Physics 123

Exam #1

October 11, 2006

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Free-Response Problems: *Please show all work in order to receive partial credit. If your solutions are illegible no credit will be given. Please use the back of the page if necessary, but number the problem you are working on. Each problem is worth 10 points.*

1. Pressure P, volume V, and temperature T are related, for a certain material, by $P = \dfrac{AT - BT^2}{V}$ where A and B are constants. What is the work done by the material if the temperature changes from T_i to T_f while the pressure remains constant?

   Since the pressure is constant, the work is given by $W = -P(V_f - V_i)$.

   The initial volume is $V_i = \dfrac{AT_i - BT_i^2}{P}$, where T_i is the initial temperature.

   The final volume is $V_f = \dfrac{AT_f - BT_f^2}{P}$, where T_f is the final temperature.

   Therefore the work done is $W = -\left[A(T_f - T_i) - B(T_f^2 - T_i^2)\right]$

2. A container encloses two ideal gases. Two moles of the first gas are present, with molar mass M_1. The second gas has molar mass $M_2 = 3M_1$, and ½ mole of this gas is present. What fraction of the total pressure on the container wall is attributable to the second gas?

   The pressure P_1 due to the first gas is $P_1 = \dfrac{n_1 RT}{V}$, and the pressure P_2 due to the second gas is $P_2 = \dfrac{n_2 RT}{V}$. Thus the total pressure on the container wall is $P = P_1 + P_2 = (n_1 + n_2)\dfrac{RT}{V}$. The fraction of the total pressure due to the second gas is then $\dfrac{P_2}{P} = \dfrac{n_2 RT}{V} \times \dfrac{1}{(n_1 + n_2)\dfrac{RT}{V}} = \dfrac{n_2}{n_1 + n_2} = \dfrac{0.5}{2 + 0.5} = \dfrac{1}{5}$.

3. A diverging lens with a focal length of magnitude 15cm and a converging lens with a focal length of magnitude 12cm have a common central axis and are separated by 12cm. An object of height 1cm is 10cm in front of the diverging lens, on the common central axis. (The ordering, from left to right is object-diverging lens-converging lens.) What are the final image properties and where is the location of the final image?

   The first image location is given by $\frac{1}{d_{o1}}+\frac{1}{d_{i1}}=\frac{1}{f_1}\rightarrow d_{i1}=\left(\frac{-1}{15cm}-\frac{1}{10cm}\right)^{-1}=-6.0cm$. The magnification of the first lens is the negative ratio of the image distance to the object distance and is *0.6*. Now the new object distance is given as *12cm + 6cm = 18cm*. Therefore the final image location is $\frac{1}{d_{o2}}+\frac{1}{d_{i2}}=\frac{1}{f_2}\rightarrow d_{i2}=\left(\frac{1}{12cm}-\frac{1}{18cm}\right)^{-1}=36cm$. This gives a magnification for the 2nd lens of *-2*. The total magnification is the product of that for lens 1 and lens 2, or *-1.2*. This means the image is *1.2cm* tall, and the final image is *real, inverted* and *enlarged*.

4. A bubble of *5.0 moles* of helium is submerged at a certain depth in liquid water when the water (and thus the helium) undergoes a temperature increase ΔT of 20^oC at *constant pressure*. As a result the monatomic and ideal gas bubble expands. How much energy is added as heat to the helium during the increase and expansion and how much work is done by the helium as it expands against the pressure of the surrounding water?

   Since the pressure is constant, the heat $Q = nC_P\Delta T$, where $C_P = C_V + R$ for an ideal gas. Therefore we have $Q=n(C_V+R)\Delta T=5mol\left(\frac{5}{2}\times 8.31\frac{J}{mol\times K}\right)(20K)=2077.5J$.

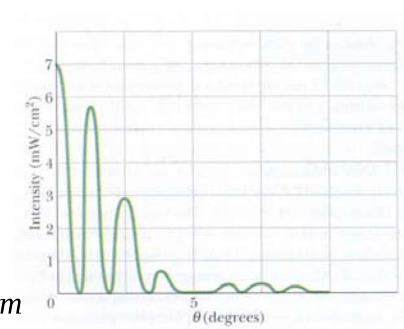
   The change in internal energy is
   $\Delta E_{\text{int}}=nC_V\Delta T=5mol\left(\frac{3}{2}\times 8.31\frac{J}{mol\times K}\right)(20K)=1246.5J$.

5. The Pyrex glass mirror in the telescope at the Mt. Palomar Observatory has a diameter of *200 inches*. The temperature ranges from *-10^{o}C* to *50^{o}C* on Mt. Palomar. Assuming that the glass can freely expand and contract, what is the maximum change in diameter of the mirror in *micrometers*? (Hints: There are *2.54cm* in *1.00 inch* and the coefficient of linear expansion for Pyrex glass is *3.2x10^{-6} /^{o}C.*)

   The change in diameter is given by $\Delta L = L\alpha\Delta T$. Therefore,

$$\Delta L = 200in \times 3.2\times10^{-6}\left({}^{o}C\right)^{-1} \times 60^{o}C = 3.84\times10^{-2}\,in \times \frac{2.54cm}{1in} \times \frac{1m}{100cm} \times \frac{1\times10^{6}\,\mu m}{1m}$$
$$= 975.4\mu m$$

6. Light of wavelength *440nm* passes through a double slit, yielding a diffraction pattern whose graph of intensity *I* versus angular position θ is shown below. What are the slit width and slit separation?



   The first diffraction minimum occurs at 5o. Thus

$$a\sin\phi_1 = \lambda \rightarrow a = \frac{440\times10^{-9}\,m}{\sin 5^o} = 5.05\times10^{-6}\,m = 5.05\mu m$$

   The number of spots is related to the ratio of d to a.
   The total number of spots is 9,

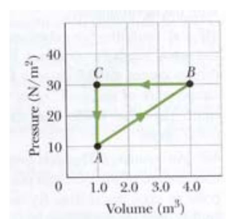
$$\text{then}\,\# = 2\left(\frac{d}{a}\right)+1 = 9 \rightarrow \frac{d}{a} = 4 \rightarrow d = 4\times5.05\mu m = 20.2\mu m$$

7. A chef, on finding the stove out of order, decides to boil the water for a customer's coffee by shaking it in a Thermos flask. Suppose that he uses tap water (*c = 4190 J/kgK*) at 15°C and that the water fall 30cm each shake and that the chef makes 30 shakes per minute. Neglecting any loss of thermal energy by the flask, how long must the chef shake the flask until the water reaches 100°C?

The mass M of water must be raised from $T_i = 15^oC$ to $T_f = 100^o$C. If c is the specific heat of water, then the energy required is $Q=Mc(Tf - Ti)$. Each shake supplies energy mgh, where h is the distance moved during the downward stroke of the shake. If N is the total number of shakes then $Nmgh = Q$. If t is the time taken to raise the water to its boiling point then $\frac{Nmgh}{t} = \frac{Q}{t}$. The number of shakes per minute is the rate, so the time t is $t = \frac{Q}{Rate \times Mgh} = \frac{Mc(T_f - T_i)}{Rate \times Mgh} = \frac{c(T_f - T_i)}{Rate \times gh} = \frac{4190 \frac{J}{kg \times K}(85K)}{30 \min^{-1} \times 9.8 \frac{m}{s^2} \times 0.30m}$

$= 3.97 \times 10^3 \min = 2.8 days$

8. Gas within a closed chamber undergoes the cycle shown below in the PV diagram. What is the net energy added to the system as heat during one complete cycle?



Over a cycle the internal energy must be the same so the heat Q absorbed equals the work done W. Over the portion of the cycle from A to B, the pressure P is a linear function of the volume V. Thus $P = \left(\frac{10}{3} \frac{N}{m^2}\right) + \left(\frac{20}{3} \frac{N}{m^3}\right) \times V$ where the coefficients were chosen so that $P = 10N/m^2$ when $V = 1m^3$ and $P = 30N/m^2$ when $V = 4m^3$. Thus the work done by the gas during this portion of the cycle is $W_{AB} = \int_1^4 PdV = \int_1^4 \left(\frac{10}{3} + \frac{20}{3}V\right)dV = \left(\frac{10}{3}V + \frac{10}{3}V^2\right)_1^4 = 60J$.

The portion BC of the cycle is at constant pressure, so the work done is $P\Delta V = -90J$. The CA portion is a t constant volume, so no work is done. Thus the total work done by the gas is $W_{net} = 60J - 90J + 0J = -30J$. The heat absorbed is $-30J$, so the gas loses 30J of heat energy.

Another, easier method, is to realize that the work done is the area under the curve, so

$W_{net} = W_{BC} - W_{AB} = \left(30 \frac{N}{m^2} \times 3m^3\right) - \left(\left(10 \frac{N}{m^2} \times 3m^3\right) + \left(\frac{1}{2} \times 20 \frac{N}{m^2} \times 3m^3\right)\right) = 90J - 60J = 30J$

and thus the gas loses 30J as heat.

9. We know that for an adiabatic process PV^{γ} = *a constant*. What is the constant for an adiabatic process involving exactly *2.0 moles* of an ideal gas passing through the state having $P = 1.0atm$ and $T = 300\ K$, if the gas is diatomic and the molecules have rotation but no oscillations?

$$PV^{\gamma} = P\left(\frac{nRT}{P}\right)^{\gamma} = P^{1-\gamma}nRT = (1.013\times10^{5}\ \tfrac{N}{m^2})^{1-\frac{7}{5}}\times\left(2mol\times8.31\tfrac{J}{mol\times K}\times300K\right)$$
$$=1.5\times10^{3}\ Nm^{2.2}$$

10. A white dwarf is the core of a dead medium sized star like the Sun. When a star like the Sun dies, it passes through several phases ending sometimes as a Red Giant. When the forces of gravity cannot hold onto the solar atmosphere, the atmosphere dissipates into space and what is left is a white hot core called a white dwarf. How long (in years) will it take for a white dwarf (at a temperature of 24,000K) to radiate away all of its energy to space (at a temperature of 2.3K) if the white dwarf has a mass of 2.2x10^{30}kg? Assume further that the white dwarf is an ideal emitter, composed entirely of carbon (with an atomic mass of 12,) and that the radius of the white dwarf is 5500 km.

The time is given by

$$t=\frac{Nk_B}{2\sigma\varepsilon A}\left(\frac{1}{T^3_{final}}-\frac{1}{T^3_{initial}}\right)=\frac{1.1\times10^{56}\times1.38\times10^{-23}\ \tfrac{J}{K}}{2\times5.67\times10^{-8}\ \tfrac{W}{m^2K^4}\times1\times3.8\times10^{14}m^2}\left(\frac{1}{(2.3K)^3}-\frac{1}{(24000K)^3}\right)$$
$$=1.26\times10^{24}\sec\times\frac{1yr}{31\times10^{6}s}=4.1\times10^{16}\ yrs$$