Name $\qquad$
Physics 121 Quiz \#5, February 12, 2016
Please show all work, thoughts and/or reasoning in order to receive partial credit. The quiz is worth 10 points total.

I affirm that I have carried out my academic endeavors with full academic honesty.

A thin filament light bulb is connected in series to two batteries (each rated at $V_{B}$ ) and $1.5 \times 10^{18}$ electrons per second pass through the filament.
a. If the filament has a length $L$, what is the electric field in the thin filament bulb in terms of the battery potential and the length of the filament? (Hint: Use the loop rule.)

$$
\begin{aligned}
& \Delta V_{B}+\Delta V_{B}+\Delta V_{W}=0 \\
& 2 V_{B}-E_{\text {thin }} L=0 \\
& E_{\text {thin }}=\frac{2 V_{B}}{L}
\end{aligned}
$$

b. Suppose now that you have the circuit wired below. The same two batteries (each of potential difference $V_{B}$ ) are connected in series to a thin filament bulb (the oval shaped bulb) and in parallel are two thick filament bulbs (the round circular bulbs.) Suppose that the filaments of each bulb are made out of the same material, the lengths of each filament are the same length, $L$, and that $A_{\text {thin }}=\frac{A_{\text {thick }}}{2}$. What is the relationship between the electric field in the thin bulb $\left(E_{\text {thin }}\right)$ and the electric field in the thick bulb $\left(E_{\text {thick }}\right)$ ? (Hint: Use the node rule and note that $E_{\text {thin }}$ is not the same as what you calculated in part a.)
$i_{\text {thin }}=i_{\text {thick }}+i_{\text {thick }}=2 i_{\text {thick }}$
$n A_{\text {thin }} \mu E_{\text {thin }}=2 n A_{\text {thick }} \mu E_{\text {thick }}$
$E_{\text {thin }}=2\left(\frac{A_{\text {thick }}}{A_{\text {thin }}}\right) E_{\text {thick }}=2\left(\frac{A_{\text {thick }}}{\frac{1}{2} A_{\text {thick }}}\right) E_{\text {thick }}=4 E_{\text {thick }}$

$E_{\text {thin }}=4 E_{\text {thick }}$
c. Applying the loop rule to the circuit shown, determine expressions for $E_{\text {thin }}$ and $E_{\text {thick }}$ in terms of the potential difference of the batteries $V_{B}$ and the length $L$ of the bulb filaments.

$$
\begin{aligned}
& \Delta V_{B}+\Delta V_{B}+\Delta V_{\text {thin }}+\Delta V_{\text {thick }}=0 \\
& 2 V_{B}-E_{\text {thin }} L-E_{\text {thick }} L=0 \\
& 2 V_{B}-4 E_{\text {thick }} L-E_{\text {thick }} L=0 \\
& \therefore E_{\text {thick }}=\frac{2 V_{b}}{5 L} \quad \& E_{\text {thin }}=4\left(\frac{2 V_{b}}{5 L}\right)=\frac{8 V_{b}}{5 L}
\end{aligned}
$$

d. What is the electron current that flows in the circuit? That is, what is $i_{\text {thin }}$ for this circuit? (Hints:

The electron current is not $1.5 \times 10^{18}$ electrons per second, but related to it. There are many unknowns in the problem, namely $n, \mu, A_{\text {thin }}, L$, and $V_{B}$. You can determine a value for all of the constants by using your result from part a.)
$i_{t h i n}=n A_{\text {thin }} \mu E_{\text {thin }}=n A_{\text {thin }} \mu\left(\frac{8 V_{B}}{5 L}\right)$ and
$i_{\text {thin }}=1.5 \times 10^{18}=n A_{\text {thin }} \mu E_{\text {thin }}=n A_{\text {thin }} \mu\left(\frac{2 V_{B}}{L}\right)$
$\therefore \frac{n A_{\text {thin }} \mu V_{B}}{L}=\frac{1.5 \times 10^{18}}{2}=0.75 \times 10^{18}$
So, $i_{\text {thin }}=n A_{\text {thin }} \mu E_{\text {thin }}=\frac{8}{5}\left(\frac{n A_{\text {thin }} \mu V_{B}}{L}\right)=\frac{8}{5}\left(0.75 \times 10^{18} s^{-1}\right)=1.2 \times 10^{18} s^{-1}$
e. What is the current $I$ produced by the battery?
$I=|e| i=1.6 \times 10^{-19} C \times 1.2 \times 10^{18} s^{-1}=0.192 A$

## Physics 121 Equation Sheet

Electricity \& Magnetism
$\vec{F}=k \frac{Q_{1} Q_{2}}{r^{2}} \hat{r} ; \quad \hat{r}=\frac{\vec{r}_{o}-\vec{r}_{s}}{\left|\vec{r}_{o}-\vec{r}_{s}\right|}$
$\vec{E}=\frac{\vec{F}}{q}$
$\vec{E}_{Q}=k \frac{Q}{r^{2}} \hat{r}$
$\vec{p}=q \vec{s}=\alpha \vec{E}$
$\left|\vec{E}_{\|}\right|=\frac{2 k q s}{r^{3}} ;$ dipole $r \gg s$
$\left|\vec{E}_{\perp}\right|=\frac{k q s}{r^{3}} ;$ dipole $r \gg s$
$\left|\vec{E}_{r o d}\right|=\frac{1}{4 \pi \varepsilon_{0}}\left[\frac{Q}{r \sqrt{r^{2}+(L / 2)^{2}}}\right] ;\left|\vec{E}_{r o d}\right| \sim \frac{1}{4 \pi \varepsilon_{0}}\left(\frac{2 Q}{r L}\right) \quad L \gg r$
$\left|\vec{E}_{\text {ring }}\right|=\frac{1}{4 \pi \varepsilon_{0}}\left[\frac{Q z}{\left(R^{2}+z^{2}\right)^{\frac{3}{2}}}\right]$
$\left|\vec{E}_{\text {disk }}\right|=\frac{Q}{2 \pi \varepsilon_{0} R^{2}}\left[1-\frac{z}{\sqrt{R^{2}+z^{2}}}\right] ; \quad\left|\vec{E}_{\text {disk }}\right| \sim \frac{Q}{2 \varepsilon_{0} A}\left[1-\frac{z}{R}\right] \quad z \ll R ; \quad\left|\vec{E}_{\text {disk }}\right| \sim \frac{Q}{2 \varepsilon_{0} A} \quad z \ll R$
$\left|\vec{E}_{\text {capacitor }}\right| \sim \frac{Q}{\varepsilon_{0} A} ; \quad\left|\vec{E}_{\text {fringe }}\right| \sim \frac{Q}{2 \varepsilon_{0} A}\left(\frac{s}{R}\right)$
$W=-q \Delta V=-\Delta U=\Delta K ; \quad U=\sum_{i \neq j} \frac{k Q_{i} Q_{j}}{r_{i j}} ;$
$V_{Q}=\frac{k Q}{r} ; \quad V_{Q^{\prime} s}=\sum_{i} \frac{k Q_{i}}{r_{i}}$
$\Delta V=-\int \vec{E} \cdot d \vec{r}$
$E_{x}=-\frac{\Delta V}{\Delta x} ; \quad E_{y}=-\frac{\Delta V}{\Delta y} ; \quad E_{z}=-\frac{\Delta V}{\Delta z} ; \quad \vec{E}=-\left\langle\frac{d V}{d x}, \frac{d V}{d y}, \frac{d V}{d z}\right\rangle$
$Q=\left(\frac{\varepsilon_{0} A}{s}\right) \Delta V$
$I=\frac{\Delta Q}{\Delta t}=i|e|=n|e| A v_{d} ; \quad \vec{v}_{d}=\mu \vec{E}$
$\vec{B}=\frac{\mu_{o}}{4 \pi}\left(\frac{q \vec{v} \times \hat{r}}{r^{2}}\right)$
$d \vec{B}=\frac{\mu_{o} I}{4 \pi}\left(\frac{d \vec{l} \times \hat{r}}{r^{2}}\right)$
$\left|\vec{B}_{\text {wire }}\right|=\frac{\mu_{0} L I}{4 \pi r \sqrt{\left(\frac{L}{2}\right)^{2}+r^{2}}} ;\left|\vec{B}_{\text {wire }}\right| \approx \frac{\mu_{0} I}{2 \pi r} L \gg r$
$\left|\vec{B}_{\text {ring }}\right|=\frac{\mu_{o} I R^{2}}{2\left(z^{2}+R^{2}\right)^{\frac{3}{2}}} ;\left|\vec{B}_{\text {ring }}\right| \approx \frac{\mu_{o} I R^{2}}{2 z^{3}} \quad z \ll R$

Constants
$g=9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}$
$1 \mathrm{eV}=1.6 \times 10^{-19} \mathrm{~J}$
$1 e=1.6 \times 10^{-19} \mathrm{C} \quad \mu_{o}=4 \pi \times 10^{-7} \frac{\mathrm{~T}_{m}}{A}$
$k=\frac{1}{4 \pi \varepsilon_{o}}=9 \times 10^{9} \frac{\mathrm{Nm}}{\mathrm{c}^{2}} \quad c=3 \times 10^{8} \frac{\mathrm{~m}}{\mathrm{~s}}$
$\varepsilon_{o}=8.85 \times 10^{-12} \frac{\mathrm{C}^{2}}{N m^{2}} \quad h=6.63 \times 10^{-34} \mathrm{JS}$
$m_{e}=9.11 \times 10^{-31} \mathrm{~kg}=\frac{0.511 \mathrm{MeV}}{c^{2}}$
$m_{p}=1.67 \times 10^{-27} \mathrm{~kg}=\frac{937.1 \mathrm{MeV}}{c^{2}}$
$m_{n}=1.69 \times 10^{-27} \mathrm{~kg}=\frac{948.3 \mathrm{MeV}}{c^{2}}$
$1 \mathrm{amu}=1.66 \times 10^{-27} \mathrm{~kg}=\frac{931.5 \mathrm{MeV}}{c^{2}}$
$N_{A}=6.02 \times 10^{23}$
$A x^{2}+B x+C=0 \rightarrow x=\frac{-B \pm \sqrt{B^{2}-4 A C}}{2 A}$
$B_{\text {Earrh }, H}=2 \times 10^{-5} \mathrm{~T}$

Loop Rule: $\sum_{i} \Delta V_{i}=\sum_{i} E_{i} \Delta l_{i}=0$
Node Rule: $\sum I_{\text {in }}=\sum I_{\text {out }}$

