

Physics 210
Medical Physics
Midterm Exam
Fall 2012
October 12, 2012

Name _____

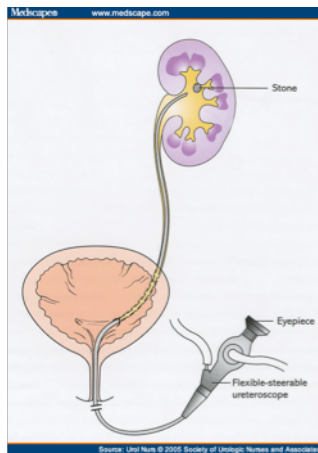
Problem 1	/32
Problem 2	/32
Problem 3	/24
Total	/88

I affirm that I have carried out my academic endeavors with full academic honesty.

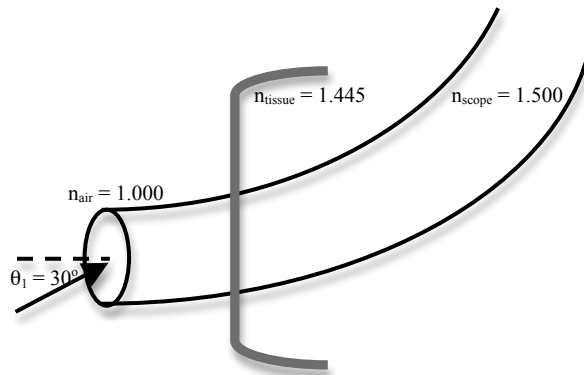
Signature

1. Optical Fibers and Lasers in Medicine - Ureteropyeloscopy

Lithotripsy is the use of high-energy shock waves to fragment and disintegrate kidney stones. This technique may be done externally using ultrasound (called *ESWL* or extracorporeal shock wave lithotripsy) or internally using a combination of optical fibers and lasers. Laser lithotripsy uses extremely high power densities to disrupt kidney stones and other painful mineralized deposits in the urinary system. High power pulsed holmium-YAG laser systems can be used in this application. A laser used for lithotripsy is transmitted into the kidneys laparoscopically using an optical fiber. The procedure *Ureteropyeloscopy* is usually performed under general anesthesia by introducing a small telescope (called a ureteroscope) through the urethra, into the bladder, and up the ureter. With direct visualization of the stone within the ureter, a laser fiber can be used to fragment the stone into smaller pieces. When laser light exits the optical fiber, the intense laser beam instantly vaporizes the water at the very tip of the optical fiber, creating a shock wave, which destroys the stone. (No focusing of the laser is needed for the high instantaneous intensities used.)



<http://www.urobriz.com.au/kidney-ureter/kidney-stones-eswl-flexible-ureteropyeloscopy-and-laser/>



- a. Suppose that the for the procedure, you use the optical fiber above was used to guide the laser light into the body through a piece of tissue. Using the information on the fiber, will the laser light be totally internally reflected? A calculation is needed to answer the question. If not, what incident angle would you need to have the laser light totally internally reflected? Is this angle a maximum or a minimum needed?

The angle of refraction for the light entering the front surface of the scope is given as $n_{air} \sin \theta = n_{pipe} \sin \theta_2 \rightarrow \theta_2 = \sin^{-1} \left(\frac{1.000}{1.500} \sin 30 \right) = 19.5^\circ$. The light here strikes the upper surface of the guide at an angle given by $90^\circ - 19.5^\circ = 70.5^\circ$. To determine if the light is totally internally reflected we need to know the critical angle on the upper surface. If 70.5° is larger than the critical angle then we have total internal reflection. The critical angle is $n_{pipe} \sin \theta_c = n_{tissue} \sin 90 \rightarrow \theta_c = \sin^{-1} \left(\frac{1.445}{1.500} \sin 90 \right) = 74.4^\circ$. Since the critical angle is larger then the angle of incidence the light will not be totally internally reflected. Thus the light incident on the exterior of the guide will have to have it's angle of incidence changed. Suppose that on the upper surface of the guide in the tissue we want the light to strike at 75° . Then the angle of refraction between the air and guide interface would be $90^\circ - 75^\circ = 15^\circ$. Then from the law of refraction

we have an angle of incidence on the front surface of $n_{air} \sin \theta = n_{pipe} \sin \theta_2 \rightarrow \theta = \sin^{-1} \left(\frac{1.500}{1.000} \sin 15 \right) = 22.8^\circ$ or smaller.

- b. To achieve this result, power densities of 10^8 watts/cm² are required. Assuming that the effective laser spot size diameter of the optical fiber is 500 microns, what must the instantaneous laser power be in order to achieve the necessary power density required to generate the shock waves needed to break up kidney stones?

$$P = IA = \left(1 \times 10^8 \frac{W}{cm^2} \right) \left(\pi \left(250 \times 10^{-6} m \times \frac{100cm}{1m} \right)^2 \right) = 19.6 \times 10^4 W$$

- c. Each pulse lasts 50 nanoseconds long and the optical fiber can carry up to 100 milliJoules without melting. How much energy is emitted per pulse and will it melt the optical fiber?

$$P = \frac{E}{t} \rightarrow E = Pt = (1.96 \times 10^5 \frac{J}{s}) (50 \times 10^{-9} s) = 0.0098 J = 9.8 mJ.$$

This is not enough energy to melt the optical fiber.

- d. Sources quote a fluence, or an energy per unit area, of 100 J/cm² being needed for this technique. How long must the laser be turned on in order to achieve this value if the pulse repetition rate is 20 Hz?

The average power is given as the energy per pulse times the pulse rate.

$$P_{avg} = \frac{\text{Energy}}{\text{pulse}} \times \text{Pulse Rate} = \frac{9.8 mJ}{1 pulse} \times \frac{20 pulses}{1 sec} = 196 \frac{mJ}{s}.$$

The average intensity is the average power divided by the area of the beam.

$$I_{avg} = \frac{P_{avg}}{A} = \frac{196 \times 10^{-36} J}{\pi \left(250 \times 10^{-6} m \times \frac{100cm}{1m} \right)^2} = 99.8 \frac{J}{s \cdot cm^2}.$$

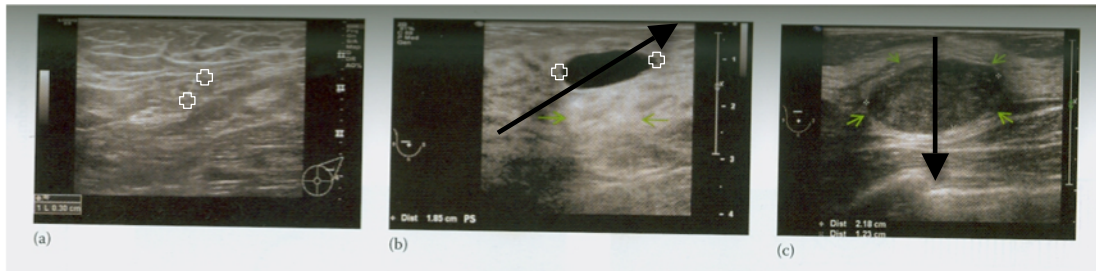
Then the fluence is the average intensity times the time needed.

$$F = I_{avg} t_{exp} \rightarrow t_{exp} = \frac{F}{I_{avg}} = \frac{100 \frac{J}{cm^2}}{99.8 \frac{J}{s \cdot cm^2}} \sim 1s$$

2. Ultrasound imaging in the breast

The following image shows ultrasound scans of a female breast. Figure A shows a normal view and the “+” signs are called *calipers* and are used for distance measurements. In figure A they are highlighting a normal lymph node. Figures B and C show structures that are of medical interest and the table below gives some materials found in the human body along with their acoustic impedances.

Medium	Speed (m/s)	Acoustic Impedance (kg/(m ² -s))
Air	343	0.000413×10^6
Amniotic fluid	1510	1.51×10^6
Blood	1570	1.61×10^6
Bone	3500	7.8×10^6
Fat	1460	1.38×10^6
Water	1480	1.48×10^6
Muscle	1,580	1.65×10^6
Cartilage	1,600	2.2×10^6
Lung	630	0.180×10^6



- a. What information do you know (if any) about the variation of the acoustic impedance along the pathway indicated by the arrows in figures B & C? What information do you know (if any) about the variation of the speed of sound along the pathway indicated by the arrows in figures B & C? What do you think the structures in figures B and C physically might be? You may indicate features using labels on the actual ultrasound scan if you would like.

The bright regions constitute regions of large changes in acoustic impedance, while the dark regions no change at all. However, you can't get the actual acoustic impedance or speed of sound from an image! The structure in B and C are physically different as shown by the changes in color. The image in B is most likely a fluid filled cyst because there are no attenuation of the sound waves as the sound wave propagates. In C it is a denser structure but more scans and studies would need to be done to determine what it is. It doesn't look as fibrous as the surrounding tissue.

- b. In figure B, what would an approximate A-mode scan look like? Assuming you are scanning along the arrow given in the picture.

The approximate A-mode scan is shown below where the intensity on the y-axis has arbitrary units.



- c. In figure B, the distance between the “+” signs is 1.85cm. If the speed of sound were taken as 1540 m/s in tissue, how long of a time delay (between echoes from the left and right side walls) would you expect from this structure if the US transducer were located at point A on the left hand side of the image.

The difference in time between the arrival of an echo off of the left surface compared to an echo off of the right surface is given as

$$v_s = \frac{2 \times \Delta d}{\Delta t} \rightarrow \Delta t = \frac{2 \times \Delta d}{v_s} = \frac{2 \times 0.0185m}{1540 \frac{m}{s}} = 2.4 \times 10^{-5} s = 24 \mu s.$$

- d. If an ultrasound wave were incident along the arrow and had an intensity of $I_{incident}$ when it struck the dark structure in figure B, what would be the reflected and transmitted intensities off of the left edge of the structure?

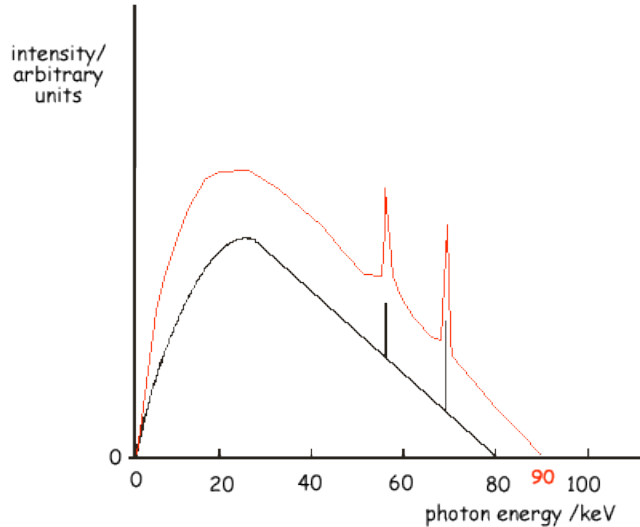
Assuming that breast tissue is made mostly out of a fat and that the cyst is water filled, the reflected intensity is

$$I_{ref} = \left(\frac{z_1 - z_2}{z_1 + z_2} \right)^2 I_{incident} = \left(\frac{1.38 - 1.48}{1.38 + 1.48} \right)^2 I_{incident} = 0.00122 I_{incident} = 0.12\% I_{incident}.$$

The transmitted intensity is the difference between what was incident and what was reflected. Thus about 99% is transmitted. This is due to fat and water having acoustic impedances almost identical, so there is no sharp echo produced and most of the sound continues to propagate. They are very similar materials.

3. X-rays generation and imaging

- a. Given the emission spectrum from a tube used to create medical x-rays below which can be operated at two different potential differences across the tube. For the tube operated at 80kV, explain why the spectrum has spikes at specific photon energies. For the tube operated at 90kV, explain the features of the curve being sure to address Bremsstrahlung and characteristic x-ray production.



The background spectra in both curves are due to Bremsstrahlung radiation as the electrons are decelerated in the anode material. The heights of the curves are indicative of the operating potential of the tube. The higher the operating potential of the tube, the more electron current and the more characteristic photons and background radiation that are produced. The spikes are the characteristic photons that are produced by the target material. The lower energy spike is most likely a K_{α} transition and the higher energy spike is the K_{β} transition for characteristic x-ray production in the anode material.

- b. Suppose that these x-rays are ultimately used to create a radiograph of a part of the body. One can try to enhance the image created on the film emulsion by using phosphors in the emulsion layer. Phosphors used in diagnostic x-ray imaging enhance the ability to film to detect x-rays by first converting the x-rays into photons of visible light. This is because the film is a better absorber of visible light photons than x-rays. How many photons of visible light will result if a 70keV x-ray photon (the right most spike) is converted into green visible light with a 10% efficiency.

The photon energy for a green photon, assuming that its wavelength is 500nm is

$$E = \frac{hc}{\lambda} = \left(\frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s} \times 3 \times 10^8 \frac{\text{m}}{\text{s}}}{500 \times 10^{-9} \text{ m}} \right) \times \frac{1 \text{ eV}}{1.6 \times 10^{-19} \text{ J}} = 2.5 \text{ eV} .$$

x-ray this corresponds to $70 \times 10^3 \text{ eV} \times \frac{1 \text{ green photon}}{2.5 \text{ eV}} = 28000 \text{ photons} .$ But the process is

only 10% efficient which means that only 10% of these will get converted into a usable photon for imaging, or ~ 2800 photons will be produced.

- c. Below is an x-ray radiograph of a normal left human *humerus* (the upper bone in the arm.) If x-rays were incident from the right with an initial intensity of I_0 , what fraction would emerge from the left side of the arm (heading towards the body along the arrow)? The thicknesses of the arm are given in the table below along with the attenuation coefficients for various tissues.



Distances (in cm) from right to left	attenuation coefficients (cm^{-1})
skin to bone – 2.5	air: 3.7×10^{-5}
bone – 2.5	fat: 0.17
bone to skin – 5.0	bone: 0.50

The intensity out of the left side of the arm is given as

$$\begin{aligned}
 I_{out} &= I_{in} e^{-\mu_{fat,1} x_{fat,1} - \mu_{bone} x_{bone} - \mu_{fat,2} x_{fat,2}} \\
 &= I_{in} e^{-(0.17 \text{ cm}^{-1} \times 2.5 \text{ cm}) - (0.50 \text{ cm}^{-1} \times 2.5 \text{ cm}) - (0.17 \text{ cm}^{-1} \times 5.0 \text{ cm})} \\
 &= 0.08 I_{in} = 8\% I_{in}
 \end{aligned}$$