Physics 220 Equations

Useful Integrals:

$$\int x^n\,dx = \frac{x^{n+1}}{n+1}$$

$$\int \sin x\,dx = -\cos x$$

$$\int \cos x\,dx = \sin x$$

$$\int \cos^2(qx)dx = \frac{x}{2} + \frac{\sin[2qx]}{4q}$$

$$\int_{-a/2}^{a/2} \sin^2\left(\frac{2n\pi}{a}x\right)dx = \frac{a}{4}\left(2 - \frac{\sin(2n\pi)}{n\pi}\right)$$

$$\int_{-a/2}^{a/2} \cos^2\left(\frac{(2n-1)\pi}{a}x\right)dx = \frac{a}{2}\left(1 + \frac{\sin(2n\pi)}{\pi - 2n\pi}\right)$$

$$\int_{-a/2}^{a/2} \cos\left(\frac{(2n-1)\pi}{a}x\right)\sin\left(\frac{2n\pi}{a}x\right)dx = 0$$

$$\int \sin^2(qx)dx = \frac{x}{2} - \frac{\sin[2qx]}{4q}$$

$$\int \cos^3(qx)dx = \frac{3\sin[qx]}{4q} + \frac{\sin[3qx]}{12q}$$

$$\int \sin^3(qx)dx = -\frac{3\cos[qx]}{4q} + \frac{\cos[3qx]}{12q}$$

$$\int_{-a/2}^{a/2} x\cos^2(qx)dx = 0$$

$$\int_{-a/2}^{a/2} x\sin^2(qx)dx = 0$$

$$\int_{-a/2}^{a/2} \sin(qx)\cos(qx)dx = 0$$

$$\int e^{ax}\,dx = \frac{e^{ax}}{a}$$

$$\int_{-\infty}^{\infty} e^{-ax^2}\,dx = \sqrt{\frac{\pi}{a}}$$

$$\int_{-\infty}^{\infty} xe^{-ax^2}\,dx = 0$$

$$\int_{-\infty}^{\infty} x^2e^{-ax^2}\,dx = \frac{\sqrt{\pi}}{2a^{\frac{3}{2}}}$$

$$\int x^n e^{bx}\,dx = \frac{e^{bx}}{b}\left[x^n - \frac{nx^{n-1}}{b} + \frac{n(n-1)x^{n-2}}{b^2} - \ldots \frac{(-1)^n n!}{b^n}\right]$$

$$\int_{400nm}^{700nm} \frac{2\pi hc^2}{\lambda^5}\left[\frac{1}{e^{\frac{hc}{\lambda kT}} - 1}\right]d\lambda = 1197\,\tfrac{W}{m^2}$$

Constants:

$g = 9.8\,\frac{m}{s^2}$

$G = 6.67\times10^{-11}\,\frac{Nm^2}{kg^2}$

$c = 3\times10^{8}\,\frac{m}{s}$

$\sigma = 5.67\times10^{-8}$

$k_B = 1.38\times10^{-23}\,\frac{J}{K}$

$1eV = 1.6\times10^{-19}\,J$

$1e = 1.6\times10^{-19}\,C$

$h = 6.63\times10^{-34}\,Js;$

$m_e = 9.11\times10^{-31}\,kg = 0.511\,\frac{MeV}{c^2}$

$m_p = 1.67\times10^{-27}\,kg = 938\,\frac{MeV}{c^2}$

$m_n = 1.69\times10^{-27}\,kg = 939\,\frac{MeV}{c^2}$

$m_E = 6\times10^{24}\,kg$

$R_E = 6.4\times10^{6}\,m$

$a = 0.5\times10^{-10}\,m$

$\alpha = 1/137$

Formulas:

$$c = \upsilon\lambda$$

$$E = h\upsilon = \frac{hc}{\lambda}$$

$$\frac{dS}{d\lambda} = \frac{2\pi hc^2}{\lambda^5}\left[\frac{1}{e^{\frac{hc}{\lambda kT}} - 1}\right]$$

$$\frac{dS}{d\upsilon} = \frac{2\pi h\upsilon^3}{c^2}\left(\frac{1}{e^{\frac{h\upsilon}{kT}} - 1}\right)$$

$$\frac{dS}{d\lambda} = \frac{2\pi ckT}{\lambda^4}$$

$$\lambda_{\max} = \frac{2.9\times10^{-3}\,m\cdot K}{T}$$

$$S = \sigma T^4$$

$$eV_{stop} = hf - \phi$$

$$\lambda' = \lambda + \frac{h}{mc}(1 - \cos\phi)$$

$$\hbar = \frac{h}{2\pi};\; k = \frac{2\pi}{\lambda};\; \omega = 2\pi f$$

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = i\hbar\frac{\partial\psi}{\partial t} = E\psi$$

$$\hat{E} = i\hbar\frac{\partial}{\partial t}$$

$$\hat{p} = -i\hbar\frac{\partial}{dx}$$

$$\hat{T} = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}$$

$$\hat{H} = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V$$

$$\hat{x} = x$$

$$\langle O\rangle = \int \psi^*\hat{O}\psi\,dr$$

$$P = \int \psi^*\psi\,dx = \langle\psi|\psi\rangle$$

$$E_n = n^2\left(\frac{\pi^2\hbar^2}{2ma^2}\right)$$

$$\Psi_n(x,t) = \sqrt{\frac{2}{a}}\sin(k_n x)e^{-i\frac{E_n}{\hbar}t}$$

$$H_n(q)=(-1)^n e^{q^2}\left(\frac{d}{dq}\right)^n e^{-q^2};\quad q=\sqrt{\tfrac{m\omega}{2\hbar}}x$$

$$Y_l^{m_l}(\theta,\phi)=\varepsilon\sqrt{\frac{(2l+1)}{4\pi}\frac{(l-|m_l|)!}{(l+|m_l|)!}}e^{im\phi}P_l^{m_l}(\cos\theta);\quad \varepsilon=\begin{cases}(-1)^{m_l} & m_l\ge 0\\ 1 & m_l\le 0\end{cases}$$

$$P_l^{m_l}(\cos\theta)=\left(1-\cos^2\theta\right)^{\frac{|m_l|}{2}}\left(\frac{d}{d\cos\theta}\right)^{|m_l|}P_l(\cos\theta)$$

$$P_l(\cos\theta)=\frac{1}{2^l l!}\left(\frac{d}{d\cos\theta}\right)^l\left(\cos^2\theta-1\right)^l$$

$$L_{n-l-1}^{2l+1}\left(\tfrac{2r}{na}\right)=(-1)^{2l+1}\left(\tfrac{na}{2}\right)^{2l+1}\left(\tfrac{d}{dr}\right)^{2l+1}L_{n+2l}\left(\tfrac{2r}{na}\right)$$

$$L_{n+2l}\left(\tfrac{2r}{na}\right)=e^{\frac{2r}{na}}\left(\tfrac{na}{2}\right)^{n+2l}\left(\tfrac{d}{dr}\right)^{n+2l}\left(e^{-\frac{2r}{na}}\left(\tfrac{2r}{na}\right)^{n+2l}\right)$$

$$\left|\psi_{nlm_l}\right\rangle=\sqrt{\left(\tfrac{2}{na}\right)^3\frac{(n-l-1)!}{2n\left[(n+l)!\right]^3}}e^{-\frac{2r}{na}}\left(\tfrac{2r}{na}\right)^l\left[L_{n-l-1}^{2l+1}\left(\tfrac{2r}{na}\right)\right]Y_l^{m_l}(\theta,\phi)$$

$$a_\pm=\frac{1}{\sqrt{2m\hbar\omega}}(\mp ip+m\omega x)$$

$$H=\left(a_\pm a_\mp\pm\tfrac{1}{2}\right)\hbar\omega$$

$$L_\pm=L_x\pm iL_y$$

$$L^2=-\hbar^2\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right]$$

$$L_z=-i\hbar\frac{\partial}{\partial\phi}$$

$$L_z\left|\psi_n\right\rangle=m_l\hbar\left|\psi_n\right\rangle$$

$$L^2\left|\psi_n\right\rangle=l(l+1)\hbar^2\left|\psi_n\right\rangle$$

$$P=\int\psi^*\psi\,d^3r=\int_0^{2\pi}\int_0^{\pi}\int_0^{\infty}\psi^*\psi r^2\,dr\sin\theta\,d\theta d\phi$$

$$S_z\left|\psi_n\right\rangle=m_s\hbar\left|\psi_n\right\rangle$$

$$S^2\left|\psi_n\right\rangle=s(s+1)\hbar^2\left|\psi_n\right\rangle$$

$$S_x=\begin{bmatrix}0 & \frac{\hbar}{2}\\ \frac{\hbar}{2} & 0\end{bmatrix};S_y=\begin{bmatrix}0 & -\frac{i\hbar}{2}\\ \frac{i\hbar}{2} & 0\end{bmatrix};S_z=\begin{bmatrix}\frac{\hbar}{2} & 0\\ 0 & -\frac{\hbar}{2}\end{bmatrix}$$

$$\left|\uparrow\right\rangle=\begin{pmatrix}1\\0\end{pmatrix};\left|\downarrow\right\rangle=\begin{pmatrix}0\\1\end{pmatrix};\left|\rightarrow\right\rangle=\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix};\left|\leftarrow\right\rangle=\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix};\left|\nearrow\right\rangle=\frac{1}{\sqrt{2}}\begin{pmatrix}1\\i\end{pmatrix};\left|\swarrow\right\rangle=\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-i\end{pmatrix};$$

$$J_z\left|\psi_n\right\rangle=m_j\hbar\left|\psi_n\right\rangle$$

$$J^2\left|\psi_n\right\rangle=j(j+1)\hbar^2\left|\psi_n\right\rangle$$

$$E_{n,j}=-13.6eV\left[1+\frac{\alpha^2}{n^2}\left(\frac{n}{j+\frac{1}{2}}-\frac{3}{4}\right)\right]$$

$$\langle\psi_m|\psi_n\rangle=\begin{cases}0 \;\; if \;\; m\neq n\end{cases}$$