Physics 220

Exam #1

April 28, 2014

Name___________________________________

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example $|\vec{p}| \approx m|\vec{v}| = (5kg)\times(2\,\tfrac{m}{s}) = 10\,\tfrac{kg\cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- All multiple choice questions are worth 4 points and each free-response part is worth 8 points

Problem #1	/28
Problem #2	/20
Problem #3	/20
Problem #4	/20
Total	/88

I affirm that I have carried out my academic endeavors with full academic honesty.

____

1. Suppose that the Earth was formed as molten rock at some very high temperature.

   a. About how much time would it take the Earth to cool to say $300K$? The radius of the Earth is $6.4\times10^{6}m$ and the average kinetic energy of a particle in the Earth is given by $E=\frac{3}{2}k_BT$.

   The total intensity is given by the Stefan-Boltzmann law, and have $S=-\frac{dE}{A_{Earth}dt}=\sigma T^4$ where the negative sign is due to the fact the Earth is losing energy as time increases. The energy is given by $E=\frac{3}{2}NkT$, so $dE=\frac{3}{2}NkdT$.

   Substituting what we have, we get

$$S=-\frac{dE}{A_{Earth}dt}=\sigma T^4\rightarrow-\frac{\frac{3}{2}NkdT}{4\pi r_{Earth}^2dt}=\sigma T^4\rightarrow\int_{\infty}^{T_F}\frac{dT}{T^4}=-\frac{8\pi r_{EArth}^2\sigma}{3Nk}\int_0^t dt$$

$$\int_{\infty}^{T_F}\frac{dT}{T^4}=-\frac{1}{3}T^{-3}\Big|_{\infty}^{T_f}=-\frac{1}{3T_f^3}$$

$$\therefore\frac{1}{3T_f^3}=\frac{8\pi r_{Earth}^2\sigma}{3Nk}t$$

$$t=\frac{Nk}{8\pi r_{Earth}^2\sigma T_f^3}=\frac{\left(\frac{6\times10^{24}kg}{1.67\times10^{-27}kg}\right)\times1.38\times10^{-23}\frac{J}{K}}{8\pi\left(6.4\times10^{6}m\right)^2\left(300K\right)^3\times5.67\times10^{-8}\frac{W}{m^2K^4}}=3.1\times10^{13}s=9.9\times10^{5}yrs$$

   b. Is this time consistent with what we know for the age of the earth ($\sim4.5\times10^{9}yr$)? If this time is not consistent with the age of the earth, what could explain the difference?

   No, this is not consistent with the accepted age of the earth. This is due to radioactive elements continually heating the earth as it cools, so the time is much longer.

c. Speaking of the Earth, there are some crazy things that go on. Suppose that, for some strange reason, a naked person at the South Pole gets out of a sauna (hot tub) with a body temperature of $311K$. The person encounters the outside air, which is at a temperature very much below the freezing point of water. How much energy per second does the person lose by radiation cooling if their surface area is $1m^2$?

From the Stefan-Boltzmann law we have

$$S = \frac{E}{At} = \sigma T^4 \rightarrow \frac{E}{t} = A\sigma T^4 = 1m^2 \times 5.67\times10^{-8}\,\tfrac{W}{m^2T^4} \times (311K)^4 = 530W\ .$$

d. By approximately what factor would this person's loss of energy to radiation cooling differ from the previous case if the person now were to step out of the sauna into a cool room with a temperature of $290K$?

From the Stefan-Boltzmann law we have

$$S = \frac{E}{At} = \sigma\left[T_{you}^4 - T_{room}^4\right]$$

$$\frac{E}{t} = A\sigma\left[T_{you}^4 - T_{room}^4\right] = 1m^2 \times 5.67\times10^{-8}\,\tfrac{W}{m^2T^4} \times \left[(311K)^4 - (290K)^4\right] = 130W\ .$$

Therefore the factor is about four times less.

2. Consider an x-ray beam, with $\lambda_x = 0.1nm$, and also a γ-ray beam from a radioactive ^{137}Cs source, with $\lambda_g = 1.88\times 10^{-3} nm$. Suppose that the radiation scattered from free electrons is viewed at 90^0 to the incident beam.

   a. What are the energies of the scattered x- and γ-rays and the speed of the scattered electron in each case?

   The energy of the scattered photon is given by:

   $$\lambda' = \lambda + \frac{h}{mc}(1-\cos\phi) \rightarrow \frac{\lambda'}{hc} = \frac{\lambda'}{hc} + \frac{(1-\cos\phi)}{mc^2} \rightarrow \frac{1}{E'} = \frac{1}{E} + \frac{(1-\cos\phi)}{mc^2}$$

   , which is

   $$E' = \frac{Emc^2}{mc^2 + E(1-\cos\phi)}$$

   the answer to part c. At 90^0 this reduces to $E' = \frac{Emc^2}{mc^2+E}$. The rest energy of an electron is $511 keV$ and the incident energies of the x- and γ-rays are

   $$E_x = \left(\frac{hc}{\lambda}\right)\times\frac{1eV}{1.6\times 10^{-19} J} = \left(\frac{6.6\times 10^{-34} Js\times 3\times 10^{8}\,\frac{m}{s}}{0.1\times 10^{-9} m}\right)\times\frac{1eV}{1.6\times 10^{-19} J} = 12.4 keV$$

   and

   $$E_\gamma = \left(\frac{hc}{\lambda}\right)\times\frac{1eV}{1.6\times 10^{-19} J} = \left(\frac{6.6\times 10^{-34} Js\times 3\times 10^{8}\,\frac{m}{s}}{1.88\times 10^{-12} m}\right)\times\frac{1eV}{1.6\times 10^{-19} J} = 288 keV$$

   respectively.

   Therefore the energies of the scattered x- and γ-rays are

   $$E_x' = \frac{E_x mc^2}{mc^2 + E_x} = \frac{(12.4keV)(511keV)}{511keV + 12.4keV} = 12.1keV$$ and

   $$E_\gamma' = \frac{E_x mc^2}{mc^2 + E_x} = \frac{(658keV)(511keV)}{511keV + 658keV} = 288keV$$ respectively.

   The speeds of the recoiling electron are given from the relativistic kinetic energy equation. We have

   $$KE = (\gamma - 1)mc^2 \rightarrow \gamma = 1 + \frac{KE}{mc^2} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \rightarrow v = \sqrt{1 - \frac{1}{\left(1+\frac{KE}{mc^2}\right)^2}}\,c$$

   and evaluating this expression for the speed of the recoiling electron in each case we have

$$v_x = v = \sqrt{1 - \frac{1}{\left(1 + \frac{KE}{mc^2}\right)^2}}\,c = \sqrt{1 - \frac{1}{\left(1 + \frac{.294KeV}{511keV}\right)^2}}\,c = 0.034c$$ and

$$v_x = v = \sqrt{1 - \frac{1}{\left(1 + \frac{KE}{mc^2}\right)^2}}\,c = \sqrt{1 - \frac{1}{\left(1 + \frac{370KeV}{511keV}\right)^2}}\,c = 0.815c$$

b. Derive an expression for the fractional loss in energy for the x- and γ-rays and then evaluate your expression in each case.

The fractional change in energy is given by $f = \frac{E - E'}{E} = 1 - \frac{E'}{E}$ and for x-rays we have $f_x = 1 - \frac{E'}{E} = 1 - \frac{12.1keV}{12.4keV} = 2.4\%$ while for γ-rays it is $f_\gamma = 1 - \frac{E'}{E} = 1 - \frac{255keV}{658keV} = 56\%$.

c. Suppose that instead of the photon scattering at 90^0, the photon scatters at an angle of θ from a stationary proton. The energy of the scattered photon is

1. E
2. $\frac{E}{2}$
3. $\frac{E^2}{mc^2}$
4. $\frac{Emc^2}{E + (1 - \cos\phi)mc^2}$
5. $\frac{Emc^2(1 - \cos\phi)}{E + mc^2}$
6. $\frac{Emc^2}{E(1 - \cos\phi) + mc^2}$

3. A potassium plate ($\phi_K = 2.9eV$) is used in a photoelectric effect experiment.

   a. If photons of wavelength $\lambda = 400nm$ are incident on the potassium plate what is the maximum kinetic energy of the ejected electron and the potential difference across the emitter-collector required to stop these electrons from striking the collector?

      The maximum kinetic energy is given by

      $$KE = \frac{hc}{\lambda} - \phi = \left(\frac{6.6 \times 10^{-34} Js \times 3 \times 10^{8} \frac{m}{s}}{400 \times 10^{-9} m} \times \frac{1eV}{1.6 \times 10^{-19} J} \right) - 2.9eV$$

      $$KE = 3.09eV - 2.9eV = 0.19eV$$

      The stopping potential is given by $KE = eV_{stop} \rightarrow V_{stop} = \frac{KE}{e} = \frac{0.19eV}{e} = 0.19V$.

   b. We've said that using a model of light as a wave that if the intensity of the beam were not large enough, then all we would have to do is wait long enough for the electron to absorb enough energy and be ejected. (Of course there is no measurable time lag between the incident photons arrival and the ejection of an electron.) However, assuming there was a lag time, how long would we have to wait for and electron to be ejected if the potassium plate were placed $1m$ away from a light source with a power output of $1\frac{J}{s}$? Assume that the ejected photoelectron may collect its energy for a circular area of the plate whose radius is say one atomic radius $r \sim 0.1nm$ and that the light energy is spread uniformly over the potassium plate. Comment on the result that you get.

      This was a poorly written question and you needed the intensity. I thought I could do the problem another way without the intensity of the light source but just assuming that the electron had to absorb at least the minimum energy to be ejected and how long would this take. You cannot answer this without an intensity. So, I've given everyone credit for this – my mistake.

      The solution should look like with the new information and wording above:

      The power at the detector (the atom) is

      $$P_{\det} = S_{source} A_{\det} = \left(\frac{P_{source}}{4\pi r^2} \right) \pi r_{atom}^2 = \left(\frac{1\frac{J}{s}}{4\pi (1m)^2} \right) \pi \left(1 \times 10^{-10} m\right)^2 = 2.5 \times 10^{-21} \tfrac{J}{s} .$$

      Then the time is given by the energy per photon divided by the power delivered at the atom. $t = \frac{E}{P} = \frac{hc/\lambda}{P} = \frac{6.6 \times 10^{-34} Js \times 3 \times 10^{8} \frac{m}{s}}{400 \times 10^{-9} m \times 2.5 \times 10^{-21} \frac{J}{s}} = 197s \sim 3.3\min$

c. Suppose instead of the violet photons used above, that red photons from a He-Ne laser ($\lambda = 633nm$) were used in this photoelectric effect experiment. Suppose that the violet photons had intensity of S_V and that the red photons were incident with intensity $S_R = S_V$. The number of photoelectrons (N_R) produced using the beam of red photons compared to the number of photoelectrons produced using the violet beam (N_V) is

1. 0
2. identically equal to $\frac{N_V}{2}$
3. identically equal to $2N_V$
4. αN_V where $0 \le \alpha \le 1$ and $\alpha \ne \frac{1}{2}$
5. αN_V where $\alpha > 1$ and $\alpha \ne 2$

The energy associated with the red photons is below the work function of potassium. Therefore no electrons will be ejected.

4. Consider the wave function $\Psi(x,t=0)=Ae^{-Cx^2}$, where A and C are positive real constants.

   a. Normalize the wave function and determine the constant A.

   The normalization condition is
   $1=\int_{-\infty}^{\infty}\psi^*\psi\,dx=\int_{-\infty}^{\infty}A^2e^{-2Cx^2}\,dx=A^2\left(\sqrt{\frac{\pi}{2C}}\right)\rightarrow A=\left(\frac{2C}{\pi}\right)^{\frac{1}{4}}$. Apparently the expression for the integral given in the table is the answer to the problem but not the to the integral. But, if you followed the formula then that's ok and your expression is fine.

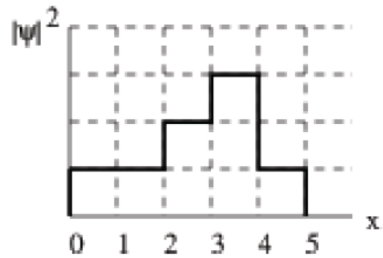
   b. Determine the expectation values of the position and the momentum operators.

   The expectation value of the position: $\langle x\rangle=\int_{-\infty}^{\infty}\psi^*x\psi\,dx=A^2\int_{-\infty}^{\infty}xe^{-2Cx^2}\,dx=0$.

   The expectation value of the momentum:
   $\langle p\rangle=\int_{-\infty}^{\infty}\psi^*\left(i\hbar\frac{d}{dx}\right)\psi\,dx=-2i\hbar CA^2\int_{-\infty}^{\infty}xe^{-2Cx^2}\,dx=0$.

   c. The quantity $|\psi|^2$ for a particle constrained to move in one dimension is shown by the graph below ($|\psi|^2=0$ for $x<0$ and $x>5$). What is the probability that the particle will be found between $x=3$ and $x=4$?

   1. $\frac{9}{64}$
   2. $\sqrt{\frac{3}{8}}$
   3. $\frac{9}{16}$
   4. $\frac{3}{16}$
   5. $\frac{3}{8}$



Physics 220 Equations

Useful Integrals:

$$\int x^n\,dx = \frac{x^{n+1}}{n+1}$$

$$\int \sin x\,dx = -\cos x$$

$$\int \cos x\,dx = \sin x$$

$$\int e^{ax}\,dx = \frac{e^x}{a}$$

$$\int_{-\infty}^{\infty} e^{ax^2}\,dx = \left(\frac{\pi}{a}\right)^{\frac{1}{2}}$$

$$\int_{-\infty}^{\infty} xe^{ax^2}\,dx = 0$$

$$\int_{-\infty}^{\infty} x^2 e^{ax^2}\,dx = \frac{\sqrt{\pi}}{2a^{\frac{3}{2}}}$$

$$\int_{-\infty}^{\infty} x^2 e^{-\frac{x}{a}}\,dx = \frac{a^3}{4}$$

Constants:

$g = 9.8\,\frac{m}{s^2}$

$G = 6.67\times10^{-11}\,\frac{Nm^2}{kg^2}$

$c = 3\times10^{8}\,\frac{m}{s}$

$\sigma = 5.67\times10^{-8}$

$k_B = 1.38\times10^{-23}\,\frac{J}{K}$

$h = 6.63\times10^{-34}\,Js;\quad \hbar = \frac{h}{2\pi}$

$m_e = 9.11\times10^{-31}\,kg = 0.511\,\frac{MeV}{c^2}$

$m_p = 1.67\times10^{-27}\,kg = 0.511\,\frac{MeV}{c^2}$

$m_n = 1.69\times10^{-27}\,kg = 0.511\,\frac{MeV}{c^2}$

$m_E = 6\times10^{24}\,kg$

$R_E = 6.4\times10^{6}\,m$

Formulas:

$c = f\lambda$

$E = hf = \frac{hc}{\lambda}$

$$\frac{dS}{d\lambda} = \frac{2\pi hc^2}{\lambda^5}\left[\frac{1}{e^{\frac{hc}{\lambda kT}}-1}\right]$$

$$\frac{dS}{d\lambda} = \frac{2\pi ckT}{\lambda^4}$$

$$\lambda_{\max} = \frac{2.9\times10^{-3}\,m\cdot K}{T}$$

$S = \sigma T^4$

$eV_{stop} = hf - \phi$

$$\lambda' = \lambda + \frac{h}{mc}(1-\cos\phi)$$

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = i\hbar\frac{\partial\psi}{dt} = E\psi$$

$$\hat{E} = i\hbar\frac{\partial}{dt}$$

$$\hat{p} = -i\hbar\frac{\partial}{dx}$$

$\hat{x} = x$

$$P = \int \psi^*\psi\,dr$$