*Name*___________________________
PIXE Homework #2 - Physics 100
Union College Fall 2014

1. What are the K_α and K_β energies for the following elements: Ni(Z=28), Fe(Z=26), Au(Z=79), and Pb(Z=82)? (Please use the equations given in the lecture and not the table of x-ray energies to do the calculation.)

The x-ray energies are given by $\Delta E = -Z^2 \frac{me^4}{2(4\pi\varepsilon_0)^2 \hbar^2}\left(\frac{1}{n_l^2} - \frac{1}{n_u^2}\right)$, *where for the* α-*transition we have* $n_u = 2 \to n_l = 1$ *and for a* β-*transition* $n_u = 3 \to n_l = 1$. *Thus for a* K_α *transition (evaluating the constants) we have*

$$\Delta E_\alpha = -Z^2 \frac{me^4}{2(4\pi\varepsilon_0)^2 \hbar^2}\left(\frac{1}{n_l^2} - \frac{1}{n_u^2}\right) = -Z^2\left[\frac{9.11\times10^{-31}kg\times\left(1.6\times10^{-19}C\right)^4}{32\pi^2\left(8.85\times10^{-12}\frac{C^2}{Nm^2}\right)^2\left(\frac{6.63\times10^{-34}Js}{2\pi}\right)^2}\right]\left(\frac{1}{1}-\frac{1}{4}\right)$$

$$\Delta E_\alpha = \left(1.626\times10^{-18}J\right)Z^2 = (10.16)Z^2$$

, while for a K_β *transition*

$$\Delta E_\beta = -Z^2 \frac{me^4}{2(4\pi\varepsilon_0)^2 \hbar^2}\left(\frac{1}{n_l^2} - \frac{1}{n_u^2}\right) = -Z^2\left[\frac{9.11\times10^{-31}kg\times\left(1.6\times10^{-19}C\right)^4}{32\pi^2\left(8.85\times10^{-12}\frac{C^2}{Nm^2}\right)^2\left(\frac{6.63\times10^{-34}Js}{2\pi}\right)^2}\right]\left(\frac{1}{1}-\frac{1}{9}\right)$$

$$\Delta E_\beta = \left(1.927\times10^{-18}J\right)Z^2 = (12.04eV)Z^2$$

Element	ΔE_α (keV)	ΔE_α (J)	ΔE_β (keV)	ΔE_β (J)
Fe (26)	6.90	1.10×10^{-15}	8.1	1.30×10^{-15}
Ni (28)	7.96	1.27×10^{-15}	9.4	1.51×10^{-15}
Au (79)	63.4	1.02×10^{-14}	74.9	1.20×10^{-14}
Pb (82)	68.3	1.09×10^{-14}	80.7	1.30×10^{-14}

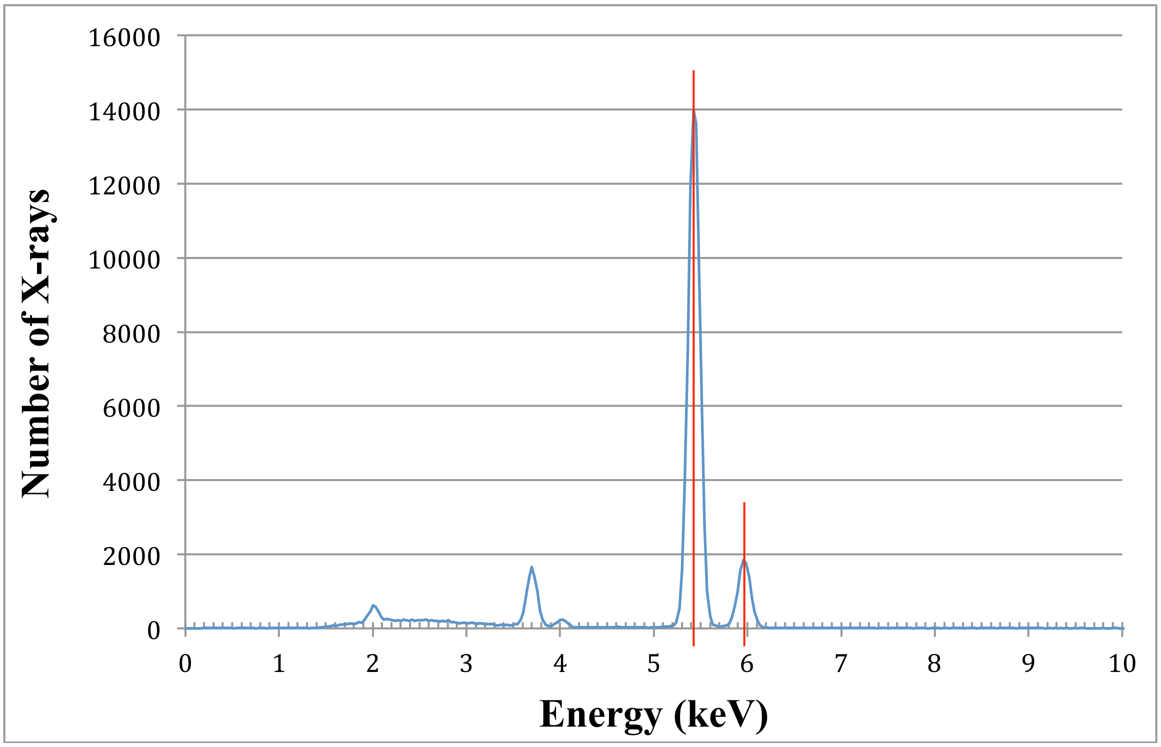
2. What is the ratio of the K_α energies of uranium(Z=92) to carbon(Z=6)?

Using the formula above: $\frac{U_{K_\alpha}}{C_{K_\alpha}} = \frac{(92)^2}{(6)^2} = 235$.

3. What is the ratio of the K_β energies of tungsten(Z=74) to calcium(Z=20)?

Based on the formula above: $\dfrac{W_{K_\beta}}{Ca_{K_\beta}} = \dfrac{(74)^2}{(20)^2} = 13.7$

4. Given the *PIXE* spectrum of a single element standard shown below, what are the elements?



Given the x-ray energies from the graph of 5.4 and 6.0keV, the element is Chromium (Cr) along with Calcium (at 3.7 and 4.0 keV) and Phosphorus (at 2.0 keV) using the table of x-rays. However, if we use the formula for the K_α transition from the Bohr theory, we find

$$\Delta E = -13.57eV\left(\frac{1}{n_{upper}} - \frac{1}{n_{lower}}\right)Z^2 \rightarrow 5400eV = -13.57eV\left(\frac{1}{2^2} - \frac{1}{1^2}\right)Z^2 \rightarrow Z = 23$$ *which*

would suggest the element Vanadium. But, if we use the modified Bohr theory then we find

$$\Delta E = -13.57eV\left(\frac{1}{n_{upper}} - \frac{1}{n_{lower}}\right)(Z-1)^2 \rightarrow 5400eV = -13.57eV\left(\frac{1}{2^2} - \frac{1}{1^2}\right)(Z-1)^2 \rightarrow Z = 24$$

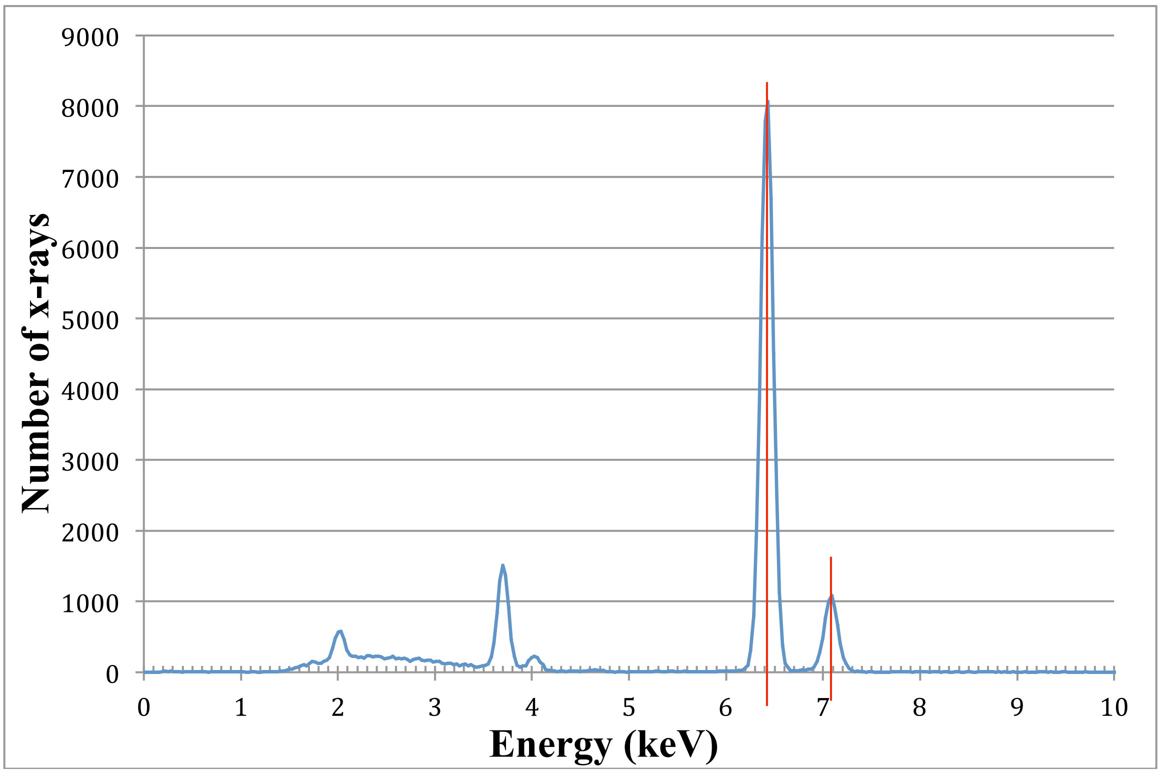
or Cr.

As a check we use the modified Bohr theory for the K_β transition and we find

$$\Delta E = -13.57eV\left(\frac{1}{n_{upper}} - \frac{1}{n_{lower}}\right)(Z-1)^2 \rightarrow 6000eV = -13.57eV\left(\frac{1}{3^2} - \frac{1}{1^2}\right)(Z-1)^2 \rightarrow Z \approx 24$$

or Cr.

5. Given the *PIXE* spectrum of a single element standard shown below, what are the elements?



Given the x-ray energies from the graph of 6.4 and 7.1keV, the element is Iron (Fe) using the table of x-rays. However, if we use the formula for the K_α transition from the Bohr theory, we find

$$\Delta E = -13.57eV\left(\frac{1}{n_{upper}} - \frac{1}{n_{lower}}\right)Z^2 \rightarrow 6400eV = -13.57eV\left(\frac{1}{2^2} - \frac{1}{1^2}\right)Z^2 \rightarrow Z = 25$$ *which*

would suggest the element Manganese. But, if we use the modified Bohr theory then we find

$$\Delta E = -13.57eV\left(\frac{1}{n_{upper}} - \frac{1}{n_{lower}}\right)(Z-1)^2 \rightarrow 6400eV = -13.57eV\left(\frac{1}{2^2} - \frac{1}{1^2}\right)(Z-1)^2 \rightarrow Z \approx 25$$

or Fe.

As a check we use the modified Bohr theory for the K_β transition and we find

$$\Delta E = -13.57eV\left(\frac{1}{n_{upper}} - \frac{1}{n_{lower}}\right)(Z-1)^2 \rightarrow 7100eV = -13.57eV\left(\frac{1}{3^2} - \frac{1}{1^2}\right)(Z-1)^2 \rightarrow Z \approx 25$$

or Fe.