In keeping with the Union College policy on academic honesty, it is assumed that you will neither accept nor provide unauthorized assistance in the completion of this work.
Part I: Free Response Problems

Please show all work in order to receive partial credit. If your solutions are illegible no credit will be given. Please use the back of the page if necessary, but number the problem you are working on.

1. A concave lens \( f_1 = 7cm \) is located 35cm to the left of a convex lens \( f_2 = 14cm \) and this system is used to form an image of an object \( h_o = 1cm \) located 24cm to the left of the concave lens.

a. Where is the image of the object located with respect to the first lens and what is height of the image formed?

\[
\frac{1}{d_{o1}} + \frac{1}{d_{i1}} = \frac{1}{f_1} \rightarrow d_{i1} = \left( \frac{1}{f_1} - \frac{1}{d_{o1}} \right)^{-1} = \left( -\frac{1}{7cm} - \frac{1}{24cm} \right)^{-1} = -5.4cm \text{ or } 5.4cm \text{ to the left of the concave lens.}
\]

\[
M_1 = -\frac{d_{i1}}{d_{o1}} = -\frac{-5.4cm}{24cm} = 0.23 \rightarrow h_{i1} = M_1 h_o = 0.23 \times 1cm = 0.23cm
\]

b. Where is the final image of the object located with respect to the second lens and what is the height of the final image formed?

\[
\frac{1}{d_{o2}} + \frac{1}{d_{i2}} = \frac{1}{f_2} \rightarrow d_{i2} = \left( \frac{1}{f_2} - \frac{1}{d_{o2}} \right)^{-1} = \left( \frac{1}{14cm} - \frac{1}{35cm + 5.4cm} \right)^{-1} = 21.4cm \text{ or } 21.4cm \text{ to the right of the convex lens.}
\]

This lens magnifies the image of the object that produced it by

\[
M_2 = -\frac{d_{i2}}{d_{o2}} = -\frac{21.4}{40.4cm} = -0.53 \rightarrow h_{i2} = M_2 h_{i1} = 0.53 \times 0.23cm = 0.12cm.
\]
2. *The Physics of Rainbows:* Suppose that you are standing outside with your back to the sun just after a rainstorm and you are looking at the rain clouds off in the distance. Consider white light (white light is composed of the colors red, orange, yellow, green, blue, and violet) incident on a spherical raindrop located in the storm clouds in front of you and that the light is incident on the raindrop at an angle of 30° as shown in the figure below. Each of the colors has its own frequency of oscillation and the index of refraction of a material varies slightly with frequency. This is known as dispersion. When white light passes from air into a material with a higher refractive index, the white light gets dispersed by the material of higher refractive index into its component colors. Thus the different colors of light get bent by varying amounts in the material and we have a rainbow of color produced, in this case in the raindrop, and this color separation is maintained as the colors exit the raindrop and we see a rainbow of color in the sky. Assume that the indices of refraction for water for red and blue light are $n_{\text{red}} = 1.331$ and $n_{\text{blue}} = 1.343$ and that a typical light ray is shown below.

a. Suppose that red and blue light enter the drop at the 30° angle shown. Determine the angles of refraction for red and blue light as well as which color (red or blue) gets bent more. On the raindrop above draw and label the rays corresponding to red and blue light in relation to the orange ray shown.

\[
 n_{\text{air}} \sin 30 = n_{\text{red}} \sin \theta_{\text{red}} \rightarrow \theta_{\text{red}} = \sin^{-1}\left(\frac{0.500}{1.331}\right) = 22.1^\circ
\]

and since these angles are measured with respect to the normal, blue light bends more than red.

\[
 n_{\text{air}} \sin 30 = n_{\text{blue}} \sin \theta_{\text{blue}} \rightarrow \theta_{\text{blue}} = \sin^{-1}\left(\frac{0.500}{1.343}\right) = 21.9^\circ
\]

b. What are the critical angles ($\theta_{c,\text{red}}$ and $\theta_{c,\text{blue}}$) for red and blue light striking the back surface of the raindrop? Draw and label on the raindrop above the rays corresponding to red and blue light in relation to the orange ray shown reflecting off of the back surface. (Note: A rainbow of light is formed from many such drops with the red light emerging at the same angle from each drop and adding together to form a bright red band, with similar effects for the other colors.)

\[
 n_{\text{red}} \sin \theta_{c} = n_{\text{air}} \sin 90 \rightarrow \theta_{c,\text{red}} = \sin^{-1}\left(\frac{1.000}{1.331}\right) = 48.7^\circ
\]

\[
 n_{\text{blue}} \sin \theta_{c} = n_{\text{air}} \sin 90 \rightarrow \theta_{c,\text{blue}} = \sin^{-1}\left(\frac{1.000}{1.343}\right) = 48.1^\circ
\]
3. The Interference of light
   a. Light from a Helium-Neon (He-Ne) laser strikes a pair of slits at normal incidence, forming the double-slit interference pattern, shown below, on a screen located 1.4m from the slits. What is the slit separation if the wavelength of the laser light is $\lambda = 632.8 \text{ nm}$?

   ![Double-slit interference pattern]

   $$d \sin \theta_m = d \tan \theta_m = d \frac{y_m}{D} = m\lambda \rightarrow d = \frac{mD\lambda}{y_m} = \frac{4 \times 1.4 \times 632.8 \times 10^{-9} m}{23 \times 10^{-3} m} = 1.54 \times 10^{-4} m = 154 \mu m$$

   b. Suppose instead of the red light, that green light ($\lambda = 505 \text{ nm}$) passes through a pair of double slits producing the interference pattern on a screen a distance $D$ from the slits as shown below. When light of a different color passes through the same pair of slits the second “grey” colored pattern is observed on the screen located at the same distance $D$ away. What is the wavelength of the second color and what color was the light source used, based on the two wavelengths referenced?

   ![Double-slit interference pattern with green light]

   ![Double-slit interference pattern with grey light]

   $$Green: d \sin \theta_m = d \tan \theta_m = d \frac{y_m}{D} = (m + \frac{1}{2})\lambda_{green}$$

   $$Grey: d \sin \theta_m = d \tan \theta_m = d \frac{y_m}{D} = m\lambda_{grey}$$

   $$\therefore m\lambda_{grey} = (m + \frac{1}{2})\lambda_{green} \rightarrow \lambda_{grey} = \left(\frac{m + \frac{1}{2}}{m}\right)\lambda_{green} = \frac{4.5}{5} \times 505 \text{ nm} = 454.5 \text{ nm}$$

   and either blue or purple colored light.
4. Experiments to study vision often need to track the movements of a subject’s eye. One way of doing this is described below. A subject is placed in an externally generated magnetic field while wearing a special contact lens that has a 6.0mm diameter coil of fine built into it, encircling the edge of the lens, as shown in the picture below. As the eye rotates a current is induced in the coil.

a. If the external magnetic field were generated by a wire, not shown, with a current of 650A flowing from the floor to the ceiling and that the wire is located 0.2 m in front the patient’s eyes, what is the externally generated magnetic field that the subject is sitting in?

\[
B = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7} \text{Tm} \times 650\text{A}}{2\pi \times 0.2\text{m}} = 6.5 \times 10^{-4} \text{T}
\]

b. If one of the eye rotates 5° in the direction of the magnetic field (in 0.2 seconds), what are the direction and magnitude of the induced current in the coil of wire?

(Hints: 1) Assume that the wire is made out of copper with resistivity \(\rho = 1.72 \times 10^{-8} \Omega\text{m}\) and cross sectional area of the slice of wire shown in the diagram is 7.85 \times 10^{-7} \text{m}^2. 2) To specify the direction, assume that you are the doctor looking at the patient.)

\[
I = \frac{\varepsilon}{R} \text{ where } R = \frac{\rho l}{A} = \frac{1.72 \times 10^{-8} \Omega\text{m} \times 2\pi \times 3 \times 10^{-3} \text{m}}{7.85 \times 10^{-7} \text{m}^2} = 4.1 \times 10^{-4} \Omega
\]

The induced emf is given by Faraday’s law as

\[
\varepsilon = N \frac{\Delta \Phi_b}{\Delta t} = NBA \left(\frac{\cos \theta_f - \cos \theta_i}{t_f - t_i}\right) = 6.5 \times 10^{-4} T \times \pi \left(3.0 \times 10^{-3} \text{m}\right)^2 \left(\frac{\cos 85 - \cos 90}{0.2\text{s}}\right) = 8 \times 10^{-6} \text{V}
\]

Thus the current flows clockwise and has a magnitude of

\[
I = \frac{\varepsilon}{R} = \frac{8 \times 10^{-6} \text{V}}{4.1 \times 10^{-4} \Omega} = 1.82 \times 10^{-2} \text{A} = 18.2 \mu\text{A}
\]

c. What power will be dissipated across the loop of wire?

\[
P = I^2 R = \left(18.2 \times 10^{-6} \text{A}\right)^2 \times 4.1 \times 10^{-4} \Omega = 1.35 \times 10^{-13} \text{W}
\]
Part II: Multiple-Choice

Circle your best answer to each question. Any other marks will not be given credit. Each multiple-choice question is worth 2 points for a total of 10 points.

1. The figure below shows an arrow pointed upward located at a distance $p$ in front of a converging lens of focal length $f$. If $p > f$ as in the figure, what kind of image of the arrow is formed by the lens?
   
   a. Real and pointing down.
   b. Real and pointing up
   c. Virtual and pointing down.
   d. Virtual and pointing up.

2. A beam of light shines from air into a transparent medium having two parallel surfaces. Part of the beam is reflected from the second surface as shown below. The index of refraction of the medium is 1.5. Which of the following describes the relationship between the angles of incidence and reflection and the relationship between the angles of incidence and refraction?
   
   a. $\theta < \theta'$ and $\theta < \alpha$
   b. $\theta = \theta'$ and $\theta > \alpha$
   c. $\theta = \theta'$ and $\theta < \alpha$
   d. $\theta > \theta'$ and $\theta > \alpha$

3. Suppose that you are sitting in your laboratory one day pondering an experiment that you are running. In the laboratory with you is a spring and while you are thinking about your experiment you decided to play with the spring. Suppose that you compress the spring. When you compress the spring, the mass of the compressed spring compared to the uncompressed spring is
   
   a. greater.
   b. equal.
   c. less.
   d. zero.
4. A ray of light strikes a material whose index of refraction is \( n \), resulting in a reflected ray and a refracted ray. The index of refraction of air is 1.0. If the incident angle \( \theta \) is decreased slightly, what happens to the reflected angle, \( \theta_1 \) and the refracted angle \( \theta_2 \)?

a. Both \( \theta_1 \) and \( \theta_2 \) increase.
b. \( \theta_1 \) increases and \( \theta_2 \) decreases.
c. \( \theta_1 \) decreases and \( \theta_2 \) increases.
d. Both \( \theta_1 \) and \( \theta_2 \) decrease.

5. Suppose that an unpolarized light with intensity \( S_o \) is incident from the left onto a polarizer whose transmission axis is oriented horizontally. A second polarizer is placed to the right of the first polarizer with its transmission axis oriented at 75° to the first. The intensity of light that emerges from the second polarizer is

a. 0.50 \( S_o \).
b. 0.41 \( S_o \).
c. 0.13 \( S_o \).
d. 0.03 \( S_o \).
Electric Forces, Fields and Potentials
\[ \vec{F} = k \frac{Q_1 Q_2}{r^2} \hat{r} \]
\[ \vec{E} = \frac{\vec{F}}{q} \]
\[ \vec{E}_Q = k \frac{Q}{r^2} \hat{r} \]
\[ PE = k \frac{Q_1 Q_2}{r} \]
\[ V(r) = k \frac{Q_1}{r} \]
\[ E_x = -\frac{\Delta V}{\Delta x} \]
\[ W_{A,B} = q \Delta V_{A,B} \]

Magnetic Forces and Fields
\[ F = qvB \sin \theta \]
\[ F = lIB \sin \theta \]
\[ \tau = \mu AB \sin \theta = \mu B \sin \theta \]
\[ PE = -\mu B \cos \theta \]
\[ B = \frac{\mu_0 I}{2\pi r} \]
\[ e_{\text{induced}} = -N \frac{\Delta \phi_B}{\Delta t} = -N \Delta \frac{B(A \cos \theta)}{\Delta t} \]

Constants
\[ g = 9.8 \text{ m/s}^2 \]
\[ 1e = 1.6 \times 10^{-19} \text{ C} \]
\[ k = 4 \pi \varepsilon_0 = 9 \times 10^9 \text{ Nm}^2 \text{C}^{-2} \]
\[ \varepsilon_0 = 8.85 \times 10^{-12} \text{ F/m} \]
\[ 1eV = 1.6 \times 10^{-19} \text{ J} \]
\[ \mu_0 = 4 \pi \times 10^{-7} \text{ Tm/A} \]
\[ c = 3 \times 10^8 \text{ m/s} \]
\[ h = 6.63 \times 10^{-34} \text{ Js} \]
\[ m_e = 9.11 \times 10^{-31} \text{ kg} = 0.511 \text{ MeV} \]
\[ m_p = 1.67 \times 10^{-27} \text{ kg} = 938 \text{ MeV} \]
\[ m_\gamma = 1.69 \times 10^{-27} \text{ kg} = 919 \text{ MeV} \]
\[ \lambda_{amu} = 1.66 \times 10^{-12} \text{ cm} = 931.5 \text{ MeV} \]
\[ N_a = 6.02 \times 10^{23} \]
\[ \text{Circles} : C = 2\pi r = \pi D \quad A = \pi r^2 \]
\[ \text{Triangles} : A = \frac{1}{2} bh \]
\[ \text{Spheres} : A = 4\pi r^2 \quad V = \frac{4}{3} \pi r^3 \]

Light as a Wave
\[ c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \]
\[ S(t) = \frac{\text{energy}}{\text{time} \times \text{area}} = c \varepsilon_0 E^2(t) = c \frac{B^2(t)}{2 \mu_0} \]
\[ I = S_{\text{avg}} = \frac{1}{2} c \varepsilon_0 E_{\text{max}}^2 = c \frac{B_{\text{max}}^2}{2 \mu_0} \]
\[ P = \frac{S}{c} = \text{Force} \quad \text{Area} \]
\[ S = S_o \cos^2 \theta \]
\[ \theta_{\text{inc}} = \theta_{\text{refl}} \]
\[ n_1 \sin \theta_1 = n_2 \sin \theta_2 \]
\[ \sin \phi = m \lambda \]
\[ a \sin \phi = m \lambda \]

Nuclear Physics
\[ E_{\text{binding}} = (Z m_p + N m_n - m_{\text{rest}})^2 \]
\[ \Delta N = -\Delta N\rightarrow \Delta N = N\rightarrow N = e^{-\Delta t} \]
\[ A(t) = A_n e^{-\Delta t} \]
\[ m(t) = m_n e^{-\Delta t} \]
\[ \Delta t = \frac{\ln 2}{\lambda} \]

Misc. Physics 110 Formulae
\[ \vec{F} = \frac{\Delta \vec{p}}{\Delta t} = \frac{\Delta (mv)}{\Delta t} = m\vec{a} \]
\[ \vec{F} = -k\vec{y} \]
\[ \vec{F}_c = m\frac{\vec{v}^2}{R} \]
\[ W = \Delta KE = \frac{1}{2} m (v_f^2 - v_i^2) = -\Delta PE \]
\[ PE_{\text{gravity}} = mgy \]
\[ PE_{\text{spring}} = \frac{1}{2} ky^2 \]
\[ x_f = x_i + v_i t + \frac{1}{2} a_i t^2 \]
\[ v_{fi} = v_{ix} + a_i t \]
\[ v_{fi} = v_{ix} + 2a_i \Delta x \]