Physics 220 Equations

Useful Integrals:

$$\int x^n\,dx = \frac{x^{n+1}}{n+1}$$

$$\int \sin x\,dx = -\cos x$$

$$\int \cos x\,dx = \sin x$$

$$\int \cos^2(qx)dx = \frac{x}{2} + \frac{\sin[2qx]}{4q}$$

$$\int \sin^2(qx)dx = \frac{x}{2} - \frac{\sin[2qx]}{4q}$$

$$\int \cos^3(qx)dx = \frac{3\sin[qx]}{4q} + \frac{\sin[3qx]}{12q}$$

$$\int \sin^3(qx)dx = -\frac{3\cos[qx]}{4q} + \frac{\cos[3qx]}{12q}$$

$$\int_{-a/2}^{a/2} x\cos^2(qx)dx = 0$$

$$\int_{-a/2}^{a/2} x\sin^2(qx)dx = 0$$

$$\int_{-a/2}^{a/2} \sin(qx)\cos(qx)dx = 0$$

$$\int e^{\pm ax}\,dx = \pm\frac{e^{\pm ax}}{a}$$

$$\int_{-\infty}^{\infty} e^{-ax^2}\,dx = \sqrt{\frac{\pi}{a}}$$

$$\int_{-\infty}^{\infty} xe^{-ax^2}\,dx = 0$$

$$\int_{-\infty}^{\infty} x^2e^{-ax^2}\,dx = \frac{\sqrt{\pi}}{2a^{\frac{3}{2}}}$$

$$2\alpha\sqrt{\tfrac{\alpha}{\pi}}\int x^2e^{-\alpha x^2}\,dx = -\sqrt{\tfrac{\alpha}{\pi}}xe^{-\alpha x^2} + \frac{\mathrm{Erf}[\sqrt{\alpha}x]}{2};\quad \mathrm{Erf}[0]\equiv 0$$

$$\int_{400nm}^{700nm} \frac{2\pi hc^2}{\lambda^5}\left[\frac{1}{e^{\frac{hc}{\lambda kT}}-1}\right]d\lambda = 1197\,\tfrac{W}{m^2}$$

Constants:

$$g = 9.8\,\tfrac{m}{s^2}$$

$$G = 6.67\times10^{-11}\,\tfrac{Nm^2}{kg^2}$$

$$c = 3\times10^8\,\tfrac{m}{s}$$

$$\sigma = 5.67\times10^{-8}$$

$$k_B = 1.38\times10^{-23}\,\tfrac{J}{K}$$

$$1eV = 1.6\times10^{-19}\,J$$

$$1e = 1.6\times10^{-19}\,C$$

$$h = 6.63\times10^{-34}\,Js;$$

$$m_e = 9.11\times10^{-31}\,kg = 0.511\,\tfrac{MeV}{c^2}$$

$$m_p = 1.67\times10^{-27}\,kg = 938\,\tfrac{MeV}{c^2}$$

$$m_n = 1.69\times10^{-27}\,kg = 939\,\tfrac{MeV}{c^2}$$

$$m_E = 6\times10^{24}\,kg$$

$$R_E = 6.4\times10^{6}\,m$$

$$a = 0.5\times10^{-10}\,m$$

Formulas :

$$c = \upsilon\lambda$$

$$E = h\upsilon = \frac{hc}{\lambda}$$

$$\frac{dS}{d\lambda} = \frac{2\pi hc^2}{\lambda^5}\left[\frac{1}{e^{\frac{hc}{\lambda kT}}-1}\right]$$

$$\frac{dS}{d\upsilon} = \frac{2\pi h\upsilon^3}{c^2}\left(\frac{1}{e^{\frac{h\upsilon}{kT}}-1}\right)$$

$$\frac{dS}{d\lambda} = \frac{2\pi ckT}{\lambda^4}$$

$$\lambda_{\max} = \frac{2.9\times10^{-3}\,m\cdot K}{T}$$

$$S = \sigma T^4$$

$$eV_{stop} = hf - \phi$$

$$\lambda' = \lambda + \frac{h}{mc}(1-\cos\phi)$$

$$\hbar = \frac{h}{2\pi};\ k = \frac{2\pi}{\lambda};\ \omega = 2\pi f$$

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = i\hbar\frac{\partial\psi}{\partial t} = E\psi$$

$$\hat{E} = i\hbar\frac{\partial}{\partial t}$$

$$\hat{p} = -i\hbar\frac{\partial}{dx}$$

$$\hat{T} = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}$$

$$\hat{H} = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V$$

$$\hat{x} = x$$

$$\langle O\rangle = \int \psi^*\hat{O}\psi\,dr$$

$$P = \int \psi^*\psi\,dx$$

$$E_n = n^2\left(\frac{\pi^2\hbar^2}{2ma^2}\right)$$

$$\Psi_n(x,t) = \sqrt{\frac{2}{a}}\sin(k_nx)e^{-i\frac{E_n}{\hbar}t}$$

$$T = \frac{k'}{k}\left|\frac{F}{A}\right|^2$$

$$T + R = 1$$

$$T = \frac{1}{1 + \frac{V_0^2}{4E(V_0 - E)}\sinh^2\left(\sqrt{\frac{8ma^2}{\hbar^2}(V_0 - E)}\right)}; \quad E < V_0$$

$$T = \frac{1}{1 + a^2k^2}; \quad E \sim V_0$$

$$T = \frac{1}{1 + \frac{V_0^2}{4E(E - V_0)}\sin^2\left(\sqrt{\frac{8ma^2}{\hbar^2}(E - V_0)}\right)}; \quad E > V_0$$

$$H_n(q) = (-1)^n e^{q^2}\left(\frac{d}{dq}\right)^n e^{-q^2}; \quad q = \sqrt{\tfrac{m\omega}{2\hbar}}x$$

$$Y_l^{m_l}(\theta,\phi) = \varepsilon\sqrt{\frac{(2l+1)}{4\pi}\frac{(l - |m_l|)!}{(l + |m_l|)!}}e^{im\phi}P_l^{m_l}(\cos\theta); \quad \varepsilon = \begin{cases} (-1)^{m_l} & m_l \geq 0 \\ 1 & m_l \leq 0 \end{cases}$$

$$P_l^{m_l}(\cos\theta) = \left(1 - \cos^2\theta\right)^{\frac{|m_l|}{2}}\left(\frac{d}{d\cos\theta}\right)^{|m_l|}P_l(\cos\theta)$$

$$P_l(\cos\theta) = \frac{1}{2^l l!}\left(\frac{d}{d\cos\theta}\right)^l\left(\cos^2\theta - 1\right)^l$$

$$L_{n-l-1}^{2l+1}\left(\tfrac{2r}{na}\right) = (-1)^{2l+1}\left(\tfrac{na}{2}\right)^{2l+1}\left(\tfrac{d}{dr}\right)^{2l+1}L_{n+2l}\left(\tfrac{2r}{na}\right)$$

$$L_{n+2l}\left(\tfrac{2r}{na}\right) = e^{\frac{2r}{na}}\left(\tfrac{na}{2}\right)^{n+2l}\left(\tfrac{d}{dr}\right)^{n+2l}\left(e^{-\frac{2r}{na}}\left(\tfrac{2r}{na}\right)^{n+2l}\right)$$

$$\left|\psi_{nlm_l}\right\rangle = \sqrt{\left(\tfrac{2}{na}\right)^3\frac{(n-l-a)!}{2n\left[(n+l)!\right]^3}}e^{-\frac{2r}{na}}\left(\tfrac{2r}{na}\right)^l\left[L_{n-l-1}^{2l+1}\left(\tfrac{2r}{na}\right)\right]Y_l^{m_l}(\theta,\phi)$$

$$a_{\pm} = \frac{1}{\sqrt{2m\hbar\omega}}(\mp ip + m\omega x)$$

$$H = \left(a_{\pm}a_{\mp} \pm \tfrac{1}{2}\right)\hbar\omega$$

$$L_{\pm} = L_x \pm iL_y$$

$$L^2 = -\hbar^2\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right]$$

$$L_z = -i\hbar\frac{\partial}{\partial\phi}$$

$$P = \int\psi^*\psi d^3r = \int_0^{2\pi}\int_0^{\pi}\int_0^{\infty}\psi^*\psi r^2\,dr\sin\theta\,d\theta d\phi$$