Name____

Physics 110 Quiz #6, May 24, 2013

Please show all work, thoughts and/or reasoning in order to receive partial credit. The quiz is worth 10 points total.

I affirm that I have carried out my academic endeavors with full academic honesty.

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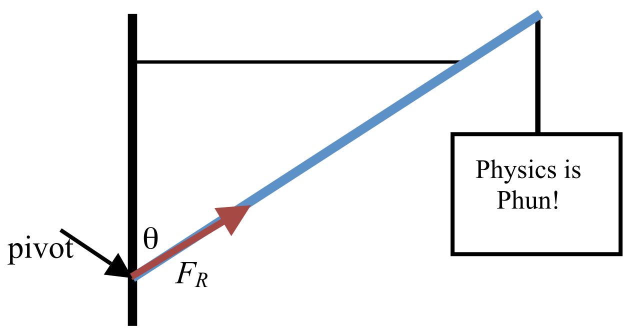
The Union College Physics Department has decided to hang a sign outside of the department office so that all passers by can read how much fun physics can be. Suppose that the sign (with a mass $m_{sign} = 220kg$) is suspended by a massless wire from a uniform blue rod ($m_{rod} = 110kg$ and $L = 2m$) and that the blue rod makes an angle of $\theta = 30^o$ with respect to the vertical as shown below. The blue rod is attached to the wall by a horizontal massless support wire. The sign acts like a point mass ($I_{sign} = m_{sign}R^2$) and the blue rod is pivoted about one end ($I_{rod} = \frac{1}{3}m_{rod}R^2$)



a. Draw the reminder of the free-body diagram, by labeling the rest of the forces. The reaction force, F_R, of the pivot is given in the diagram. Then write down Newton's 2nd law in the horizontal and vertical directions and determine the tension in the horizontal support wire.

$$\sum F_x: \quad F_R \sin\theta - F_T = 0 \rightarrow F_T = F_R \sin\theta = 3734.3N \sin 30 = 1867.2N$$

$$\sum F_y: \quad F_R \cos\theta - m_{rod}g - m_{sign}g = 0 \rightarrow F_R = \frac{(m_{rod} + m_{sign})g}{\cos\theta} = \frac{(330kg \times 9.8\,\frac{m}{s^2})}{\cos 30} = 3734.3N$$

b. Write the expression for the sum of the torques about the pivot by choosing clockwise rotations as the positive direction for torque.

$$\sum \tau: \quad m_{rod} g \frac{L}{2} \sin\theta + m_{sign} g L \sin\theta - F_T R \sin(90 - \theta) = 0$$

c. How far from the pivot is the horizontal wire attached to the blue rod?

$$m_{rod} g \frac{L}{2} \sin\theta + m_{sign} g L \sin\theta - F_T R \sin(90 - \theta) = 0$$

$$\left(110kg \times 9.8 \tfrac{m}{s^2} \times 1m \times \sin 30\right) + \left(220kg \times 9.8 \tfrac{m}{s^2} \times 2m \times \sin 30\right) - 1867.2N(R)\sin 60 = 0$$

$$R = 1.67m$$

d. Suppose that the cable snaps and the system rotates about the pivot. What is the initial angular acceleration of the system about the pivot?

$$\sum \tau: \quad m_{rod} g \frac{L}{2} \sin\theta + m_{sign} g L \sin\theta = I\alpha = \left(\tfrac{1}{3} m_{rod} L^2 + m_{sign} L^2\right)\alpha$$

$$\alpha = \frac{m_{rod} g \frac{L}{2} \sin\theta + m_{sign} g L \sin\theta}{\left(\frac{1}{3} m_{rod} L^2 + m_{sign} L^2\right)} = \frac{\left(\frac{110kg}{2} + 220kg\right) \times 9.8 \frac{m}{s^2} \times 2m \times \sin 30}{\left(\frac{110kg}{3} + 220kg\right) \times 4m^2} = 2.63 \tfrac{rad}{s^2}$$

e. If the blue rod rotates from the initial angle of $\theta_i = 30^o$ to $\theta_f = 90^o$, work is done on the system. The torque is not constant, but changes with angle, and as the blue rod rotates through the 60^oangle $3500J$ of work is done. If the system starts from rest, what is the angular velocity of the boom as it passes through the horizontal?

$$W_R = \Delta KE_R = \tfrac{1}{2} I \omega_f^2 \rightarrow \omega_f = \sqrt{\frac{2W_R}{\left(\frac{m_{rod}}{3} + m_{sign}\right)L^2}} = \sqrt{\frac{2 \times 3500J}{\left(\frac{110kg}{3} + 220kg\right)4m^2}} = 2.6 \tfrac{rad}{s}$$

Useful formulas:

Motion in the r = x, y or z-directions

$$r_f = r_0 + v_{0r}t + \tfrac{1}{2}a_r t^2$$

$$v_{fr} = v_{0r} + a_r t$$

$$v_{fr}^2 = v_{0r}^2 + 2a_r\Delta r$$

Uniform Circular Motion

$$a_r = \frac{v^2}{r}$$

$$F_r = ma_r = m\frac{v^2}{r}$$

$$v = \frac{2\pi r}{T}$$

$$F_G = G\frac{m_1 m_2}{r^2}$$

Geometry /Algebra

Circles: $C = 2\pi r$, $A = \pi r^2$

Triangles: $A = \tfrac{1}{2}bh$

Spheres: $A = 4\pi r^2$, $V = \tfrac{4}{3}\pi r^3$

Quadratic equation: $ax^2 + bx + c = 0$,

whose solutions are given by: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Vectors

$$magnitude\ of\ a\,vector = \sqrt{{v_x}^2 + {v^2}_y}$$

$$direction\,of\ a\,vector \rightarrow \phi = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

Useful Constants

$g = 9.8\,{}^{m}/_{s^2}$ $\quad G = 6.67\times10^{-11}\,{}^{Nm^2}/_{kg^2}$

$N_A = 6.02\times10^{23}\,{}^{atoms}/_{mole}$ $\quad k_B = 1.38\times10^{-23}\,{}^{J}/_{K}$

$\sigma = 5.67\times10^{-8}\,{}^{W}/_{m^2K^4}$ $\quad v_{sound} = 343\,{}^{m}/_{s}$

Linear Momentum/Forces

$$\vec{p} = m\vec{v}$$

$$\vec{p}_f = \vec{p}_i + \vec{F}\Delta t$$

$$\vec{F} = m\vec{a}$$

$$\vec{F}_s = -k\vec{x}$$

$$F_f = \mu F_N$$

Work/Energy

$$K_t = \tfrac{1}{2}mv^2$$

$$K_r = \tfrac{1}{2}I\omega^2$$

$$U_g = mgh$$

$$U_S = \tfrac{1}{2}kx^2$$

$$W_T = FdCos\theta = \Delta E_T$$

$$W_R = \tau\theta = \Delta E_R$$

$$W_{net} = W_R + W_T = \Delta E_R + \Delta E_T$$

$$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = 0$$

$$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = -\Delta E_{diss}$$

Heat

$$T_C = \tfrac{5}{9}\left[T_F - 32\right]$$

$$T_F = \tfrac{9}{5}T_C + 32$$

$$L_{new} = L_{old}\left(1 + \alpha\Delta T\right)$$

$$A_{new} = A_{old}\left(1 + 2\alpha\Delta T\right)$$

$$V_{new} = V_{old}\left(1 + \beta\Delta T\right):\ \beta = 3\alpha$$

$$PV = Nk_BT$$

$$\tfrac{3}{2}k_BT = \tfrac{1}{2}mv^2$$

$$\Delta Q = mc\Delta T$$

$$P_C = \frac{\Delta Q}{\Delta t} = \frac{kA}{L}\Delta T$$

$$P_R = \frac{\Delta Q}{\Delta T} = \varepsilon\sigma A\Delta T^4$$

$$\Delta U = \Delta Q - \Delta W$$

Rotational Motion

$$\theta_f = \theta_i + \omega_i t + \tfrac{1}{2}\alpha t^2$$

$$\omega_f = \omega_i + \alpha t$$

$$\omega^2{}_f = \omega^2{}_i + 2\alpha\Delta\theta$$

$$\tau = I\alpha = rF$$

$$L = I\omega$$

$$L_f = L_i + \tau\Delta t$$

$$\Delta s = r\Delta\theta:\ v = r\omega:\ a_t = r\alpha$$

$$a_r = r\omega^2$$

Fluids

$$\rho = \frac{M}{V}$$

$$P = \frac{F}{A}$$

$$P_d = P_0 + \rho g d$$

$$F_B = \rho g V$$

$$A_1 v_1 = A_2 v_2$$

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

$$P_1 + \tfrac{1}{2}\rho v^2{}_1 + \rho g h_1 = P_2 + \tfrac{1}{2}\rho v^2{}_2 + \rho g h_2$$

Simple Harmonic Motion/Waves

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$T_S = 2\pi\sqrt{\frac{m}{k}}$$

$$T_P = 2\pi\sqrt{\frac{l}{g}}$$

$$v = \pm\sqrt{\frac{k}{m}}A\left(1 - \frac{x^2}{A^2}\right)^{\frac{1}{2}}$$

$$x(t) = A\sin\left(\tfrac{2\pi t}{T}\right)$$

$$v(t) = A\sqrt{\frac{k}{m}}\cos\left(\tfrac{2\pi t}{T}\right)$$

$$a(t) = -A\frac{k}{m}\sin\left(\tfrac{2\pi t}{T}\right)$$

$$v = f\lambda = \sqrt{\frac{F_T}{\mu}}$$

$$f_n = nf_1 = n\frac{v}{2L}$$

$$I = 2\pi^2 f^2 \rho v A^2$$

Sound

$$v = f\lambda = (331 + 0.6T)\,\tfrac{m}{s}$$

$$\beta = 10\log\frac{I}{I_0};\quad I_o = 1\times10^{-12}\,\tfrac{W}{m^2}$$

$$f_n = nf_1 = n\frac{v}{2L};\quad f_n = nf_1 = n\frac{v}{4L}$$