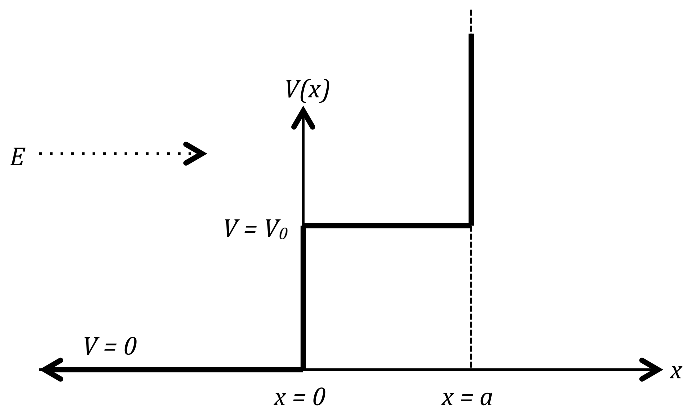
Physics 220
Homework #2
Spring 2017
Due Wednesday, 4/19/17

1. Consider reflection from a step potential of height V_0 with $E > V_0$ but now with an infinitely high wall added at a distance a from the step as shown below.
   a. What is $\psi(x)$ in each region?
   b. Show that the reflection coefficient at $x = 0$ is $R = 1$. This is different than the previously derived reflection coefficient without the infinite wall? What is the physical reason that $R = 1$ in this case?
   c. Which part of the wave function represents a left moving particle at $x \leq 0$? Show that this part of the wave function is an eigenfunction of the momentum operator and calculate the eigenvalue. Is the total wave function for $x \leq 0$ an eigenfunction of the momentum operator?



2. Griffith's 2.35
3. Griffith's 2.52
4. Griffith's 2.53
5. Starting with the expression for the transmission coefficient for the case of $E > V_0$, show that
   a. in the limit that $E \sim V_0$, the transmission coefficient can be written as
   $$T \sim \frac{1}{1 + a^2 k^2}.$$
   b. in the limit that $E < V_0$, the transmission coefficient can be written as
   $$T = \frac{1}{1 + \frac{V_0^2}{4E(V_0 - E)} \sinh^2\left(\sqrt{\frac{8ma^2}{\hbar^2}(V_0 - E)}\right)}.$$ Hint, you will need to use the fact that $\sinh(x) = -i\sin(ix)$.

6. Suppose that you have a potential barrier of height $V_0 = 40eV$ and that a beam of electrons is incident on the barrier. At what incident energies greater than the barrier height will there be no reflected particles? That is, at what incident energies grater than the barrier height will $T = 1$? Assume that the barrier has a width of $a = 2.3\times10^{-10}m$ and determine the first 5 energies.

7. Fusion reactions are important in solar energy production and this process involves the capture of a proton by a carbon nucleus of radius about $2\times10^{-15}m$.
   a. What is the Coulomb potential experienced by the proton if it is at the nuclear surface of carbon? Express your answer in MeV.
   b. The proton is incident upon the nucleus because of its thermal motion, which is given approximately as $E \sim 10kT$, where the temperature in the interior of the sun is about $T \sim 1\times10^{7}K$. How does this energy compare to the height of the Coulomb barrier?
   c. Calculate the probability that the proton can penetrate a rectangular barrier potential of height V extending from r to $2r$, the point at which the Coulomb barrier potential drops to $\frac{V}{2}$. Hint: When we derived the transmission coefficient our barrier had a width of $2a$. In this problem the width is only a. You need to change the transmission coefficient appropriately to take this into account. You do not need to redo the analysis of the finite barrier if you're cleaver and think about it.