Name____

Physics 110 Quiz #3, April 26, 2013

Please show all work, thoughts and/or reasoning in order to receive partial credit. The quiz is worth 10 points total.

I affirm that I have carried out my academic endeavors with full academic honesty.

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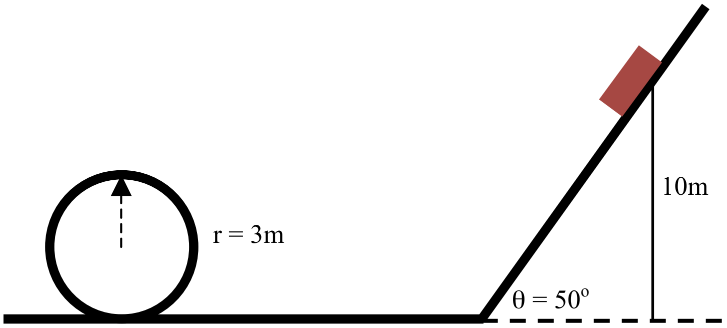
A thrill is constructed out of a $500kg$ cart that can hold six $70kg$ passengers (for a total mass of $920kg$) that is connected to a cable and the cable is connected to a wall as shown in the figure below. There is friction between the cart and riders and the ramp and the reminder of the track is frictionless. (It can't really be frictionless but we'll assume it is.)



1. What is the magnitude of the tension force in the cable?

$$\sum F_x: \quad F_w \sin\theta - F_T = ma_x = 0$$

$$\rightarrow F_T = F_w \sin\theta = mg\sin\theta = 920kg \times 9.8\tfrac{m}{s^2} \times \sin 50 = 6907N$$

2. When the cable is released from the cart, the riders and card start from rest $10m$ above the ground. If there is friction between the cart and the ramp with coefficient $\mu_k = 0.4$, what is the acceleration and how fast are the cart and riders going at the bottom of the ramp?

$$\sum F_y: \quad F_N - F_w \cos\theta - F_T = ma_y = 0$$

$$\rightarrow F_N = F_w \cos\theta = mg\cos\theta = 920kg \times 9.8\tfrac{m}{s^2} \times \cos 50 = 5795N$$

$$\sum F_x: \quad F_w \sin\theta - F_{fr} = ma_x$$

$$\rightarrow a = \frac{F_w \sin\theta - \mu_k F_N}{m} = \frac{mg\sin\theta - \mu_k mg\cos\theta}{m} = g(\sin\theta - \mu_k \cos\theta)$$

$$a = 9.8\tfrac{m}{s^2}(\sin 50 - 0.4 \times \cos 50) = 5\tfrac{m}{s^2}$$

The speed of the cart at the bottom of the track is given as:

$$v_f^2 = v_i^2 + 2a\Delta x \rightarrow v_f = \sqrt{2a\left(\frac{y}{\sin\theta}\right)} = \sqrt{2\times 5\tfrac{m}{s^2}\left(\frac{10m}{\sin 50}\right)} = 11.4\tfrac{m}{s}.$$

3. If the cart and riders travel at the constant speed found in *part 2* around the circular loop, will the riders and cart make it around the loop? Assume that engines attached to the cart that are not shown are used maintain the constant speed of the cart around the loop.

   The cart and riders leave the track when the normal force vanishes at the top. At the top we have: $-F_N - F_w = -m\frac{v^2}{r} \rightarrow v_{\min} = \sqrt{rg} = \sqrt{3m\times 9.8\tfrac{m}{s^2}} = 5.4\tfrac{m}{s}$. Since the speed of the cart (assumed constant all the way around) is greater than the minimum speed needed to stay on the track, the cart will make it around.

4. If the cart and riders make it around the loop, how long would it take to complete one loop? If they do not make it around, by how much would the speed of the cart and riders need to be increased so that they do make it around the loop?

   The time it takes is given from: $v = \frac{2\pi r}{T} \rightarrow T = \frac{2\pi r}{v} = \frac{2\pi\times 3m}{11.4\tfrac{m}{s}} = 1.7s$.

5. To get the ride started, the cart and riders need to be pushed up the ramp by a mechanism, which is not shown. The mechanism is oriented parallel to the ramp and pushes the cart up the ramp so that the cable can be attached. Then the mechanism is released from the cart and riders. The cart is pushed up the ramp at a rate of $1\tfrac{m}{s}$. The magnitude of the force applied to the cart by the mechanism follows the relation that

   a. $F_{applied} < F_w \sin\theta$.
   b. $F_{applied} = F_w \sin\theta$.
   [✓ c.] $F_{applied} > F_w \sin\theta$. (because of the frictional force that opposes the motion up the ramp.)
   d. There is not enough information given in order to solve this problem

Useful formulas:

Motion in the r = x, y or z-directions

$$r_f = r_0 + v_{0r}t + \tfrac{1}{2}a_r t^2$$

$$v_{fr} = v_{0r} + a_r t$$

$$v_{fr}{}^2 = v_{0r}{}^2 + 2a_r \Delta r$$

Uniform Circular Motion

$$a_r = \frac{v^2}{r}$$

$$F_r = ma_r = m\frac{v^2}{r}$$

$$v = \frac{2\pi r}{T}$$

$$F_G = G\frac{m_1 m_2}{r^2}$$

Geometry /Algebra

Circles	*Triangles*	*Spheres*
$C = 2\pi r$	$A = \tfrac{1}{2}bh$	$A = 4\pi r^2$
$A = \pi r^2$		$V = \tfrac{4}{3}\pi r^3$

Quadratic equation : $ax^2 + bx + c = 0$,

whose solutions are given by : $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Vectors

$$\textit{magnitude of a vector} = \sqrt{v_x{}^2 + v^2{}_y}$$

$$\textit{direction of a vector} \rightarrow \phi = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

Useful Constants

$g = 9.8\,{}^{m}\!/\!_{s^2}$ $\quad G = 6.67\times10^{-11}\,{}^{Nm^2}\!/\!_{kg^2}$

$N_A = 6.02\times10^{23}\,{}^{atoms}\!/\!_{mole}$ $\quad k_B = 1.38\times10^{-23}\,{}^{J}\!/\!_{K}$

$\sigma = 5.67\times10^{-8}\,{}^{W}\!/\!_{m^2K^4}$ $\quad v_{sound} = 343\,{}^{m}\!/\!_{s}$

Linear Momentum/Forces

$$\vec{p} = m\vec{v}$$

$$\vec{p}_f = \vec{p}_i + \vec{F}\Delta t$$

$$\vec{F} = m\vec{a}$$

$$\vec{F}_s = -k\vec{x}$$

$$F_f = \mu F_N$$

Work/Energy

$$K_t = \tfrac{1}{2}mv^2$$

$$K_r = \tfrac{1}{2}I\omega^2$$

$$U_g = mgh$$

$$U_S = \tfrac{1}{2}kx^2$$

$$W_T = FdCos\theta = \Delta E_T$$

$$W_R = \tau\theta = \Delta E_R$$

$$W_{net} = W_R + W_T = \Delta E_R + \Delta E_T$$

$$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = 0$$

$$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = -\Delta E_{diss}$$

Heat

$$T_C = \tfrac{5}{9}\left[T_F - 32\right]$$

$$T_F = \tfrac{9}{5}T_C + 32$$

$$L_{new} = L_{old}(1 + \alpha\Delta T)$$

$$A_{new} = A_{old}(1 + 2\alpha\Delta T)$$

$$V_{new} = V_{old}(1 + \beta\Delta T) : \beta = 3\alpha$$

$$PV = Nk_BT$$

$$\tfrac{3}{2}k_BT = \tfrac{1}{2}mv^2$$

$$\Delta Q = mc\Delta T$$

$$P_C = \frac{\Delta Q}{\Delta t} = \frac{kA}{L}\Delta T$$

$$P_R = \frac{\Delta Q}{\Delta T} = \varepsilon\sigma A\Delta T^4$$

$$\Delta U = \Delta Q - \Delta W$$

Rotational Motion

$$\theta_f = \theta_i + \omega_i t + \tfrac{1}{2}\alpha t^2$$

$$\omega_f = \omega_i + \alpha t$$

$$\omega^2{}_f = \omega^2{}_i + 2\alpha\Delta\theta$$

$$\tau = I\alpha = rF$$

$$L = I\omega$$

$$L_f = L_i + \tau\Delta t$$

$$\Delta s = r\Delta\theta : \ v = r\omega : \ a_t = r\alpha$$

$$a_r = r\omega^2$$

Sound

$$v = f\lambda = (331 + 0.6T)\,\tfrac{m}{s}$$

$$\beta = 10\log\frac{I}{I_0}; \quad I_o = 1\times10^{-12}\,\tfrac{W}{m^2}$$

$$f_n = nf_1 = n\frac{v}{2L}; \ f_n = nf_1 = n\frac{v}{4L}$$

Fluids

$$\rho = \frac{M}{V}$$

$$P = \frac{F}{A}$$

$$P_d = P_0 + \rho gd$$

$$F_B = \rho gV$$

$$A_1v_1 = A_2v_2$$

$$\rho_1A_1v_1 = \rho_2A_2v_2$$

$$P_1 + \tfrac{1}{2}\rho v^2{}_1 + \rho gh_1 = P_2 + \tfrac{1}{2}\rho v^2{}_2 + \rho gh_2$$

Simple Harmonic Motion/Waves

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$T_S = 2\pi\sqrt{\frac{m}{k}}$$

$$T_P = 2\pi\sqrt{\frac{l}{g}}$$

$$v = \pm\sqrt{\frac{k}{m}}A\left(1 - \frac{x^2}{A^2}\right)^{\frac{1}{2}}$$

$$x(t) = A\sin\left(\tfrac{2\pi t}{T}\right)$$

$$v(t) = A\sqrt{\frac{k}{m}}\cos\left(\tfrac{2\pi t}{T}\right)$$

$$a(t) = -A\frac{k}{m}\sin\left(\tfrac{2\pi t}{T}\right)$$

$$v = f\lambda = \sqrt{\frac{F_T}{\mu}}$$

$$f_n = nf_1 = n\frac{v}{2L}$$

$$I = 2\pi^2 f^2 \rho v A^2$$