Physics 220
Homework #1
Spring 2017
Due Wednesday, 4/5/17

1. Consider the function $y = Axe^{-\frac{x}{2a}}$ over the region $0 \le x \le \infty$. Perform the following integral on Mathemtatica ($\int_0^\infty y^2 x^2\, dx$) and set the result equal to one and determine the constant A.

2. Consider the following functions $y = 4x - 2$ and $y = x^2 + 2x - 3$ over the region $-5 \le x \le 5$. Using Mathematica, plot both of these functions on the same graph over the region of $-5 \le x \le 5$. Label the axes and title the plot. Using the *FindRoot* command, what are the points of intersection? Lastly, using the *Solve* command, what are the points of intersection? How do your two results compare?

3. A particle of mass m is moving in one dimension in a potential $V(x,t)$. The wave function for the particle is $\Psi(x,t) = Axe^{-\left(\frac{\sqrt{km}}{2\hbar}\right)x^2} e^{-i\left(\frac{3}{2}\sqrt{\frac{k}{m}}\right)t}$ for $-\infty < x < \infty$, where A and k are constants.

   a. Normalize the wave function and determine the constant A.
   b. Using the normalized wave function, calculate $\langle x \rangle$, $\langle x^2 \rangle$, $\langle p \rangle$, and $\langle p^2 \rangle$.
   c. Show that V is independent of t, and determine $V(x)$.

4. Determine which of the following one-dimensional wave functions represent state of definite momentum. For each wave function that does correspond to a state of definite momentum, determine the momentum.
   a. $\psi(x) = e^{ikx}$
   b. $\psi(x) = xe^{ikx}$
   c. $\psi(x) = \sin(kx) + i\cos(kx)$
   d. $\psi(x) = e^{ikx} + e^{-ikx}$

5. Griffith's problem 1.4.

6. Griffith's problem 1.5.

7. Suppose that ψ_1 and ψ_2 are two different solutions of the time-independent Schrodinger wave equation with the same energy E.
   a. Show that $\psi_1 + \psi_2$ is also a solution with energy E.
   b. Show that $c\psi_1$ is also a solution of the Schrodinger equation with energy E.

8. A particle of mass m is moving in one dimension near the speed of light so that the relation for the kinetic energy $E = \frac{p^2}{2m}$ is no longer valid. Instead, the total energy is given by $E^2 = p^2c^2 + m^2c^4$. So, we can no longer use the Schrodinger equation. Suppose that the wave function for the particle $\Psi(x,t)$ is an eigenfunction of the energy operator ($i\hbar\frac{d}{dt}|\Psi\rangle = E|\Psi\rangle$) and an eigenfunction of the momentum operator ($-i\hbar\frac{d}{dx}|\Psi\rangle = p|\Psi\rangle$). If there is no potential energy V, derive a linear differential equation for $\Psi(x,t)$.

9. Griffith's Problem 1.15