varepsilon_o = 8.85\times10^{-12}\,\frac{C^2}{Nm^2}$

$m_e = 9.11\times10^{-31}kg = \frac{0.511MeV}{c^2}$

$m_p = 1.67\times10^{-27}kg = \frac{937.1MeV}{c^2}$

$m_n = 1.69\times10^{-27}kg = \frac{948.3MeV}{c^2}$

$1amu = 1.66\times10^{-27}kg = \frac{931.5MeV}{c^2}$

$N_A = 6.02\times10^{23}$

$Ax^2 + Bx + C = 0 \rightarrow x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

$1eV = 1.6\times10^{-19}J$

$\mu_o = 4\pi\times10^{-7}\,\frac{Tm}{A}$

$c = 3\times10^{8}\,\frac{m}{s}$

$h = 6.63\times10^{-34}Js$