Name_____

Physics 121 Quiz #4, October 14, 2022

Please show all work, thoughts and/or reasoning in order to receive partial credit. The quiz is worth 10 points total.

I affirm that I have carried out my academic endeavors with full academic honesty.

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A spherical capacitor is made of two concentric metal spherical plates of inner radius $r_{inner} = 10cm$ while the outer plate has a radius of $r_{outer} = 80cm$. The space between the spheres is filled with Teflon of dielectric constant $\kappa = 2.1$. The capacitor is connected to a $100V$ battery and charges instantaneously.

1. What is the capacitance of this spherical capacitor?

$$C = \frac{4\pi\kappa\varepsilon_0 R_{outer}R_{inner}}{R_{outer} - R_{inner}} = \frac{4\pi \times 2.1 \times 8.85 \times 10^{-12}\frac{C^2}{Nm^2} \times 0.8m \times 0.1m}{0.8m - 0.1m} = 2.7 \times 10^{-11}F$$

2. Suppose that the fully charged spherical capacitor is connected to a $R = 10k\Omega = 10000\Omega$ resistor and allowed to discharge. After three time-constants (3τ), what percent of the initial stored charge has been lost from the plates of this capacitor?

   $Q(t) = Q_{max}e^{-\frac{t}{RC}} \rightarrow Q(t = 3RC) = Q_{max}e^{-\frac{3RC}{RC}} = Q_{max}e^{-3} = 0.05Q_{max}$

   Thus 5% of the maximum charge, Q_{max}, remains on the capacitor, so 95% of Q_{max} must have been lost.

3. What is the magnitude of the current at a time $t = 3\tau$, where τ is the time constant of the circuit?

$$I = \frac{dQ}{dt} = \frac{d}{dt}\left(Q_{max}e^{-\frac{t}{RC}}\right) = \frac{Q_{max}}{RC}e^{-\frac{t}{RC}} = \frac{CV_{max}}{RC}e^{-\frac{t}{RC}} = \frac{V_{max}}{R}e^{-\frac{t}{RC}}$$

$$I = \frac{100V}{10000\Omega}e^{-\frac{3RC}{RC}} = 0.01A \times e^{-3} = 5 \times 10^{-4}A$$

4. Suppose that the wires connecting the resistor to the capacitor were made of gold, with a diameter of $1mm$. How many charge carriers are there per unit volume are there if gold donates one free charge carrier per atom? Gold is known to have Gold is known to have a density of $19300\frac{kg}{m^3}$ and a molecular mass $197\frac{g}{mol}$.

$$n = \frac{\rho_{Au}}{m_{Au}} \times N_A = \frac{19300\frac{kg}{m^3}}{0.197\frac{kg}{mol}} \times 6.02 \times 10^{23} = 5.9 \times 10^{28}m^{-3}$$

5. What is the drift velocity of the charge carriers at a time $t = 3\tau$, where τ is the time constant of the circuit?

$$I = neAv_d \rightarrow v_d = \frac{I}{neA} = \frac{5 \times 10^{-4}A}{5.9 \times 10^{28}m^{-3} \times 1.6 \times 10^{-19}C \times \pi(0.5 \times 10^{-3}m)^2}$$

$$v_d = 6.7 \times 10^{-8}\frac{m}{s}$$