Name_______________________________________

Physics 121 Quiz #6, February 19, 2016

Please show all work, thoughts and/or reasoning in order to receive partial credit. The quiz is worth 10 points total.

I affirm that I have carried out my academic endeavors with full academic honesty.

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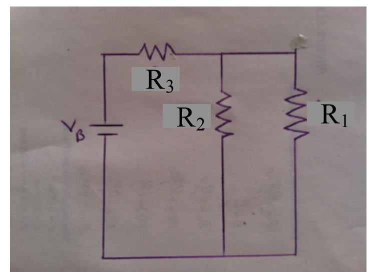
1. A $24V$ battery is connected to some resistors as shown below. What is the value of the unknown resistor, labeled R_3 if a current of $I = 0.2A$ is to be produced by the battery? Let $R_1 = 50\Omega$ and $R_2 = 100\Omega$.



Resistors R_1 and R_2 are in parallel so the equivalent resistance of this combination is $R_{12} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} = \left(\frac{3}{100\Omega}\right)^{-1} = 33.3\Omega$. Then resistors R_{12} and R_3 are in series and the equivalent resistance is $\mathrm{R}_{eq} = R_{12} + R_3 = 33.3\Omega + R_3$. Applying Ohm's law we can determine the unknown resistance. We find that

$V = IR_{eq} \rightarrow 24V = 0.2A\left(33.3\Omega + R_3\right) \Rightarrow R_3 = 86.7\Omega$.

Now suppose that resistor R_3 is wired to an air-filled parallel-plate capacitor. The capacitor has a length of $L = 60cm$ and a width of $W = 75cm$ and the plates are separated by $s = 0.5mm$. The capacitor is connected to a battery $V = 24V$, a resistor R_3 and a switch S. At time $t = 0$ the switch is closed and the capacitor begins to charge.

2. What is the capacitance of the capacitor and what is the maximum charge that can be placed on the capacitor?

$$C = \frac{\varepsilon_0 A}{d} = \frac{8.85\times10^{-12}\,\frac{C^2}{Nm^2}\times\left(0.6m\times0.75m\right)}{0.5\times10^{-3}m} = 7.97\times10^{-9}F$$
$$Q_{\max} = CV_{\max} = 7.97\times10^{-9}F\times24V = 1.91\times10^{-7}C$$

3. At what time $t > 0$ is the potential across the capacitor equal to that across the resistor?

Method 1: $$V_C(t) = V_R(t) \rightarrow V_{\max}\left(1 - e^{-\frac{t}{RC}}\right) = V_{\max}e^{-\frac{t}{RC}} \rightarrow t = -RC\ln\left(\frac{1}{2}\right)$$
$$t = -\left(86.7\Omega\times7.97\times10^{-9}F\right)\ln\left(\tfrac{1}{2}\right) = 4.79\times10^{-7}s = 0.48\mu s$$

Method 2: $$V_R(t) = \frac{V_{\max}}{2} = V_{\max}e^{-\frac{t}{RC}} \rightarrow t = -RC\ln\left(\frac{1}{2}\right) = 0.48\mu s$$

Method 3: $$V_C(t) = \frac{V_{\max}}{2} = V_{\max}\left(1 - e^{-\frac{t}{RC}}\right) \rightarrow t = -RC\ln\left(\frac{1}{2}\right) = 0.48\mu s$$

4. How much energy has been stored in the capacitor after one time constant?

$$E(t)=\tfrac{1}{2}CV^2(t)=\tfrac{1}{2}C\left(V_{\max}\left(1-e^{-\frac{t}{RC}}\right)\right)^2=\tfrac{1}{2}CV_{\max}^2\left(1-e^{-\frac{t}{RC}}\right)^2$$

$$E(t=RC)=\tfrac{1}{2}\times 7.97\times 10^{-9}F\times(24V)^2\left(1-e^{-\frac{RC}{RC}}\right)^2=\left(1-e^{-1}\right)^2 5.5\times 10^{-5}J$$

$$E(t=RC)=0.632\times 2.295\times 10^{-6}J=1.45\times 10^{-6}J=1.45\mu J$$

5. The table below gives four sets of values for different resistor-capacitor circuits. Assume that at time $t=0$ all capacitors are fully charged. Which circuit would take the least amount of time for the potential to decrease to one-half of its initial amount?

a. Circuit #1
b. Circuit #2
c. Circuit #3
d. Circuit #4
e. There is not enough information available to answer the question.

Circuit #	1	2	3	4
$V(V)$	12	12	10	10
$R(\Omega)$	2	3	10	5
$C(\mu F)$	3	2	0.5	2
$\tau = RC(\times 10^{-6}s)$	6	6	5	10

Physics 121 Equation Sheet

Electricity & Magnetism

$$\vec{F} = k\frac{Q_1Q_2}{r^2}\hat{r}; \quad \hat{r} = \frac{\vec{r}_o - \vec{r}_s}{|\vec{r}_o - \vec{r}_s|}$$

$$\vec{E} = \frac{\vec{F}}{q}$$

$$\vec{E}_Q = k\frac{Q}{r^2}\hat{r}$$

$$\vec{p} = q\vec{s} = \alpha\vec{E}$$

$$\left|\vec{E}_{\parallel}\right| = \frac{2kqs}{r^3}; \text{ dipole } r >> s$$

$$\left|\vec{E}_{\perp}\right| = \frac{kqs}{r^3}; \text{ dipole } r >> s$$

$$\left|\vec{E}_{rod}\right| = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{r\sqrt{r^2+\left(L/2\right)^2}}\right]; \quad \left|\vec{E}_{rod}\right| \sim \frac{1}{4\pi\varepsilon_0}\left(\frac{2Q}{rL}\right) \quad L >> r$$

$$\left|\vec{E}_{ring}\right| = \frac{1}{4\pi\varepsilon_0}\left[\frac{Qz}{\left(R^2+z^2\right)^{\frac{3}{2}}}\right]$$

$$\left|\vec{E}_{disk}\right| = \frac{Q}{2\pi\varepsilon_0 R^2}\left[1-\frac{z}{\sqrt{R^2+z^2}}\right]; \quad \left|\vec{E}_{disk}\right| \sim \frac{Q}{2\varepsilon_0 A}\left[1-\frac{z}{R}\right] \quad z << R; \quad \left|\vec{E}_{disk}\right| \sim \frac{Q}{2\varepsilon_0 A} \quad z << R$$

$$\left|\vec{E}_{capacitor}\right| \sim \frac{Q}{\varepsilon_0 A}; \quad \left|\vec{E}_{fringe}\right| \sim \frac{Q}{2\varepsilon_0 A}\left(\frac{s}{R}\right)$$

$$W = -q\Delta V = -\Delta U = \Delta K; \quad U = \sum_{i\neq j}\frac{kQ_iQ_j}{r_{ij}};$$

$$V_Q = \frac{kQ}{r}; \quad V_{Q's} = \sum_i \frac{kQ_i}{r_i}$$

$$\Delta V = -\int \vec{E}\cdot d\vec{r}$$

$$E_x = -\frac{\Delta V}{\Delta x}; \quad E_y = -\frac{\Delta V}{\Delta y}; \quad E_z = -\frac{\Delta V}{\Delta z}; \quad \vec{E} = -\left\langle \frac{dV}{dx}, \frac{dV}{dy}, \frac{dV}{dz}\right\rangle$$

$$Q = \left(\frac{\varepsilon_0 A}{s}\right)\Delta V$$

$$I = \frac{\Delta Q}{\Delta t} = i|e| = n|e|Av_d; \quad \vec{v}_d = \mu\vec{E}$$

$$\vec{B} = \frac{\mu_o}{4\pi}\left(\frac{q\vec{v}\times\hat{r}}{r^2}\right)$$

$$d\vec{B} = \frac{\mu_o I}{4\pi}\left(\frac{d\vec{l}\times\hat{r}}{r^2}\right)$$

$$\left|\vec{B}_{wire}\right| = \frac{\mu_o LI}{4\pi r\sqrt{\left(\frac{L}{2}\right)^2+r^2}}; \quad \left|\vec{B}_{wire}\right| \approx \frac{\mu_o I}{2\pi r} \quad L >> r$$

$$\left|\vec{B}_{ring}\right| = \frac{\mu_o IR^2}{2\left(z^2+R^2\right)^{\frac{3}{2}}}; \left|\vec{B}_{ring}\right| \approx \frac{\mu_o IR^2}{2z^3} \quad z << R$$

Constants

$g = 9.8\,\frac{m}{s^2}$ $\qquad$ $1eV = 1.6\times10^{-19}J$

$1e = 1.6\times10^{-19}C$ $\qquad$ $\mu_o = 4\pi\times10^{-7}\,\frac{Tm}{A}$

$k = \frac{1}{4\pi\varepsilon_o} = 9\times10^9\,\frac{Nm^2}{C^2}$ $\qquad$ $c = 3\times10^8\,\frac{m}{s}$

$\varepsilon_o = 8.85\times10^{-12}\,\frac{C^2}{Nm^2}$ $\qquad$ $h = 6.63\times10^{-34}Js$

$m_e = 9.11\times10^{-31}kg = \frac{0.511MeV}{c^2}$

$m_p = 1.67\times10^{-27}kg = \frac{937.1MeV}{c^2}$

$m_n = 1.69\times10^{-27}kg = \frac{948.3MeV}{c^2}$

$1amu = 1.66\times10^{-27}kg = \frac{931.5MeV}{c^2}$

$N_A = 6.02\times10^{23}$

$Ax^2+Bx+C=0 \rightarrow x = \frac{-B\pm\sqrt{B^2-4AC}}{2A}$

$B_{Earth,H} = 2\times10^{-5}T$

$$\text{Loop Rule: } \sum_i \Delta V_i = \sum_i E_i\Delta l_i = 0$$

$$\text{Node Rule: } \sum I_{in} = \sum I_{out}$$

$$V = IR; \qquad R = \frac{L}{A} = \frac{\rho L}{\sigma A}$$

$$\left|\vec{J}\right| = \frac{I}{A} = \sigma\left|\vec{E}\right|$$

$$\mathrm{R}_{eq,series} = \sum_{i=1}^{N} R_i$$

$$\frac{1}{\mathrm{R}_{eq,parallel}} = \sum_{i=1}^{N}\frac{1}{R_i}$$

$$P = IV = I^2R = \frac{V^2}{R}$$

$$E = \tfrac{1}{2}qV = \tfrac{1}{2}CV^2 = \frac{q^2}{2C}$$

$$Q(t) = Q_{\max}\left(1-e^{\frac{-t}{RC}}\right)$$

$$I(t) = I_{\max}e^{\frac{-t}{RC}}$$